University of Southern Queensland

Faculty of Engineering and Surveying

Verification of Simplified Optimum Designs for Reinforced Concrete Beams

A dissertation submitted by

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Abstract

Current demand on resources have forced engineering sector to look at more efficient design and construction methods. Methods that will yield better designs that are cost effective and puts less demand on decreasing resources. In this dissertation the use of topology optimisation for the design of concrete beams is investigated. The method uses topology optimisation to obtain the optimum strut-and-tie model (STM) and then uses the STM provisions of AS3600:2009 to design the beam. As a control a similar beam is designed using the conventional design methods and both beams are tested.

Test results showed that the conventional beam performed better then the optimum beam and it was concluded that construction methods utilised maybe the reason for this results. It has been recommended that further research in this area is required with better construction procedures implemented.

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ENG4111 Research Project Part 1 &
ENG4112 Research Project Part 2

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Certification

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Chapter 1
INTRODUCTION

Overview

This chapter gives a brief background of the topic, outlines the aim and objectives, anticipated potential outcomes, methodology, project resources used, the risk assessment, project timeline and an outline of the dissertation.

1.1 Background

Reinforced concrete was invented in the mid 1800's and there have been enormous advances in its design and use. Many design techniques and procedures have been developed over the years, which have been included in design codes and standards. One of the latest design techniques that is being researched here is topology optimization. Even though the first paper on topology optimization was published in 1904 (Rozvany 2009), major development in this research field has happened only in the last few decades. It's an extremely rapidly expanding research field, which has interesting theoretical implications in mathematics, mechanics, multi-physics and computer science, but also important practical applications by the manufacturing industries such as car and aerospace (Rozvany 2009). Numerous researchers are continuingly developing new techniques in this field and some of the more prominent ones will be discussed in Chapter 3.

Literature review has found that though there are many papers on topology optimisation techniques, there is little research being done on its application in reinforced concrete design and verifying these methods through physical testing. Liang et al (2002) have used topology optimisation for structural concrete design. It should be noted that these optimisation techniques are well developed methods that have been verified through vigorous numerical analysis. Yet, physical testing is essential as it has been well documented that physical behaviour of reinforced concrete is hard to model using numerical modelling.

The focus of this project is to optimise the stiffness of a reinforced concrete beam using one of the topology optimisation techniques and verify this simplified optimised design through testing. For this project the design domain will be optimized using a MATLAB code to obtain the optimum layout and then strut and tie method will be used to design the reinforcement. To compare results, the same beam will be designed using conventional reinforced concrete design method. Beam samples for both designs will be constructed and then tested to compare results.

1.2 Aims and Objectives

The aims and specific objectives of this project are as follows:

- To verify optimum designs obtained using a simplified linear elastic model for reinforced concrete.
- To test the efficiency and accuracy of the Matlab code.
- To be able to provide reinforced concrete designers a simple and effective method of finding optimum strut-and-tie layout.

1.3 Anticipated Potential Outcomes

Prior to the commencement of this project it was envisaged that the potential outcomes of this project would include:

- Test results comparing well with theoretical values.

- Using verified test results to prepare a method of designing deep beams using simplified optimization.
- Using the data from this project together with other similar projects done by other USQ students to present a technical paper.

1.4 Methodology

The methodology used to complete this dissertation is described in the following steps:

1. Research background information relating to topology optimisation, especially the SIMP (Solid Isotropic Microstructure with Penalisation) method.
   A well focused literature review enabled the author to understand the theory behind the optimization techniques as well as ensure that similar research has not already been done.
2. Research on concrete beam design by strut-and-tie method. Design methods were researched including those provided in codes and methods proposed by other researchers. The design procedure used in this dissertation is explained later in this dissertation.
3. Select a design problem including beam dimensions.
4. Design the given problem using conventional design methods.
5. Using MATLAB optimisation code determine optimum layout and design using strut-and-tie method.
6. Prepare test samples for both designs.
7. Test samples and compare results.
8. Conclusions.
9. Recommendations for further research.

1.5 Project Resources

The resources utilised in this project consisted of computer softwares, concrete testing equipment and materials for the construction of reinforced concrete beams. Table 1.1 below contains a list of resources required to complete this project and its uses.

Table 1.1: List of resources and its uses

Resource	**Use**
MATLAB Code	For topology optimization
MATLAB Software	Numerical analysis software
Microstran Software	For finite element analysis
USQ Lab	For construction and testing of beam samples
Cement, Aggregate and Reinforcement	For construction of reinforced concrete deep beams
Testing Equipment	For testing concrete compressive strength and loading test samples
USQ Lab Technician	For supervising and operating test equipment

The MATLAB code mentioned in the table above is one of the most important resource for this project and it was provided by Dr. Kazem Ghabrie. The MATLAB software was purchased as part of the requirement for another course undertaken at USQ in 2009. In the USQ lab, equipments such as electric concrete mixer, concrete vibrator, concrete compression testing machine and reinforcement cutting and bending tools were utilised.

1.6 Risk Assessment and Consequential Effects

Potential risks associated with this project were identified and these risks were analysed using tables 1.2, 1.3 and 1.4. The major one is the risk of injury during beam sample construction and testing. This risk can be minimised by following all the lab safety procedures and following testing equipment operating procedures.

Table 1.2 Measures of Likelihood

Rating	Description
Almost Certain	The event is *expected* to occur in most circumstances
Likely	The event will *probably* occur in most circumstances
Moderate	The event *should* occur at some time
Unlikely	The event *could* occur at some time
Rare	The event *may* occur only in exceptional circumstances

Table 1.3 Measures of Consequence

Rating	Description
Catastrophic	Death, huge financial loss
Major	Extensive injuries requiring hospitalisation, major financial loss
Moderate	Medical treatment required, high financial loss
Minor	First Aid treatment, high financial loss
Insignificant	No injuries, low financial loss

Table 1.4 Risk analysis matrix

LIKELIHOOD	CONSEQUENCES				
	1 Insignificant	2 Minor	3 Moderate	4 Major	5 Catastrophic
Almost Certain	Low + 4	Medium + 16	High 32	Very High 64	Extreme 128
Likely	Low – 2	Medium – 8	Medium + 8	High 32	Very High 64
Moderate	Negligible 1	Low + 4	Medium – 8	Medium + 16	High 32
Unlikely	Negligible 0.5	Low – 2	Low + 4	Medium – 8	Medium + 8
Rare	Negligible 0	Negligible 0.5	Negligible 1	Low – 2	Low + 4

A risk summary of all risks indentified whilst undertaking this projected is given in table 1.5 below.

Table 1.5 Risk summary

Hazard	Consequence	Likelihood	Risk	Control
Sharp Tools	Moderate	Unlikely	Low	Follow user guide
Electric Tools	Moderate	Unlikely	Low	Operate with care
Breaking Concrete	Moderate	Almost Certain	High	Wear safety glass

The potential consequential effects of this project are minimal. There is no sustainability issue that needs to be considered for this project. The ethical issues related to this project include firstly crediting other researchers where it is due for using their ideas and secondly, responsibly conducting and reporting this project as the results could be used by designers and other researchers who will have an expectation that this project has been done diligently. The safety issues while undertaking this project have been discussed above. The safety issues after the completion of the project include incorrect use of project results. That is, designers incorrectly or inappropriately using the results of this project to design reinforced concrete beams. If the results of this project are not properly and independently verified then it should not be used by designers.

1.7 Project Schedule

As means to track progress and manage time for this project a schedule was prepared for completing various sections of this dissertation and requirements of ENG4111 and ENG4112 and it is tabulated in table 1.6 below.

Table 1.6 Project Schedule

Phase of Work	Completion Date
Project Proposal	9th March 2011
Project Specification	22nd March 2011
Literature Review	30th June 2011

Project Appreciation	23rd May 2011 (One week Extension Approved – 30th May 2011)
Finalisation of Deep Beam Layout and Loadings	10th June 2011
Design of Deep Beam (Strut-and-tie Method)	20th June 2011
Design of Deep Beam (Optimization)	25th June 2011
Preparation of Test Samples	July 2011 (exact date depends on lab availability)
Testing of Samples	August 2011 (exact date depends on lab availability)
Project Conference Presentation	11th September 2011
Professional Practice 2	12th-16th September 2011
Partial Draft Dissertation	16th September 2011
Project Performance (Final Dissertation)	27th October 2011

The above table was an indicative timeline which changed as progress was made throughout the year. There were some delays in sample design, construction and testing which was expected.

1.8 Dissertation Structure

The dissertation contains four major sections namely, literature review, methodology, results and conclusion.

Literature review

This section contains results of literature review done on strut-and-tie modelling and topology optimisation. It presents calculation method and Australian standard's requirements for use of strut-and-tie modelling. Also presented in this section is a brief overview of different optimisation techniques available and explanation of the particular method used in this dissertation.

Methodology

Presented in this section are the details of the Matlab code used for optimisation. The sample problem chosen for this dissertation is presented, together with its design method and calculations. A brief explanation of the testing method and equipment used is also given.

Results

In this section the results from testing and numerical analysis is discussed and compared with each other.

Conclusion

From the results obtained conclusions are drawn and brief explanation of the results is given. Recommendations for future works are also made in this section.

Chapter 2
STRUT-AND-TIE MODELLING

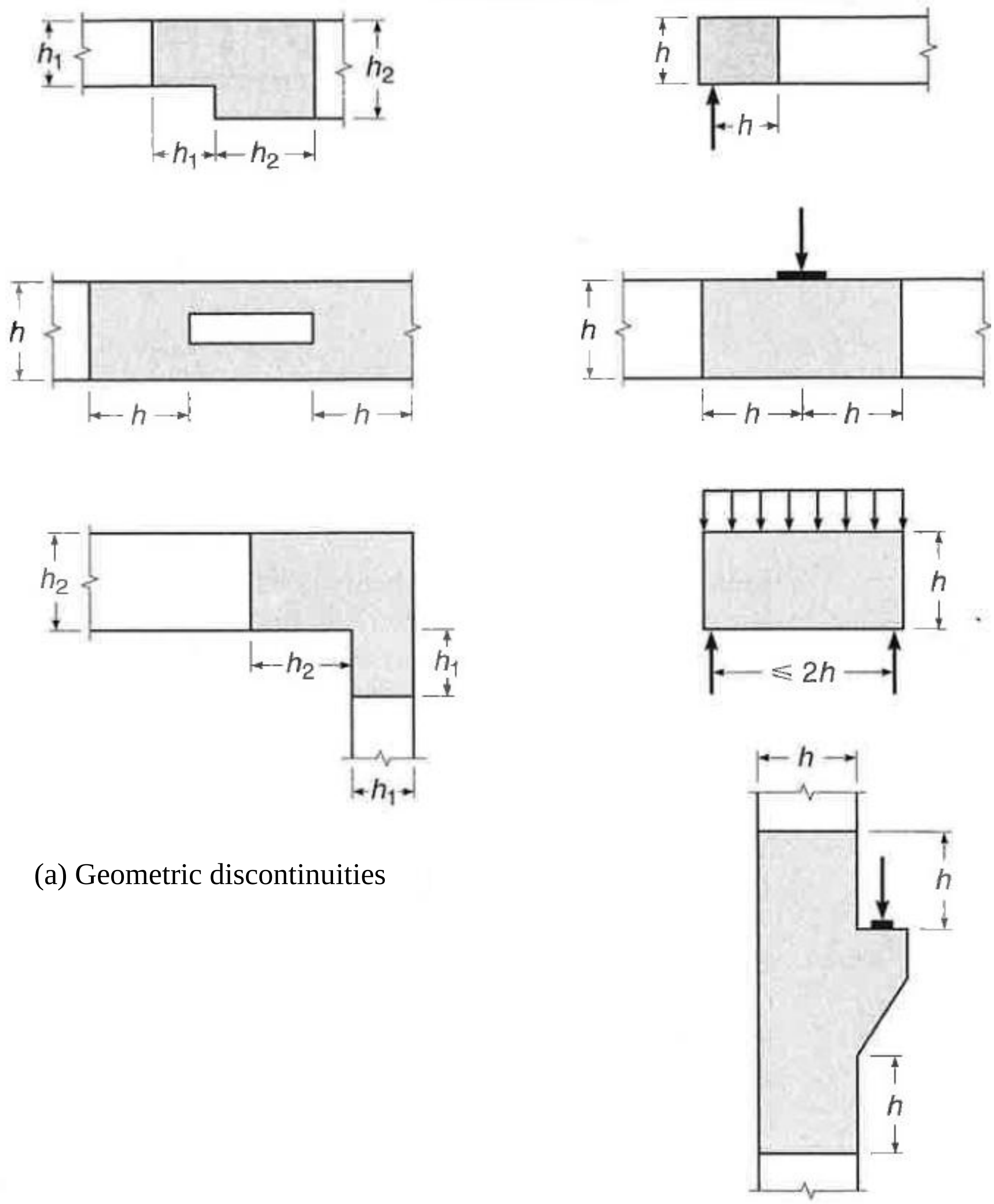
Overview

This chapter summaries the literature review done on the strut-and-tie modelling. It includes the history, development methods, key components, advantages and limitations of the strut-and-tie modelling.

2.1 Introduction

Reinforced concrete beam theory is based on equilibrium and the constitutive behaviour of the materials, steel and concrete. Particularly important is the assumption that strain varies linearly through the depth of a member and that, as a result plane sections remain plane. St. Venant’s principle validated this assumption by stating that strains around load or member cross section discontinuity vary in an approximately linear fashion at distance greater than or equal to the greatest cross sectional dimension h from the point of load application as shown in Figure 2.1.

At points closer than the distance h to discontinuous load or member dimensions, St Venant’s principle is not applicable. Reinforced concrete structures can be divided into regions where beam theory is valid and regions where discontinuities affect member behaviour. A region where beam theory is valid is referred to as B-regions and a region with discontinuities is referred to as D-regions.



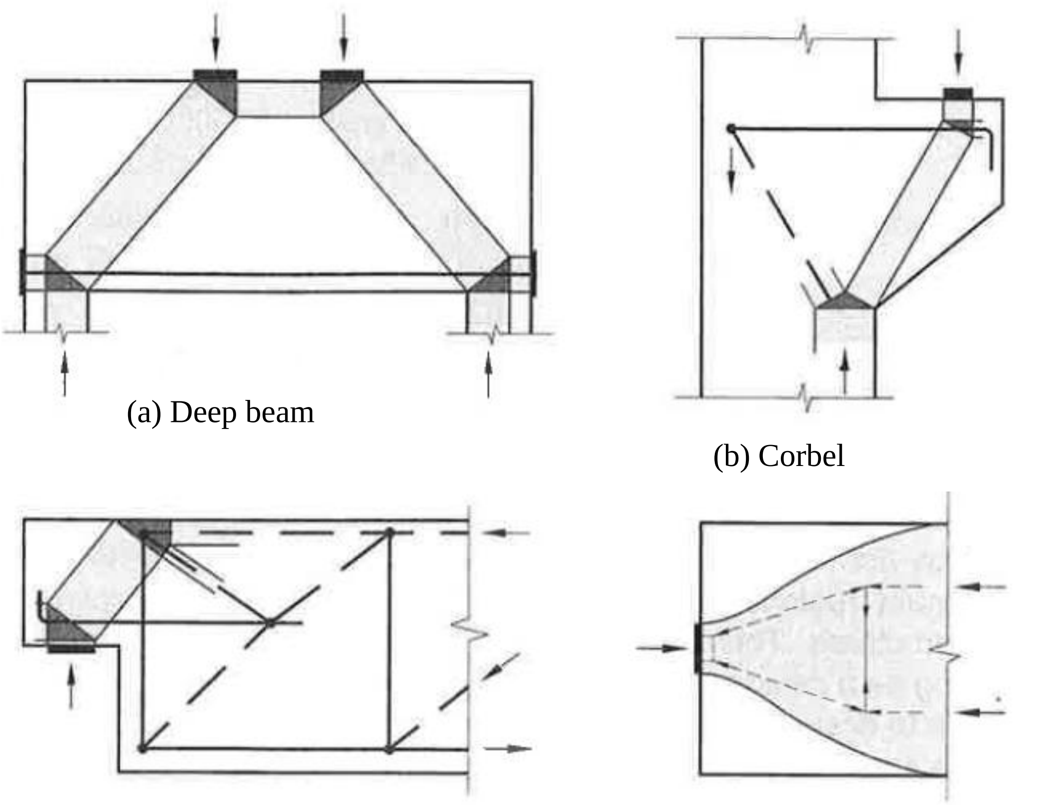
(a) Geometric discontinuities

(b) Loading discontinuities

Figure 2.1 Geometric and load discontinuities for D-regions

(Source: Nilson et al, 2004)

When the concrete is elastic and uncracked, the stresses in D-regions can be determined using finite element analysis and elastic theory. After concrete cracks the strain field is disrupted and internal forces are redistributed. The internal force can be represented by a statically determinate truss known as the strut-and-tie model, which allows the complex problem to be simplified. Figure 2.2 shows examples of strut-and-tie model in typical reinforced concrete members.



(a) Deep beam

(b) Corbel

(c) Dapped connection

(d) Prestressing anchorage

Figure 2.2 Examples of strut-and-tie models
(Source: Warner et al, 2007)

2.2 Definitions

The following terms are used in this section (Warner et al, 2007):

B-region – A portion of a structure in which the Bernoulli-Euler assumption that plane sections remain plane can be applied.

Discontinuity – An abrupt change in member's geometry or loading.

D-region – The portion of a member within a distance equal to the member depth *h* from a force discontinuity or a geometry discontinuity. In D-regions Bernoulli-Euler assumption is not valid after the concrete cracks.

Node – A point in a strut-and-tie model where the axes of the struts, ties and concentrated forces acting on the joint intersect.

Nodal zone – The volume of concrete surrounding a node that transfers strut-and-tie forces through the node.

Strut – A compressive member in a strut-and-tie model. A strut represents the resultant of a parallel or fan-shaped compressive field.

Bottle-shaped strut – A strut that is wider at mid-length than at its ends.

Strut-and-tie model – A truss model of a structural member, made up of struts and ties connected at nodes that is capable of transforming the factored loads to the supports.

Tie – A tension member in a strut-and-tie model.

2.3 Development of Strut-and-Tie Model

Strut-and-tie modelling has increased in popularity since it was promoted by Marti (1985a, 1985b) and Schlaich et al (1987). Though much development of strut-and-tie method occurred after the ground breaking paper by Schlaich et al (1987), the authors of that paper acknowledge that they were not the first to present the idea of using truss analogy to design structural concrete. According to them it was at the turn of the last century when Ritter and Morsch introduced the truss analogy.

Ritter found that a reinforced concrete beam after cracking due to diagonal tensile stresses could be idealized as a parallel chord truss with compressive diagonals inclined at 45^{o} with respect to the longitudinal axis of the beam. Morsch (1920, 1922) extended the truss models to the design of reinforced concrete members under torsion (Liang, 2005). This method was later refined and expanded by Leonhardt, Rusch, Kupfer and others until Thurlimann’s Zurich school, with Marti and Mueller, created its scientific basis for a rational application in tracing the concept back to the theory of plasticity.

The standard truss model was developed to be used for designing regions of concrete structure where the Bernoulli hypothesis of plane strain distribution was assumed to be valid. But this model could not be applied in regions where the strain distribution was non linear. This led to the refining of the truss model to the now called strut-and- tie model (Liang, 2005).

Strut-and-tie modelling was developed to provide a consistent design method for structural concrete because at that time existing methods only catered for the design of B-regions in structures. D-regions of structures then were designed based on the so-called ‘detailing,’ ‘past experience’ or ‘good practice’. It had become apparent that a design method must be developed that considered both B- and D-regions of a structure without contradiction (Schlaich et al, 1987).

Over the years the method has been continuingly developed and as such it is a mature design method included in most codes of practice including Australian standard (AS3600-2009), European standard (EUROCODE 2.2004), Canadian standard (A23.3-94), Indian standard (IS 456) and American standard (ACI 318-08).

2.4 Conventional Approach for Developing Strut-and-Tie Models

In their ground breaking paper on strut-and-tie modelling Schlaich et al (1987) suggested using the elastic stress distribution method. In using strut-and-tie method the structure is designed according to the lower bound theorem of plasticity. Since concrete permits only limited plastic deformations, the internal structural system has to be chosen in a way that the deformation limit is not exceeded at any point. This ductility requirement is fulfilled by constructing a strut-and-tie model where the struts and ties are oriented to the mean direction of the principle stress trajectory, which are obtained by performing a linear elastic finite element analysis on an uncracked homogeneous concrete member. Tensile forces, usually transverse to the direction of loads may cause premature cracking and failure. Hence, orienting the geometry of the model to the elastic stress distribution is also a safety requirement because the tensile strength of concrete is only a small fraction of the compressive strength. Schlaich and Schafer (1991) reported that since uncracked concrete is used in the linear elastic finite element analysis, the strut-and-tie model obtained from elastic stress distribution method may differ from the actual load transfer mechanism at the ultimate limit states.

The load path method can also be used to develop strut-and-tie models in structural concrete. The first step in this method is to ensure that the external forces are in equilibrium, that is, the loads and support reactions. The load paths are then traced using the corresponding stress diagrams. After tracing load paths in the direction of loads, further struts and ties must be added for transverse equilibrium between nodes. In selecting the model, it is helpful to realise that loads try to use the path with the least forces and deformations. Since reinforced ties are much more deformable than concrete struts, the model with the least and shorted ties are the best (Schlaich et al, 1987). This criterion can be formulated as follows;

$$\sum F_i l_i \varepsilon_{mi} = Minimum \tag{2.1}$$

where:

F_i = force in strut or tie i

l_i = length of member i

ε_{mi} = mean strain in member i

This equation is derived from the principle of minimum strain energy for linear elastic behaviour of struts and ties.

For complicated cases Schlaich et al (1987) recommended using a combination of finite element analysis and load path method for developing new strut-and-tie models. However, it is difficult to find the optimum models in structural concrete members with complex loading and geometry using these conventional methods, which usually involve a trial and error process or requires some prior experience in modelling.

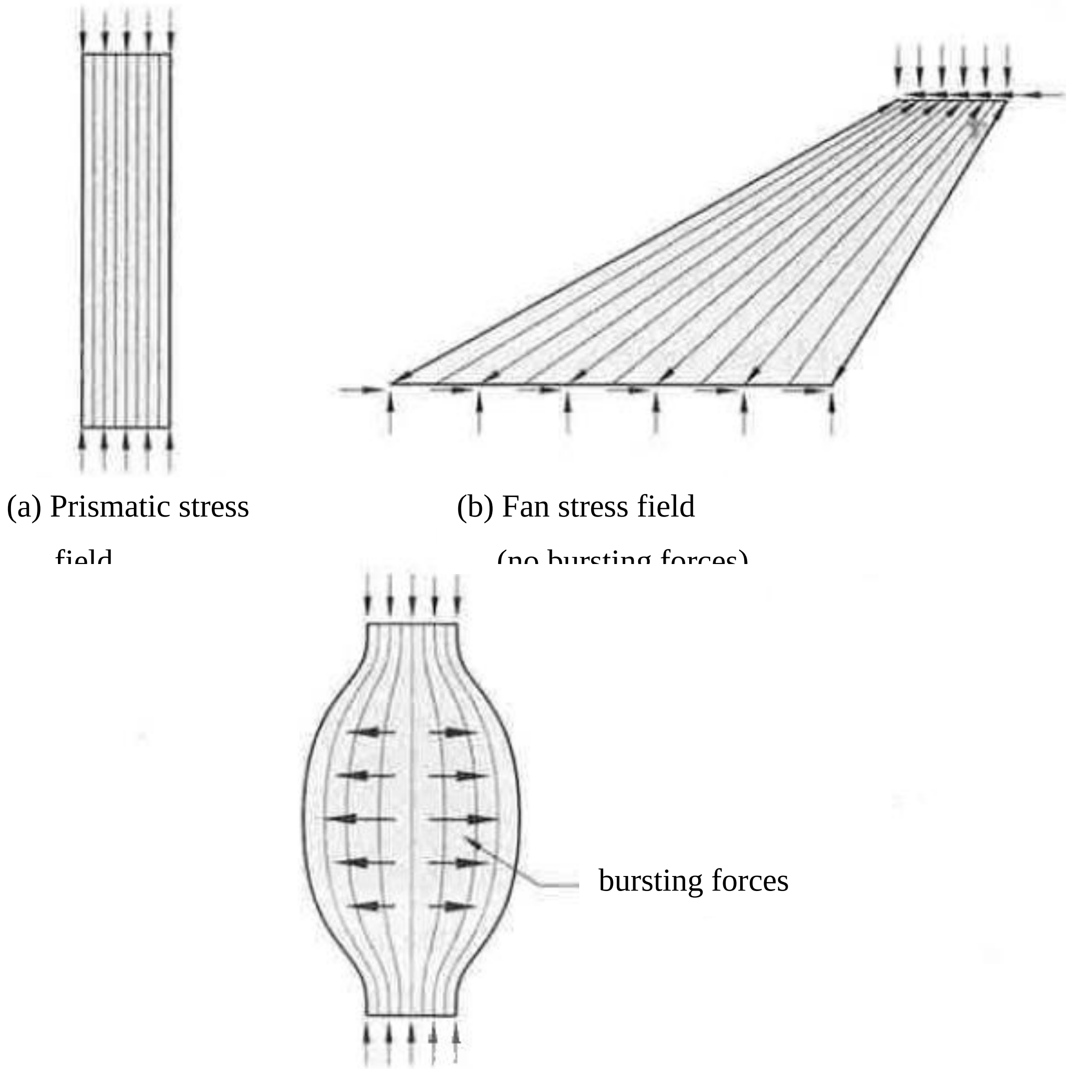
Marti (1985) realized the limitations of conventional methods for developing strut-and-tie models and suggested that there is a potential for applying iterative computer programs with graphical input and output routines which could replace the traditional drawing board method for developing strut-and-tie models. In Chapter 3 such a method is presented.

2.5 Key Components of Strut-and-Tie Models

Strut-and-tie modelling is considered the basic tool in the design and detailing of structural concrete under bending, shear and torsion. The designer specifies a load path and then designs and details the structure such that this load path is sufficiently strong to carry the applied loads. The loads applied to the structural concrete member are transferred through a set of compressive stress fields that are distributed and interconnected by tension ties. The compression stress fields are idealised using compression members called struts while tensile stress fields are idealised using tension members called ties. Tension ties can be reinforcing steel bars or prestressed tendons or concrete in tension. Concrete's tensile strength is considerably less than its compressive strength and normally concrete's tensile resistance is ignored.

2.5.1 Struts

A strut is an internal compression member. It may have a prismatic, fan or bottle shape as shown in Figure 2.3. Prismatic shape is an idealised representation of fan or bottle shaped struts. The dimensions of the cross section of the strut are established by the contact area between the strut and the nodal zone.



(a) Prismatic stress field

(b) Fan stress field (no bursting forces)

(c) Bottle stress field

Figure 2.3 Types of concrete struts and related stress fields
(Source: Nilson et al, 2004)

Bottle shaped struts are wider at the centre than the ends and as the compression zone spreads along the length of bottle shaped struts, tensile stresses perpendicular to the axis of the strut may cause longitudinal cracking. For simplicity in design, bottle shaped struts are idealised as having linearly tapered ends and uniform centre sections as shown in Figure 2.4 (b). The capacity of the struts is proportional to the concrete compressive strength and it is affected by the lateral stresses in bottle shaped struts. Because of longitudinal splitting, bottled shaped struts are weaker than prismatic struts, even though they possess a larger cross section at mid-length.

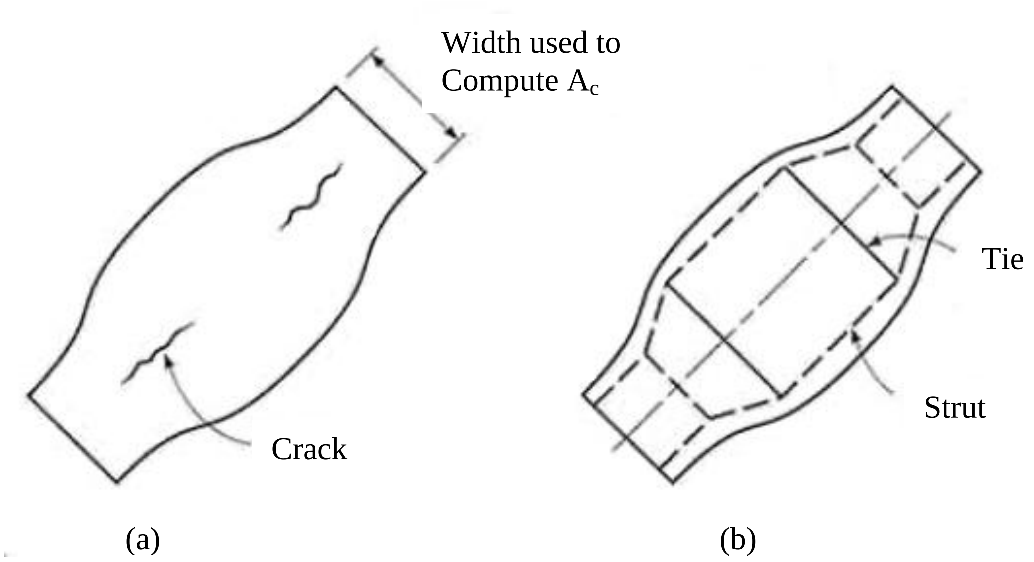


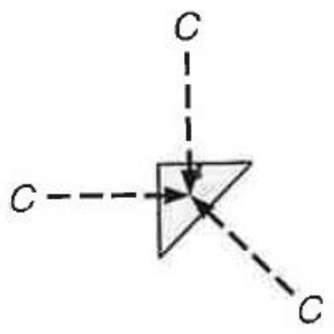
Figure 2.4 Bottle-shaped strut
(Source: Nilson et al, 2004)

2.5.2 Ties

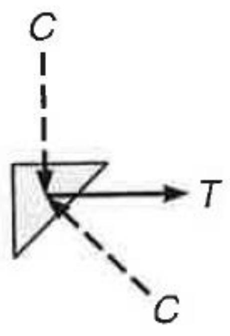
A tie is a tension member in a strut-and-tie model. The ties consist of either steel bar or a prestressed tendon. For design purpose, it is assumed that the concrete within the tie does not carry any tensile force. Concrete does assist in reducing tie deformation at service load.

2.5.3 Nodes

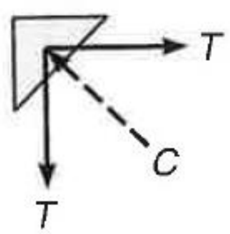
Nodes are points within strut-and-tie models where the axis of struts, ties and concentrated loads intersect. For equilibrium, at least three forces must act on a node. Nodes are defined by the sign of forces acting at it. Therefore, a CCC node resists three compressive forces, a CCT resists two compressive forces and a one tensile force. There can be multiple forces acting at a node but care must be taken to ensure there is room for anchorage of tie reinforcements. Figure 2.5 illustrates some common node classifications.



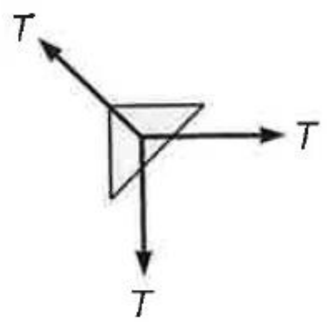
(a) C-C-C node



(b) C-C-T node



(c) C-T-T node

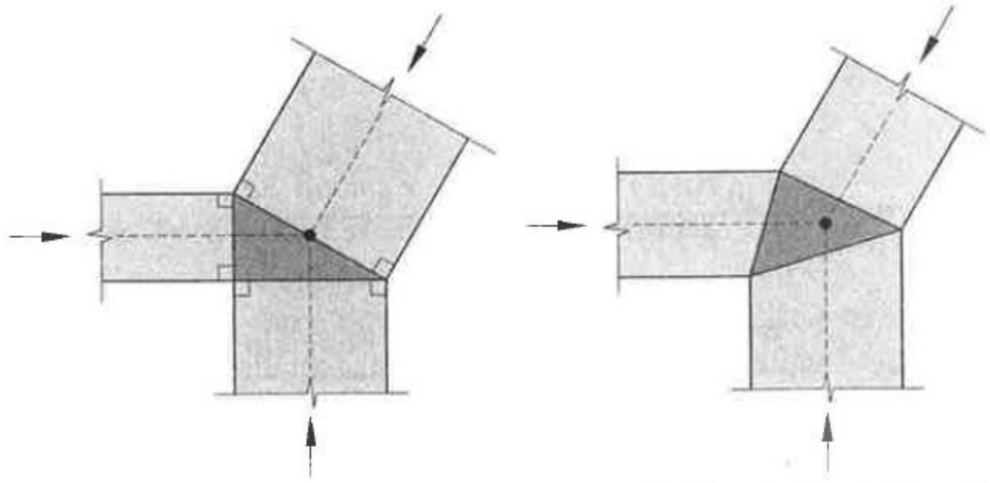


(d) T-T-T node

Figure 2.5 Classification of nodes

(Source: Nilson et al, 2004)

Both tensile and compressive forces place nodes in compression because tensile forces are treated as if they pass through the node and apply compression in the anchorage face. There are two types of nodes, non hydrostatic and hydrostatic nodes. A node is hydrostatic if all members are at right angles to the adjacent node face, as shown in Figure 2.6 (a). If one or more of the members enter the node at an angle other than right angle, the node is non hydrostatic as shown in Figure 2.6 (b) (Warner et al, 2007).



(a) Hydrostatic node (b) Non-hydrostatic node

Figure 2.6 Node types
(Source: Warner et al, 2007)

2.6 Advantages of Using Strut-and-Tie Modelling

Liang (2005) summarised the advantages of using strut-and-tie modelling as follows:

- The designer can easily idealise the flow of internal forces in a structural concrete member.
- The influence of shear and moment can be accounted for simultaneously and directly in one model.
- The designer can give special attentions to the potential weak spots indicated by the strut-and-tie model.
- It offers a unified, rational and safe design procedure for structural concrete.

2.7 Limitations of Strut-and-Tie Modelling

Strut-and-tie modelling is good for structures at overload, that is, after extensive cracking and large deformations have occurred. It is not suited to representing transitional behaviour when the structure is changing from uncracked to the fully cracked condition (Warner et al, 2007). The strut-and-tie model is a conservative, design approach which means that it is almost always over designed.

There is no single design solution and the designer has the flexibility to choose the shape and dimensions of the strut-and-tie model. This fact requires the designer to have some experience in the use of strut-and-tie modelling so that they can choose an effective model.

The strut-and-tie modelling offers the designer the flexibility to focus on performance design while also providing a safe design. Different performance criteria may be achieved with strut-and-tie modelling, however, the ultimate failure mode and load cannot be predicted by strut-and-tie modelling.

Chapter 3
TOPOLOGY OPTIMISATION TECHNIQUES

Overview

This chapter summaries the literature review done on topology optimisation techniques. It includes the history, uses, and types of topology optimisation techniques available.

3.1 Introduction

The efficient use of material is important in many different settings. For example, the aerospace industry and the automotive industry use sizing and shape optimisation to design structures and mechanical elements. Efficient use of materials is not only cost effective but it helps to maintain a sustainable future.

Topology optimisation involves the determination of features such as the number, location and shape of holes and the connectivity of the domain (Bendsoe and Sigmund, 2003). This method distributes the specified amount of material in a design domain depending on the design variables. The optimisation of geometry and topology has great impact on the performance of structures such as increasing the structures stiffness. Topology optimisation is the newest of different types’ of structural optimisation techniques available, which include shape and size optimisation. In shape optimisation the overall layout of the members is known but the best shape is required, where as in size optimisation the optimum member dimensions is determined. Figure 3.1 shows these three structural optimisation categories.

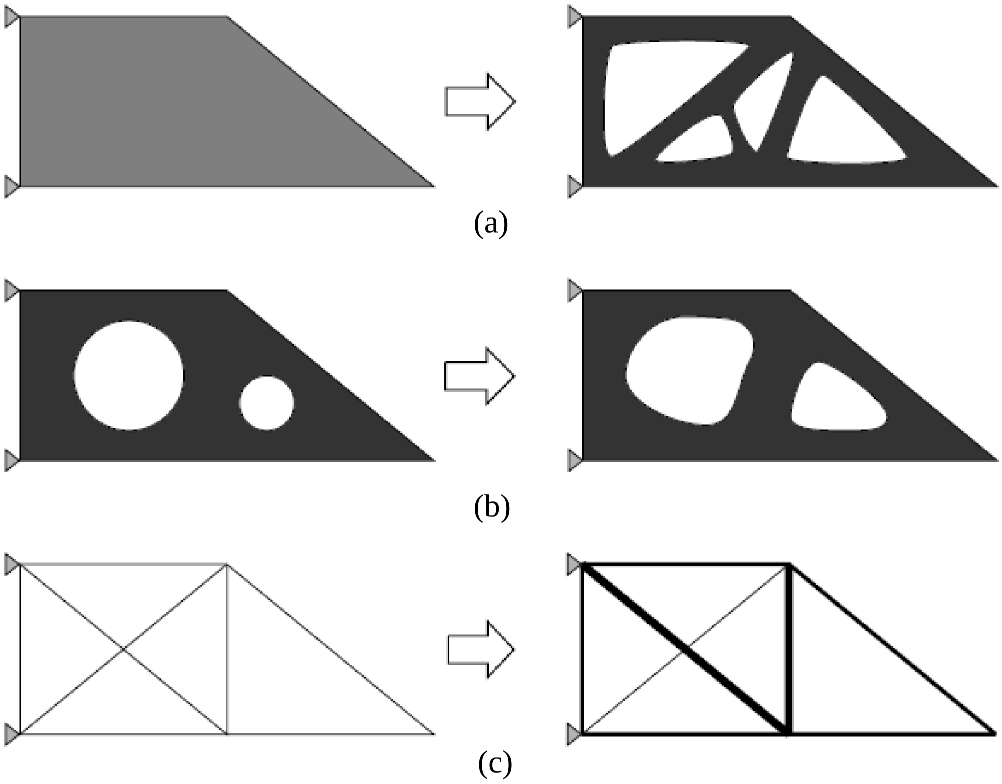


Figure 3.7 Structural optimisation categories.

a) Topology optimisation; b) Shape optimisation; c) Size optimisation. The initial problems are shown at the left and the optimal solutions are shown at the right.

(Source: Ghabrie, 2010)

Topology optimisation is used for optimising the stiffness of the design problem in this dissertation hence shape and size optimisation will not be discussed further here.

3.2 Brief History

There are two broad classes of techniques that can be applied to optimize shape and topology of a structural system:

- Discrete optimization of the structural system.
- Continuum optimization of the structural system.

In discrete optimization methods, a structure is modelled with discrete truss or beam/column elements and the optimum number, positions and mutual connectivity of the structural members is determined. In continuum optimisation methods, a structure is modelled as a continuous domain and involves simultaneous optimisation of the shape of external as well as internal boundaries and the number of inner holes (Eschenauer and Olhoff 2001).

The history of shape and topology optimization of discrete structural systems can be classified into three periods (Burns (Ed), 2002):

During the initial period Maxwell (1894) and Michell (1904) made their pioneering studies in the field. Michell developed the theory for determining the optimum topology by minimising the weight of a thin-bar, truss like structure (Eschenauer and Olhoff 2001). Following these initial works, research in the field of topology optimisation fell dormant for many decades.

The second period occurred during the 1960's and 1970's in which time interest in optimization was re-kindled by the initial developments of high speed computers. Very important generalisation of Michell's theory was made by Prager (1969, 1974), Rozvany (1972a, b) and Rozvany and Prager (1976). They extended Michell's theory to beam systems in these papers and formulated the first general theory of topology optimisation termed 'optimum layout theory' (Rozvany, 2007). During this period, many important theoretical results for general optimization methods and numerical implementations were first presented, and difficulties in structural topology optimization were given extensive attention. In addition, methods for discrete optimization were exercised on very small test problems due to computing limitations (Eschenauer and Olhoff, 2001).

The third period during the 1980's and 1990's saw a revitalised interest in topology optimisation research mainly due to the extremely dramatic growth in computing technologies. While theoretical work has continued, numerical techniques have been further refined, developed and applied to larger scale, more realistic structures (Eschenauer and Olhoff, 2001).

Also in this third period continuum structural topology optimization techniques were developed. It was first proposed by Cheng and Olhoff (1981) and some further research was done by Kohn and Strang (1986). First practical approach to topology optimisation was demonstrated by Bendsoe and Kikuchi (1988) utilising a homogenization approach. Flowing this work, Xie and Steven (1993) proposed a simple finite element based topology optimisation technique, in which inefficient elements in the design domain is gradually removed based on some optimality criteria. These two works attracted numerous researchers to the field and it has seen major development of the theory, techniques and its application in industry. The great potential of topology optimisation in Civil engineering has not yet been realised but there is growing consensus to further research into this area.

3.3 Homogenisation Method

In their ground breaking paper, Bendsoe and Kikuchi (1988) presented the homogenisation method. Subsequent research on the field of structural topology optimisation has been on the basis of their work. The homogenisation method works on the basis of replacing materials in a composite domain with a kind of equivalent material model. This is done because “even with the help of high-speed modern computers, the analysis of the boundary value problems consisting of composite media with a large number of heterogeneities is extremely difficult” (Hassani and Hinton 1998a). Such a procedure is called homogenisation. It is assumed that the design domain is made of periodic microstructures, hence this type of materials are called composites with periodic microstructures.

The above mentioned microstructures can be introduced in the design domain using two methods; the rank laminate composite method or the microcells with internal voids (Hassani and Hinton, 1998b). The geometric parameters of these microstructures are the design variables and by adjusting them it is possible that the void area inside the microstructure remains a void or changes to solid.

3.3.1 Types of microstructures

As mentioned before there are two types of microstructures. The square cell with centrally placed rectangular voids is the simplest form of microcell with internal voids. It is important that the voids in the cell are defined with the least number of variables possible as they are the design variables. Hence rectangular voids require only three variables, length (b), width (a) and rotation (θ) relative to design domain as shown in Figure 3.2.

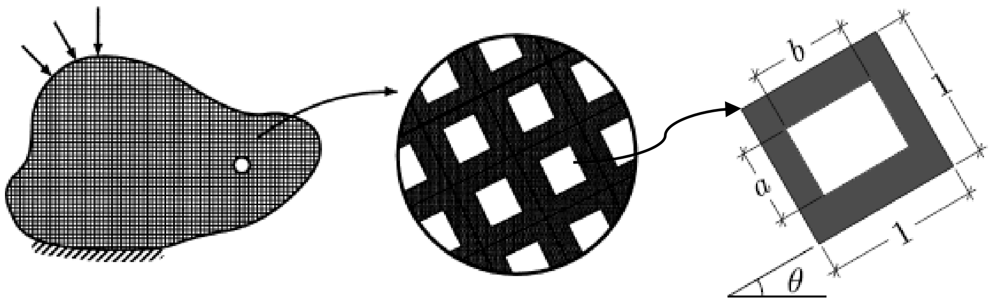


Figure 3.8 Microcell with rectangular holes.
(Source: Hassani and Hinton, 1998b)

Rank laminate cells are another type of microstructure that can be used to define the design domain. Each cell is constructed of layers of different material and voids. To avoid singularity in the stiffness matrix the voids are replaced with very soft material (Hassani and Hinton, 1998b). Higher ranked cells can be constructed by using stiff material and rank-1 composite. The directions of different layers are orthogonal to each other. Like microstructures with rectangular holes, rank-2 laminate cells have three design variables which include density of solid material in rank-1 layer (γ), density of solid material in rank-2 layer (μ), and rotation (θ) relative to design domain as shown in Figure 3.3.

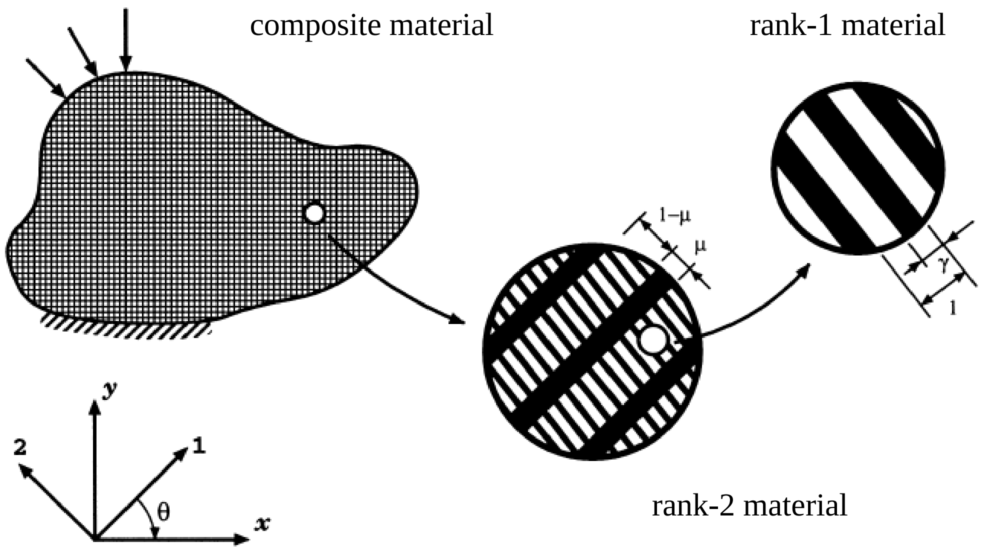


Figure 3.9 Rank-2 layered (laminate) material.

(Source: Hassani and Hinton, 1998b)

Studies have shown that in plane elasticity problems rank-2 composites yield the stiffest material. The homogenisation equations for rank laminate materials can be solved analytically but for microcells with voids the solution can only be obtained through numerical methods such as finite element analysis (Hassani and Hinton, 1998b).

3.3.2 Optimally criteria

In late 1960s optimality criteria methods were developed for solving structural optimisation problems as an alternative to mathematical programming (Hassani and Hinton 1998c). Optimality criteria methods have been considered to solve structural optimisation problem mainly because these problems have a large number of variables which take time to solve using mathematical programming. Optimality criteria methods are much more efficient for problems with large number of variables and few constraints (Hassani and Hinton, 1998c).

3.3 SIMP Method

The **SIMP** (**S**olid **I**sotropic **M**icrostructures with **P**enalisation) was proposed by Bendsoe (1989) which he called the direct approach method. Rozvany introduced the term 'SIMP' in 1992, which was not accepted by the research schools until recently (Rozvany, 2001). It is also known as the 'power law' method (Rovany, 2001). The relationship between the elasticity tensor and the density of the base material is referred to as material interpolation scheme (Bendsoe and Sigmund, 1999).

The basic concept of SIMP method is that 'grey' elements are penalised and removed from the domain to obtain a black (ρ = 1) and white (ρ = 0) topology. That is any element that has density within $0 < \rho < 1$ is removed from the design domain. The first step in this method is to choose a suitable design domain or reference domain which allows the definition of surface tractions and other boundary conditions (Bendsoe, 1989). It is assumed that the domain is made of an artificial material and its density can be related to structures stiffness by the following power law (Bendsoe and Sigmund, 1999):

$$s = \rho^p \tag{3.1}$$

where: s = stiffness of structure

ρ = density of artificial material

$p > 1$, penalty parameter

The density variable is within the limits $0 \leq \rho \leq 1$ but to avoid singular finite element matrix a small lower bound, $0 < \rho_{\min} \leq \rho$ is imposed. As the penalty parameter is increased the element with intermediate densities is penalised as it's structurally less effective and doesn't contribute the structural stiffness of the design domain. The algorithm will redistribute the material of given volume within the design domain (Burns (Ed), 2005). A penalty parameter of $p \geq 3$ should be used to obtain a good topology.

The advantage of SIMP method over other similar methods is that it only requires one variable per element in the ground structure and also it requires no homogenisation.

3.4 Evolutionary Structural Optimisation

The Evolutionary Structural Optimisation (ESO) method was proposed by Xie and Steven (1993). The basic idea of the method is that inefficient elements are removed from the design domain based on a material removal criterion. Such a criterion function or parameter value is calculated for each element and in each iteration some elements with the lowest criterion value that do not meet the minimum criterion set are eliminated (Rozvany, 2001). By progressively removing such elements the structure will evolve towards an optimum. This method totally removes inefficient elements and as such is sometimes referred to as the 'hard kill' method.

3.4.1 ESO based on stress level

The stress level of the elements in the design domain can be found using finite element analysis and low stress levels can be interpreted as underutilized materials. This concept has been used in this method to remove underutilized materials with stress levels below a threshold value. When all the elements below the threshold values have been removed the threshold value is increased and the iteration started again. This procedure of increasing the threshold value continues until a desired optimum is obtained, for example, when there is no material in the final structure that has a stress level below 25% of the maximum stress (Huang and Xie, 2010).

3.4.2 ESO for stiffness optimisation

In the design of structures such as building and bridges the stiffness is one of the key factors to consider. Keeping this in mind the compliance based method was developed. Mean compliance is the inverse measure of the overall stiffness of the structure (Huang and Xie, 2010). That is by minimising the compliance, the stiffness in maximised. The compliance can be defined by the total strain energy of the structure or the external work done by the loads on the structure.

The element with the lowest sensitivity number is removed in each iteration. The sensitivity number is an approximation of the change in the compliance as a result of removing an element. At each iteration the number of elements removed is restricted by the element removal ratio which is the ratio of the number of elements removed in each iteration to the total number of initial or current elements.

According to Rozvany (2001) ESO is an inappropriate name for this method as 'evolutionary' means a genetic algorithm whereas 'optimisation' means to find optimum solutions. He proposed the name SERA (**S**equential **E**lement **R**ejections and **A**dmissions) for such methods.

3.5 Bi-directional Evolutionary Structural Optimisation (BESO)

Two major deficiencies present in early versions of ESO method was solution time and uniqueness (Querin et al, 1998). Since elements were only removed in the ESO method, it was questioned if the method ensured that it was not a local optimum solution that was obtained and could the elements removed, be returned. The 'Bi-directional Evolutionary Structural Optimisation' method presented by Querin et al (1998) provided an improved version of the ESO algorithm. The improved method was able to remove inefficient material to eliminate low stress as well as add materials to efficient areas to alleviate high stress.

The element efficiency in BESO is measured the same way as in ESO but the adding and removing uses a different procedure. A control parameter named 'Inclusion Ratio' is used to control the amount of material that is added. When no more elements is removed or added that is at steady state, the inclusion ratio is decreased and the rejection ratio is increased.

3.5 Other Available Techniques

There are other techniques available that are extension of the methods outlined above. Some of them are briefly described below.

BESO utilising SIMP

This method incorporates the BESO method with the SIMP method for determining the sensitivity number of the elements. See Huang and Xie (2010) for further details.

Performance-base optimisation (PBO)

The PBO method combines the topology and sizing optimisation into a single scheme to achieve the optimal topology and thickness design of continuum structures. The performance of the structure is the objective criteria for the method that is it uses realistic performance criteria. These performance criteria include structures stiffness, strain, shear, etc. See Liang (2005) for further details.

Chapter 4
METHODOLOGY – DESIGN & TESTING

Overview

This chapter outlines the design and testing methods used in this dissertation. It includes brief design procedure of strut-and-tie (STM) modelling, summary of requirements of AS3600:2009 for STM methods and conventional beam design method. The testing procedure used is summarised including brief description of testing equipment used and its functions.

4.1 Design Problem

Before explaining the methodology used in this dissertation, it's best to present the design problem first. Figure 4.1 shows the beam geometry, the support conditions and loading. As shown the beam is 1400mm long with a depth of 250mm. The width of the beam is 100mm. On the right end, for a length of 500mm the beam depth has been reduced to only 150mm. This was done to create a D-region (see chapter 2 for definition) in the beam. The beam supports have been placed 100mm from each end so that there is some bearing for the supports as it's obvious that the beam cannot be supported at the edge of the beam. Hence the effective beam span is only 1200mm. A single point load at the mid-span of the beam is applied which makes the design problem quite simple and also makes setting up the experiment fairly simple.

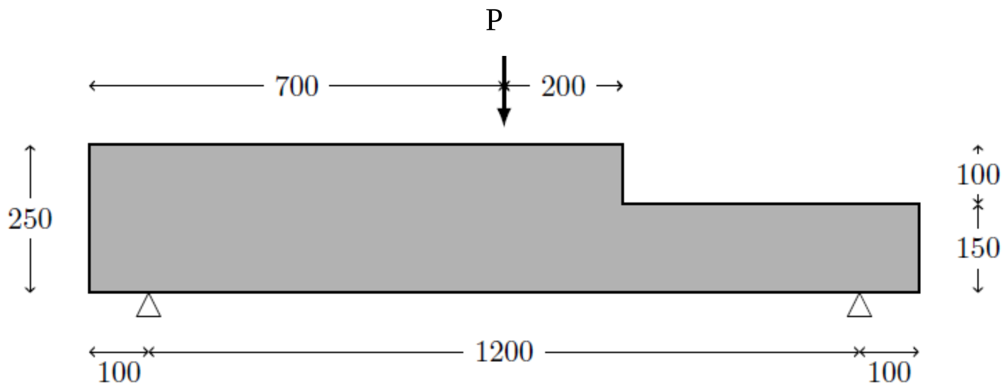


Figure 4.10 Design problem – beam geometry, support and loading conditions.

4.2 MATLAB Code

One of the most important parts of this dissertation is the topology optimisation code written in MATLAB. This code determines the optimum layout of a design domain for a given loading and support condition. The origin of the code and its functionality is explained briefly in this section.

The code used in this dissertation is a revised version of the 88 line code written by Andreassem et al (2010). The 88 line code is a improved version of the 99 line code presented by Sigmund in 2001. The initial 99 line code implemented topology optimisation for compliance minimisation of statically loaded structures (Sigmund 2001). The code is created as a function file that can be called from the command window in MATLAB by the line:

top (nelx, nely, volfrac, penal, rmin)

where: nelx = the number of elements in the horizontal direction

nely = the number of elements in the vertical direction

volfrac = is the volume fraction of solids in the final topology

penal = is the penalisation power

rmin = is the filter size

The support conditions and loading have to be defined in the code in lines 79-82 (see Appendix B.1).

The improvement in the 88 line code is that the original sensitivity filter is extended by a density filter and the efficiency has been considerably improved by preallocating arrays and vectorizing loops. The code is called up in MATLAB in a similar manner to the 99 line code but with the line:

top (nelx, nely, volfrac, penal, rmin, ft)

where the additional argument **ft** specifies whether sensitivity filter (ft = 1) or density filter (ft = 2) is used. When sensitivity filter is used the topology obtained is identical to that obtained by the 99 line code. Readers are referred to papers by Sigmund (2001) and Andreassen et al (2010) for a comprehensive detail of the two codes.

4.2.1 Topology of design problem

As mentioned earlier the 88 line code was revised for use in this dissertation. The noticeable changes were the removal of the density filter and hence the argument **ft** was no longer needed. A number of lines were added to improve the output and the new code:

- shows initial topology and prints it to the file T0000.pdf;
- prints the topology after every 10 iterations to the files T0010.pdf, T0020.pdf, T0030.pdf, ...;
- stores the values of the objective function at each iteration and writes them to a Comma Separated Value (CSV) file named his.csv, and;
- plots the evolution of the values of the objective function and prints it to the file his.pdf.

The revised code had two new arguments added as shown below:

beam (nelx, nely, xv, yv, volfrac, penal, rmin)

where xv and yv define the void in the design domain. In the case of the design problem this void is the top right hand portion of the beam where the beam depth reduces from 250mm to 150mm. The lines 7-9 (see Appendix B.3) define the passive elements by assigning these elements the value 1 which the code recognises as being void. The support conditions and loading is defined in lines 22-26 and the design problem is solved using the following prompt line:

beam (140,25, (90:140), (1:10), 0.2, 3, 1.3)

where a 140x25 mesh is used to define the design domain, the intersection of elements 90-140 in horizontal direction (xv) and elements 1-10 in the vertical direction (yv) define the void area. It is assumed that the reinforcement is 20 percent of the total volume hence volfrac is 0.2. A penalisation factor of 3 is used and the filter radius is 1.3. The optimum topology obtained is presented in Chapter 5.

4.3 Design Procedure

To be able to compare results the design problem was designed using two methods, namely the topology optimisation method in conjunction with strut-and-tie modelling (STM) and the conventional beam design method. These two methods are defined further in the next sections.

4.3.1 Conventional method

The conventional method is a well established method for design of reinforced concrete beams in bending and shear. The objectives of this method is to determine the maximum bending moment and shear forces being carried by the beam and then reinforce the beam accordingly to resist these forces. The theory behind this method is that concrete in the compression side carries the compressive forces (C_c) as illustrated in Figure 4.2. Steel reinforcement bars placed in the tensile zone resists the tensile forces (T). If total compressive forces are greater than compressive strength of concrete then steel reinforcement bars can be placed in the compression zone to resist additional compressive forces (C_s). Readers can lookup Warner et al (2007) for further information on reinforced concrete design basics.

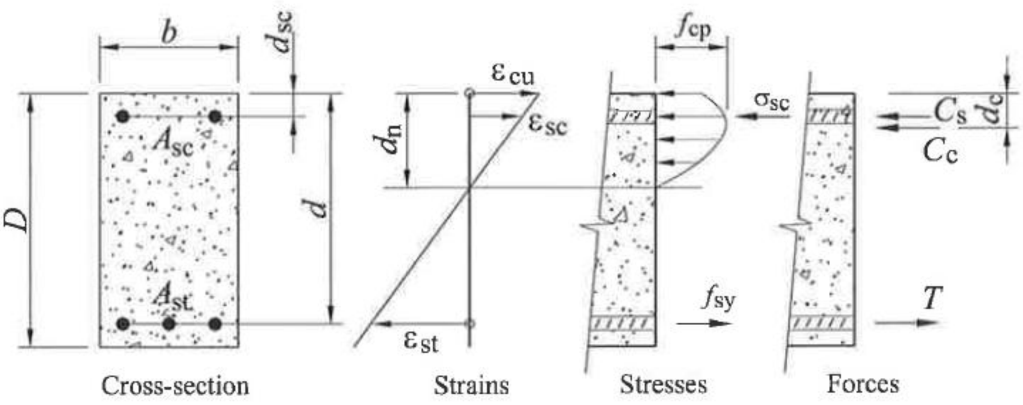


Figure 4.11 Conditions at ultimate moment in a doubly reinforced concrete section.

For this dissertation two spreadsheets were setup that the was used to calculate the reinforcement required in the beam based on maximum bending moment and shear force in the beam. One spreadsheet calculated bending reinforcement while the other calculated shear reinforcement and both were setup to adhere to the requirements of AS3600:2009. Since the beam is simply supported a simple formula (4.1) was used to determine maximum bending moment.

$$M^* = \frac{PL}{4} \tag{4.1}$$

where: M^* = maximum bending moment

P = applied load

L = beam span

Due to the reduced beam depth in the design problem, two shear calculations were done as section depth influences the beams capacity to resist shear forces. Deeper beams resist greater shear forces. Spreadsheet output and reinforcement layout in Appendix D shows that shear reinforcement at the end with reduced depth is at closer centres than the other end. Calculating bending reinforcement was fairly straightforward.

4.3.2 Strut-and-tie modelling method

The optimum topology that was obtained from the MATLAB code was modelled in Microstran which is a finite element analysis software, to determine the internal forces in the truss layout. Microstran outputs are presented in Appendix D.1. Once the internal member forces were known the beam was designed using the provisions of Section 7, AS3600:2009.

Provisions of AS3600:2009

Section 7 of this standard outlines the design of concrete structures using strut-and-tie modelling method. The strut capacity C is:

$$C = \emptyset_{st}\beta_s 0.9 f_c' A_c \qquad (4.2)$$

where: $\emptyset_{st}$ = is the strength reduction factor

β_s = is the strut efficiency factor calculated by equation 4.3 below

f_c' = is the characteristic strength of concrete

A_c = is the cross section area of the strut.

The strut efficiency factor of prismatic strut (see figure 2.3a) is taken as 1.0 and for fan or bottle-shaped strut is taken as;

$$\beta_s = \frac{1}{1+0.66\cot^2\theta} \qquad (4.3)$$

where: θ = is the angle between the strut and tie axis

According to AS3600, prismatic struts should only be used where the compressive stress cannot diverge, otherwise bottle-shaped strut should be used. The bursting forces (figure 2.3c) in bottle-shaped struts need to be determined as given in section 7.4.2 of AS3600 and transverse reinforcement provided if needed.

The design strength of ties is similar to strength of tensile reinforcement in conventional beam design. Hence;

$$T = A_{st} f_{sy} \tag{4.4}$$

where: A_{st} = is the cross sectional area of reinforcement

f_{sy} = is the yield strength of steel reinforcement

For unconfined nodal region the design strength shall be such that compressive stress on any nodal face is not greater than $\emptyset_{st}\beta_n 0.9 f_c'$, where:

- for CCC node β_n = 1.0;
- for CCT node β_n = 0.8;
- for CTT node β_n = 0.6.

Where confinement is provided the design strength shall not exceed maximum compressive stress on any face of $\emptyset_{st} 1.8 f_c'$. The flow diagram summaries the design process for this method.

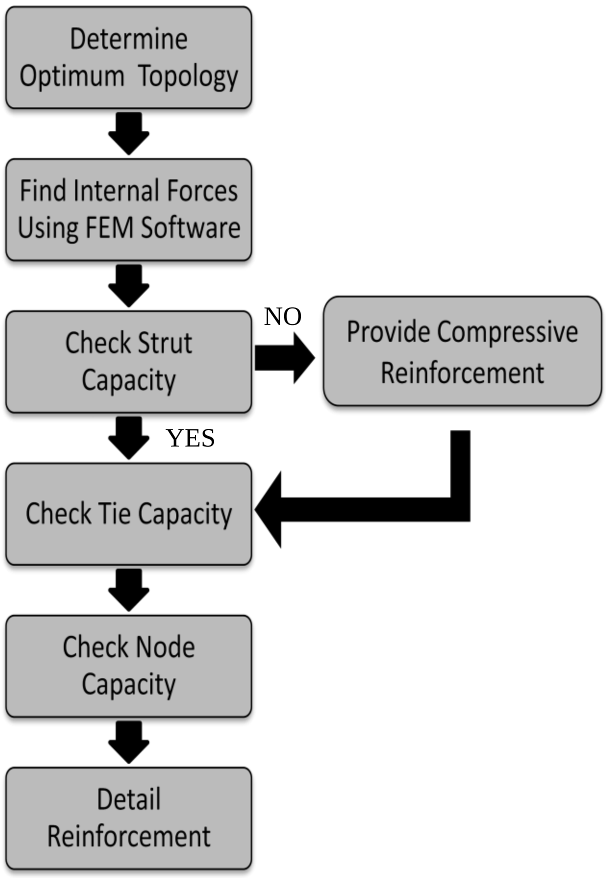


Figure 4.12 Design process, combining topology optimisation and STM.

4.3.3 Concrete mix design

It was decided to use 40MPa concrete in the beam specimen and to achieve this the following ratios were used.

- Water/cement raito = 0.5
- Aggregate/cement ratio = 3.5
- Fine aggregate/course aggregate = 0.5

Based on the beam geometry it was calculated that about 0.035m^3 of concrete would be required and the water, cement and aggregate volumes were determined using the above ratios. Sand and 10mm aggregate was used for fine aggregate while 15mm aggregate was used as course aggregate. The concrete was mixed in a automated mechanical mixer which ensured that the mix was consistent.

4.4 Finding Optimum Topology of Problems from Literature

To test the versatility of the topology optimisation code, the dissertation scope was extended to determine optimum topology of standard problems found in literature. These problems were mainly taken from papers on strut-and-tie modelling. This gave a good opportunity to test if the optimum topology was comparable to the STM layout the authors of those papers proposed.

The first problem was the deep beam problem from the paper by Ley et al (2007). In Figure 4.4 the beam is simply supported with a void in the middle and a point load is acting above the void. In the paper by Ley et al (2007) 5 specimens are designed by graduate students for different criteria such as using minimum steel, or limit deflection. In the rest of the dissertation this beam will be referred as Ley beam.

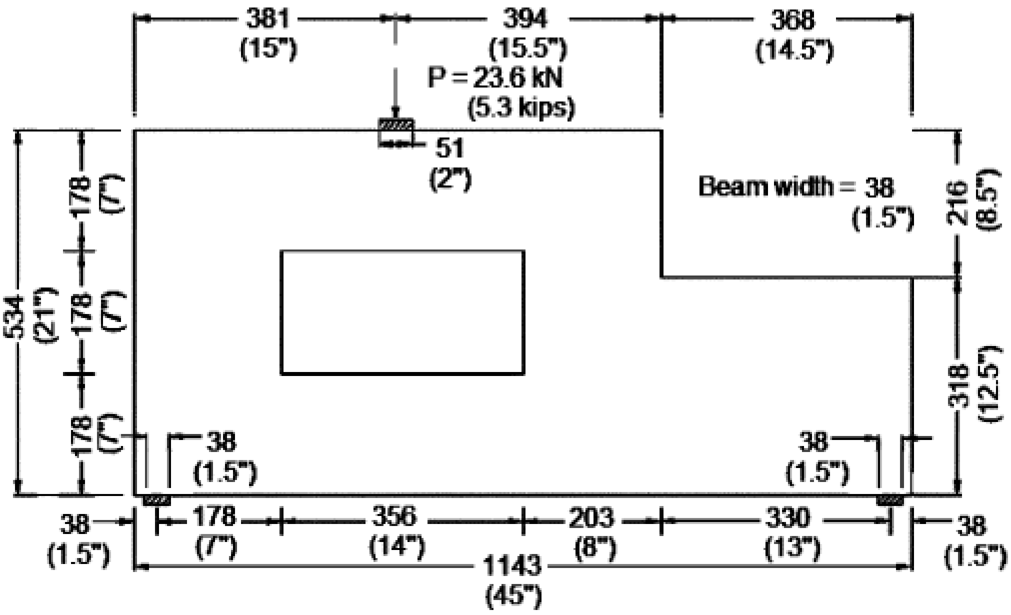


Figure 4.13 Ley beam geometry and loading. (Dimensions are in mm [in.])

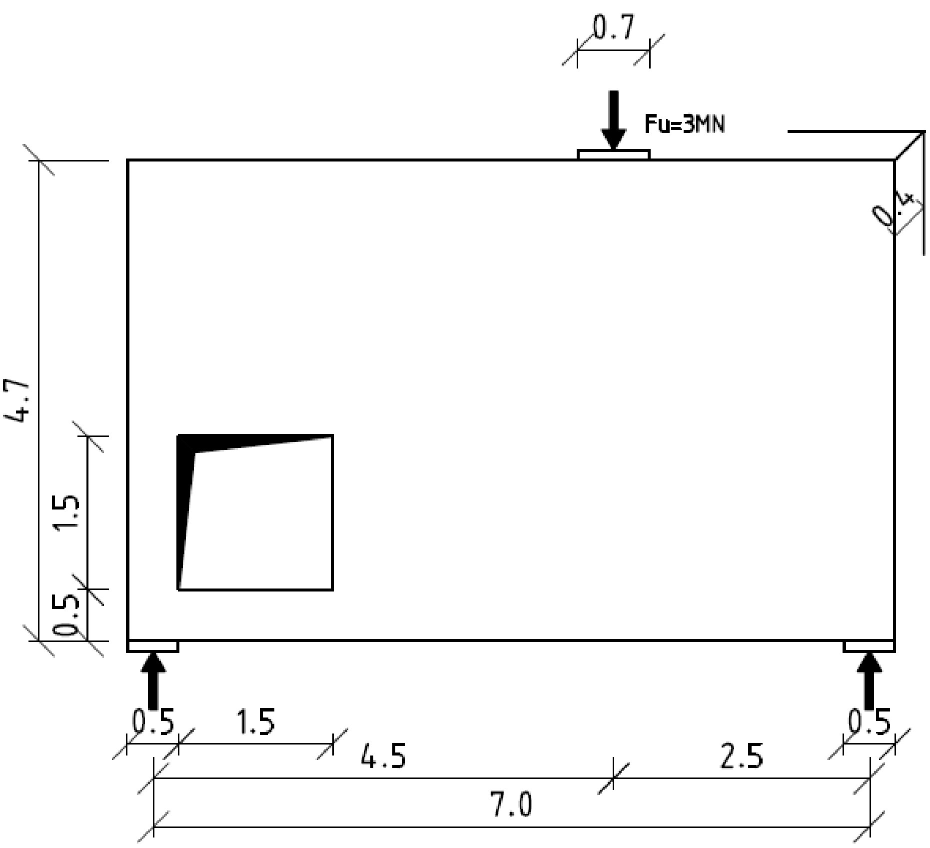


Figure 4.14 Schlaich beam geometry and loading.

The second problem was the deep beam presented by Schlaich et al (1987). It is similar to Ley beam but it doesn't have any reduction in beam depth as can be seen in Figure 4.5. The void is at the bottom left corner close to the support and a point load is acting at about two thirds of the span.

4.5 Testing Equipment & Procedure

The objective of testing is to determine the ultimate load and maximum deflection of the two specimens and compare results. Also the mode of failure of the beams and cracking pattern would be compared. The testing was done in Centre of Excellence in Engineering Fibre Composite (CEEFC) lab P11. The tests were done using SANS compression testing machine model YAW-6206 (see figure 4.6) which can perform compression and bending tests. It's fully automated with precise, full digital and graphic display instruments. With its powerful testing software PowerTest, the system can acquire, dispose automatically testing data, display real-time stress-strain curve, load-deformation curve, load-time curve and other related curves and at the same time save, output and print test report and data with customised format. The machine is also equipped with a three point flexure grip that was used to test the beam in bending. The flexure grip is set-up on a track and once the specimen is set-up, it can be rolled into position (Figure 4.7).

First the compressive strength of the concrete was determined by testing the cylinder specimens that were prepared. Two 100Ø x 200mm cylinders were cast and one 150Ø x 300mm cylinder was prepared. There was a slight problem while testing the 150Ø specimen so the test had to be done twice. In the first test the machine's maximum load was set at only 500kN so the test stopped before the specimen had failed. Since the output from the first test showed that stress in specimen was still in elastic range, it meant that there was no cracking of the specimen and this was confirmed by visual inspection of the specimen. The maximum load was increased to 1500kN and the test was redone.

Once the cylinders had been tested, the beam specimens were loaded onto the flexure grip as shown in Figure 4.7 and rolled into position as shown in Figure 4.8.

Figure 4.15 SANS (YAW-6206) Compression Testing Machine

Figure 4.16 Specimen set-up on flexure grip

Figure 4.17 Beam set-up in compression testing machine

The specimen was loaded at a constant rate until failure and loading was continued till a maximum deflection of 35mm was reached. The load rate and maximum deflection of 35mm were the control for both the specimens and the test were stopped when the specimen had deflected 35mm. The crack patterns were noted and other results are presented in the next chapter.

Chapter 5

RESULTS

Overview

This chapter presents the results of the testing done and corresponding analysis and interpretation of these results. Also provided here are the optimum topology obtained for some of the beams found in literature and some discussion on its similarity or differences to the original design.

5.1 Optimum topology of beam

The optimum topology for the design problem was obtained using the modified code written in MATLAB. The outputs from the code are presented in Figure 5.1 to Figure 5.3.

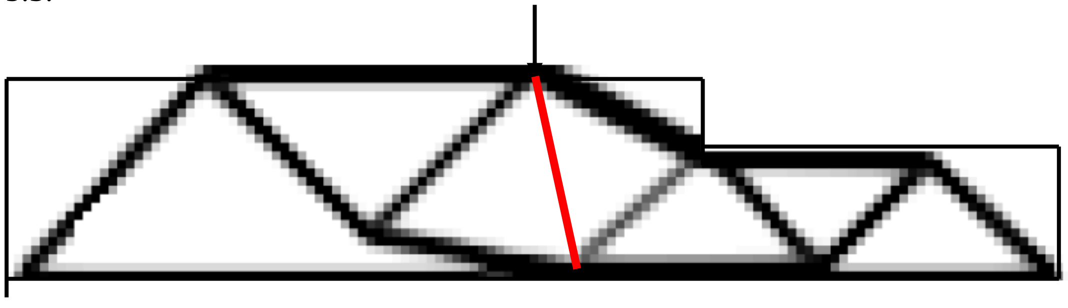
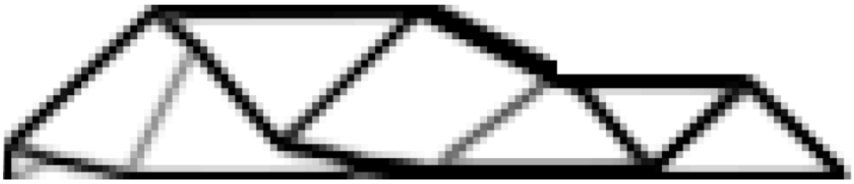


Figure 5.18 Optimum topology of design problem after 159 iterations
(Source: Matlab code)

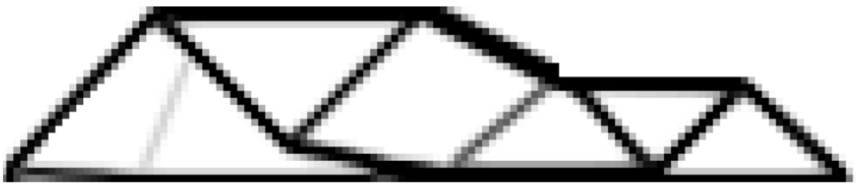
The above optimum topology was used as the base layout for the STM method and was analysed in Mircostran to determine the internal forces. The member shown in red was introduced to make the truss stable for easy analysis.

After 20 iterations

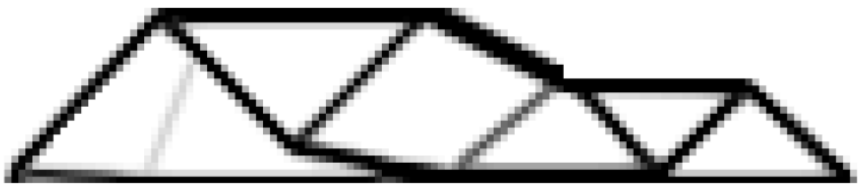


After 40 iterations

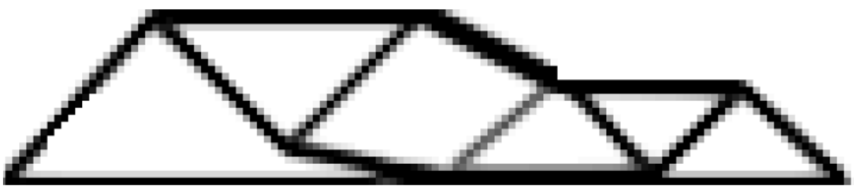
After 60 iterations



After 80 iterations



After 100 iterations



After 120 iterations

After 140 iterations

Figure 5.19 Optimisation output at every 20 iteration

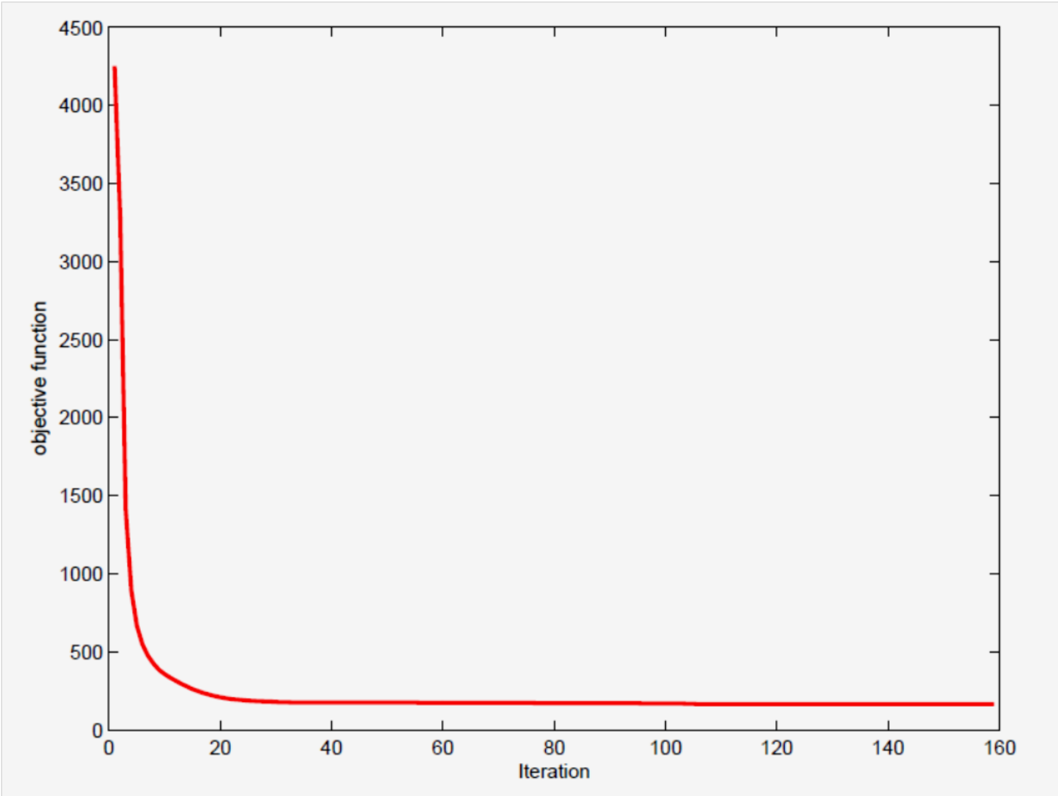


Figure 5.20 Graph of objective function vs iteration

(Source: Matlab code)

5.2 Test Results

The results presented in this section are from the physical testing of the beam specimens. As mentioned before the tests were carried out at Centre of Excellence in Engineering Fibre Composite (CEEFC). The tests carried out included compression test of concrete cylinder to determine concrete compressive strength and flexural test on beam specimens to determine its ultimate load and failure mode.

5.1.1 Compression test of cylinder samples

The three samples that were prepared were tested to determine the concrete compressive strength. The failure load of each sample is given in table 5.1 together with its compressive strength.

Table 5.7 Cylinder test results

Specimen	Dimension (mm)	Failure load (kN)	Compressive strength, f_c’ (MPa)
1	100Øx200	422.355	53.76
2	100Øx200	257.6	32.8
3	150Øx300	873.358	49.42

The average of the three samples comes to 45MPa which is still higher than the assumed 40MPa used in the design. The second sample had the lowest compressive strength and one possible reason for this would be because it was the last cylinder cast using the left over concrete. The concrete used to fill this cylinder was mostly scrapped off the bottom of the wheelbarrow so it had started to set. The load versus deflection graphs for the three samples are given in the following figures.

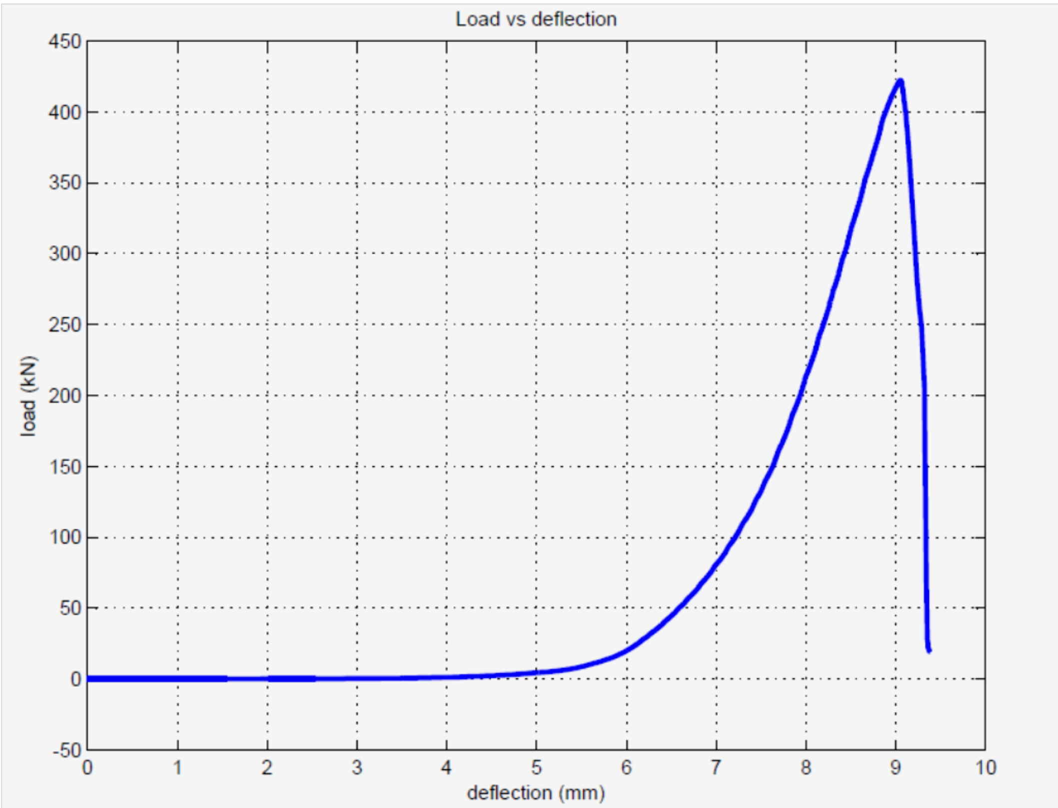


Figure 5.21 Load versus deflection graph for specimen 1

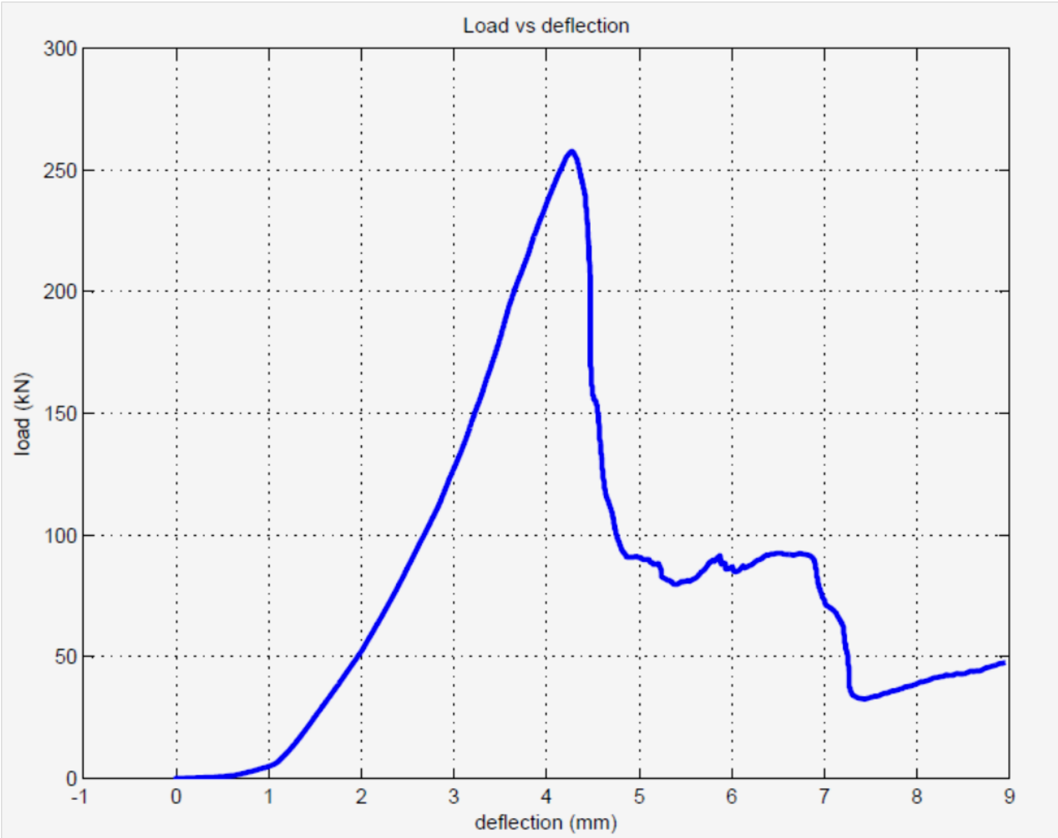


Figure 5.22 Load versus deflection graph for specimen 2

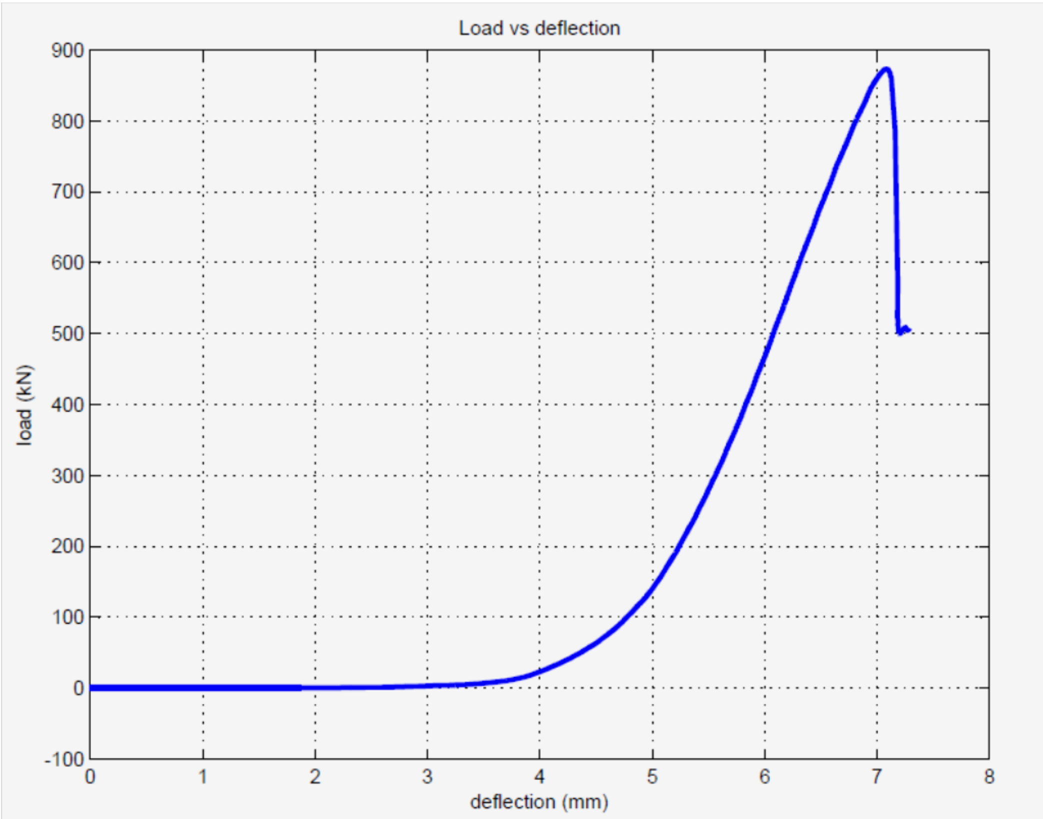


Figure 5.23 Load versus deflection graph for specimen 3

5.1.2 Flexural testing of beams

The two beam specimens were tested in bending as explained earlier in chapter 4 and its ultimate load, mode of failure and crack pattern were noted. This section presents the results of the tests and discusses the possible reasons for the results obtained.

Optimised Beam

As can be seen in Figure 5.4 the ultimate load for this beam was 70.96kN which was less than the design load of 75kN. This irregularity can be explained by a number reason most of which has to do with the construction of the beam. The first problem was with the construction of the reinforcement cage. Due to the unconventional placement of bars it was difficult to get the bars to line up as in the optimum truss layout.

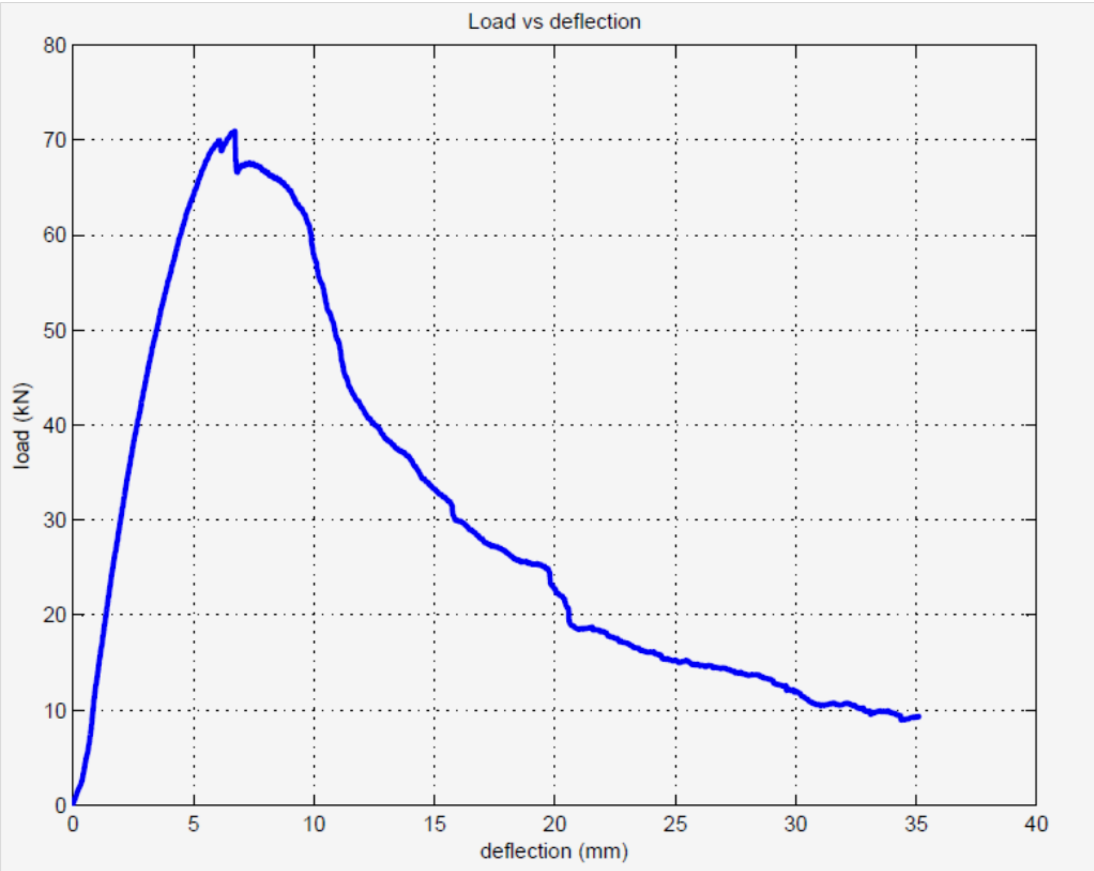


Figure 5.24 Load versus deflection graph for optimum beam

The other potential problem could have been the steel reinforcement ratios used in the design of the two beams. For them to be comparable the total weight of reinforcement used in both beam were approximately same but perhaps the ratios of longitudinal and shear reinforcements used should have been same. This is something that needs to be investigated further.

The graph in Figure 5.7 shows a sudden drop in load carrying capacity with continued deflection which can be interpreted as shear failure. This was also evident in the crack pattern forming in the beam which can be seen in Figure 5.8. Also refer to photos 5-7 in Appendix E. The fact that this beam failed in shear was surprising as the STM method has been proven to handle both shear and bending simultaneously. Again this could be due to the reinforcement placement during construction.

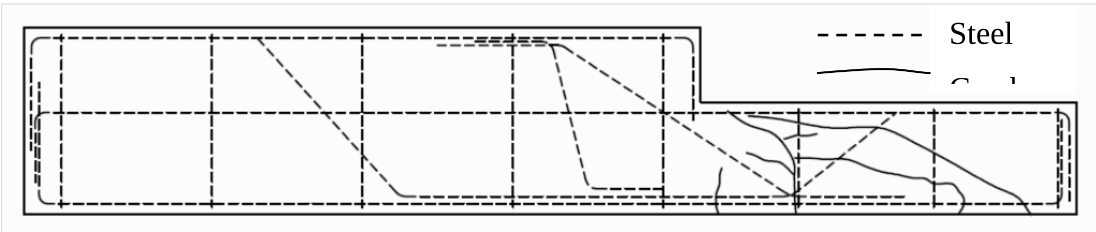


Figure 5.25 Crack pattern in optimum beam

Conventional Beam

This beam performed exceptionally well, failing at an ultimate load of 104.5kN which is 39.33% higher than the design load of 75kN. This shows that conventional design based on assumptions, simplifications and rational formulae is a conservative design method. The design load wasn't multiplied by any factor of safety but design standards require that loads be factored and if that was done the design would have been more conservative. The need for a better design method is clear as such conservative methods are neither cost effective nor is it sustainable.

Figure 5.9 shows the load versus deflection graph for this beam and it can seen from the graph that the beam was able to sustain considerable load for a while before completely failing. This can be interpreted as a balanced flexural failure which is the preferred mode of failure of concrete as it gives time to rectify the problem of vacate structure before complete failure. A larger deformation before failure is a good indicator that something is not right with the structure.

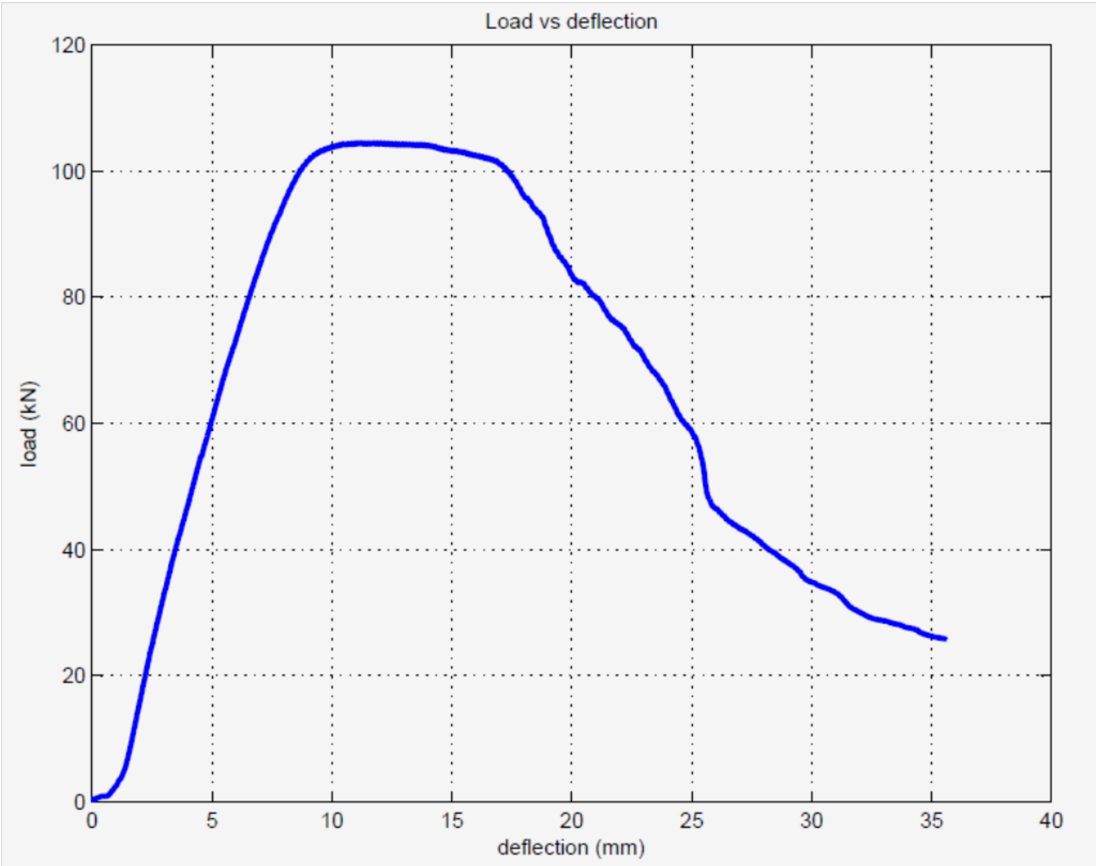


Figure 5.26 Load versus deflection graph for conventional beam

Flexural failure mode is also confirmed by the crack pattern in the beam with crack first appearing at the bottom of the beam and then extending to the top. Figure 5.10 shows the crack pattern in relation to the reinforcement. For reinforcement details refer to Appendix D and also refer to photos 9-10 in Appendix E.

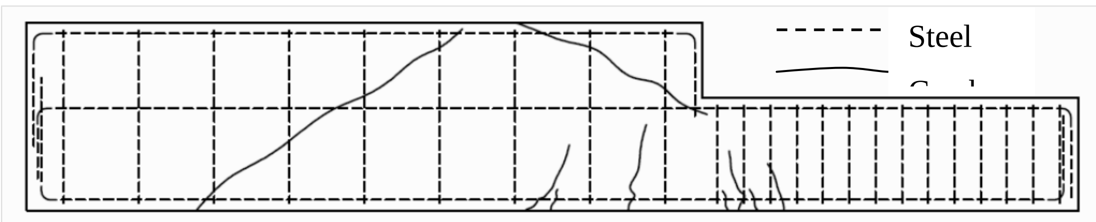


Figure 5.27 Crack pattern in conventional beam

5.3 Optimum Topology of Problems from Literature

As mentioned in chapter 4, two beams found in the literature were selected and its optimum topology determined by the code written in MATLAB (refer Appendix B.3).

Ley beam

In the paper presented by Ley et al (2007), five specimens were independently designed by graduate students. For the purpose of this dissertation the STM produced for specimen 1 was used to compare with the optimum topology determined by the code. The focus of specimen 1 design was to minimise the weight of reinforcement used by minimising the strain energy, which in turn maximises the stiffness. The code used in this dissertation also maximises the stiffness of the structure. Figure 5.11 shows the STM presented by Ley et al (2007) and figure 5.12 shows the optimum topology determined by the code.

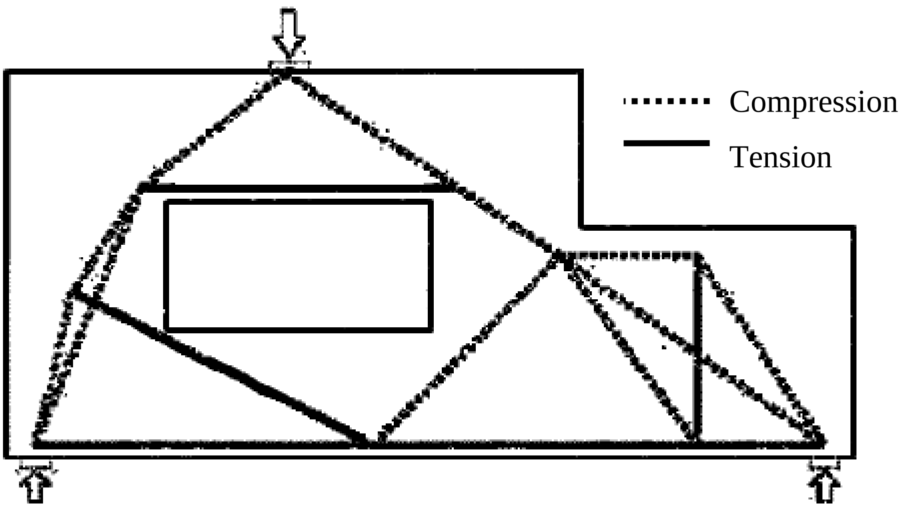


Figure 5.28 STM presented by Ley et al (2007)

(Source: Ley et al 2007)

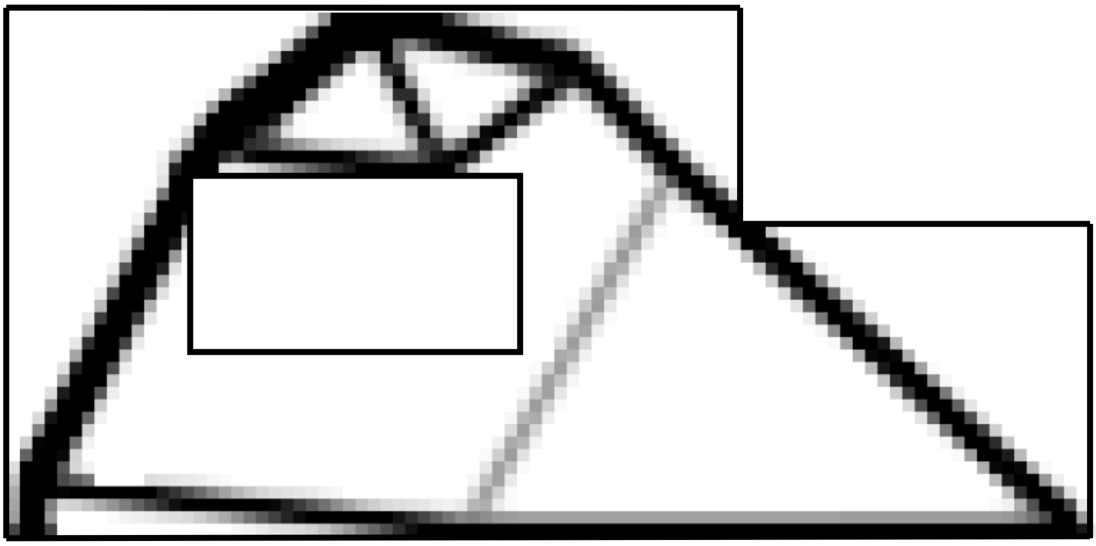


Figure 5.29 Optimum Topology for Ley beam
(Source: Schlaich et al 1987)

There are general similarities in the two STM produced in the sense that both use tied arch behaviour to transfer the loads to the supports. Depending on the load, the strut on the right side of the beam will required additional reinforcement and to cater for this the STM presented by Ley et al (2007) has subdivided the strut into smaller struts and for equilibrium have introduced a vertical tie.

Schlaich beam

In their paper Schlaich et al (1987) presented a numerical example of a deep beam with a large hole to illustrate the method of STM. Figure 5.13 shows the complete STM model that they produced and Figure 5.14 shows the optimum topology for the same problem determined by the code. It can be seen that there are some similarities in the two layouts with the large strut on the right side of the beam being one such similarity. The layout on the right side similar general form but the STM proposed by Schlaich et al (1987) is much more complex. As mentioned earlier in the dissertation, STM method can have numerous layouts that will effectively transfer the loads to the supports; hence the two layouts are not identical. Further testing of both layouts will be needed to confirm which is the best.

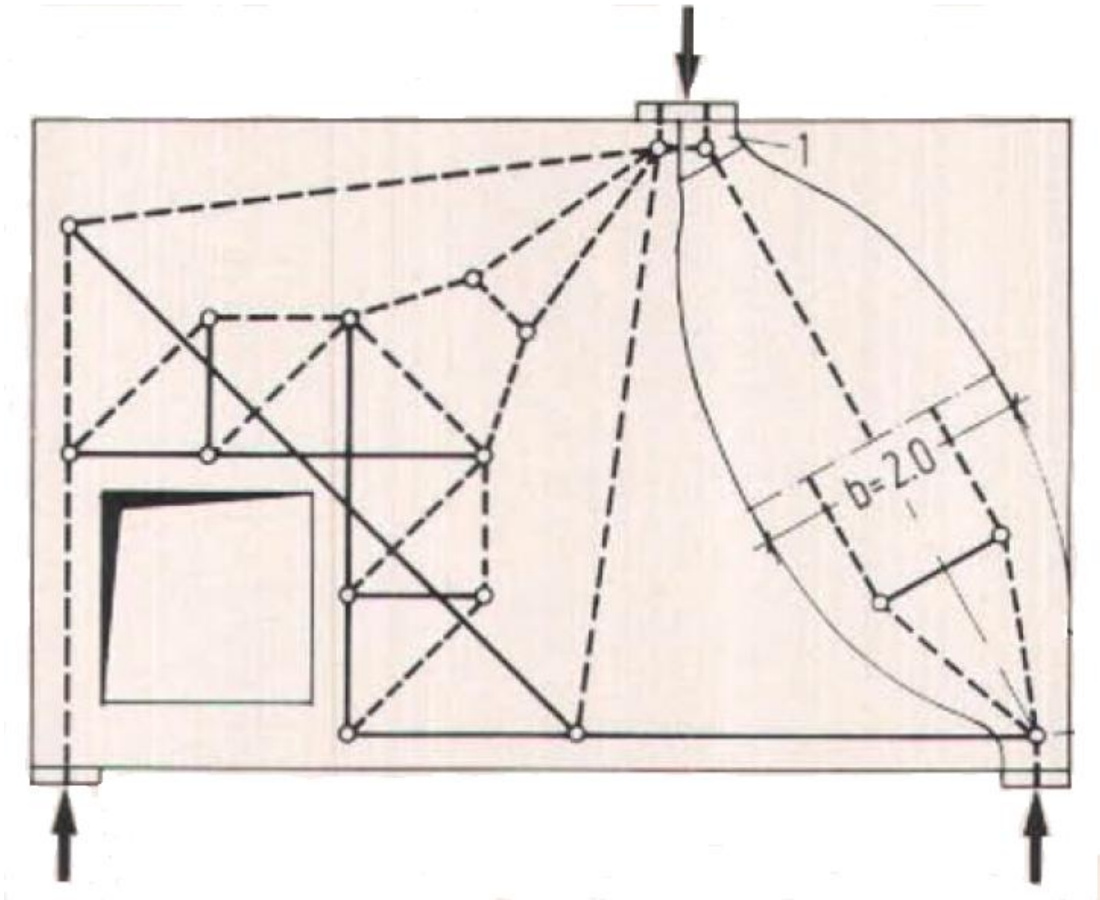


Figure 5.30 STM presented by Schlaich et al (1987)

(Source: Schlaich et al 1987)

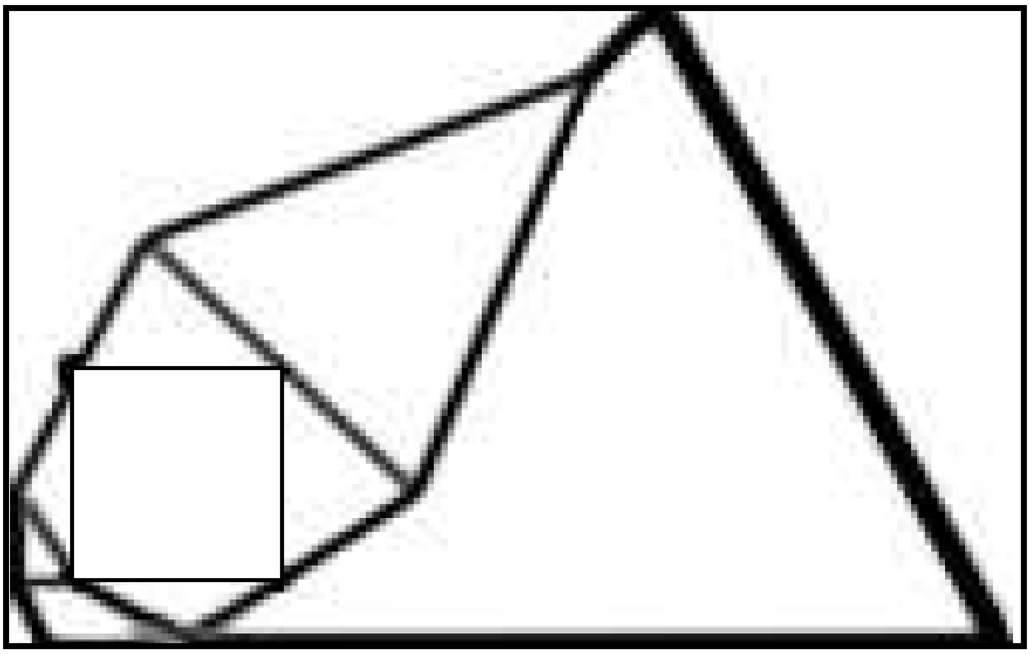


Figure 5.31 Optimum topology for Schlaich beam

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Chapter 6

CONCLUSION & RECOMMENDATIONS

Overview

Presented here are the conclusions drawn based on the test results and recommendations for further work are given.

6.1 Conclusion

Two beam samples were built and tested and the results obtained were not as expected. It was thought that the optimum design would perform better that the conventional design but the results were opposite. As mentioned in pervious chapter, quality control during construction may have been the reason for the results obtained. It should be noted that the author has had no prior experience in steel works relating to beam construction and this lack of experience has to be taken into account. I can be seen that the optimum design did not fail by much and better construction will give better results. On the other hand conventional design was over designed and as such optimum designs will give more realistic results.

To conclude, further work needs to be done to obtain conclusive results that clearly show that optimum design are better. Recommendations for further work are suggested in the next section.

6.2 Recommendations for Future Work

The optimum beam may need to be redesigned, built and testing to get conclusive results. Due to time constraints this was not done for this dissertation but this may be done by the author at a later stage as part of his further research in this topic.

As mentioned better construction techniques need to be used so that reinforcement can be easily placed in the steel cage. It’s was also noted that the ratios for the total steel reinforcement in the two beams were same but for a better comparison the ratios of longitudinal and shear reinforcements need to be similar.

It may also be of great advantage to use strain gauges to determine deflections at various locations along the beam to analyse the beam behaviour under load.

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Appendix A – Specification

University of Southern Queensland

FACULTY OF ENGINEERING AND SURVEYING

ENG4111/4112 Research Project
PROJECT SPECIFICATION

FOR: Nitesh Nitin PRASAD

TOPIC: VERIFICATION OF SIMPLIFIED OPTIMUM DESIGNS FOR REINFORCED CONCRETE DEEP BEAMS

SUPERVISOR: Dr Kazem Ghabraie

PROJECT AIM: To verify optimum designs obtained by using a simplified linear elastic model for reinforced concrete.

PROGRAMME:

1. Research background information relating to topology optimisation, especially the SIMP method.
2. Research on concrete beam design by strut-and-tie method.
3. Design the given deep beam using strut-and-tie method.
4. Optimise design obtained in (3) using the Matlab optimisation code.
5. Prepare samples based on strut-and-tie design and optimised design.
6. Test samples and compare results.
7. Conclusions.

Appendix B – Topology Optimisation Codes

B.1 99 Line Code

```
%%%% A 99 LINE TOPOLOGY OPTIMIZATION CODE BY OLE SIGMUND, JANUARY 2000 %%%
function top(nelx,nely,volfrac,penal,rmin);
% INITIALIZE
x(1:nely,1:nelx) = volfrac;
loop = 0;
change = 1.;
% START ITERATION
while change > 0.01
  loop = loop + 1;
  xold = x;
% FE-ANALYSIS
  [U]=FE(nelx,nely,x,penal);
% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
  [KE] = lk;
  c = 0.;
  for ely = 1:nely
    for elx = 1:nelx
      n1 = (nely+1)*(elx-1)+ely;
      n2 = (nely+1)* elx   +ely;
      Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
      c = c + x(ely,elx)^penal*Ue'*KE*Ue;
      dc(ely,elx) = -penal*x(ely,elx)^(penal-1)*Ue'*KE*Ue;
    end
  end
% FILTERING OF SENSITIVITIES
[dc]   = check(nelx,nely,rmin,x,dc);
% DESIGN UPDATE BY THE OPTIMALITY CRITERIA METHOD
[x]    = OC(nelx,nely,x,volfrac,dc);
% PRINT RESULTS
change = max(max(abs(x-xold)));
disp([' It.: ' sprintf('%4i',loop) ' Obj.: ' sprintf('%10.4f',c) ...
    ' Vol.: ' sprintf('%6.3f',sum(sum(x))/(nelx*nely)) ...
    ' ch.: ' sprintf('%6.3f',change )])
% PLOT DENSITIES
colormap(gray); imagesc(-x); axis equal; axis tight; axis off;pause(1e-6);
end
%%%%%%%%%% OPTIMALITY CRITERIA UPDATE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [xnew]=OC(nelx,nely,x,volfrac,dc)
l1 = 0; l2 = 100000; move = 0.2;
while (l2-l1 > 1e-4)
  lmid = 0.5*(l2+l1);
  xnew = max(0.001,max(x-move,min(1.,min(x+move,x.*sqrt(dc./lmid)))));
```

```
  if sum(sum(xnew)) - volfrac*nelx*nely > 0;
    l1 = lmid;
  else
    l2 = lmid;
  end
end
%%%%%%%%%% MESH-INDEPENDENCY FILTER %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [dcn]=check(nelx,nely,rmin,x,dc)
dcn=zeros(nely,nelx);
for i = 1:nelx
  for j = 1:nely
    sum=0.0;
      for k = max(i-floor(rmin),1):min(i+floor(rmin),nelx)
        for l = max(j-floor(rmin),1):min(j+floor(rmin),nely)
          fac = rmin-sqrt((i-k)^2+(j-l)^2);
          sum = sum+max(0,fac);
          dcn(j,i) = dcn(j,i) + max(0,fac)*x(l,k)*dc(l,k);
        end
      end
    dcn(j,i) = dcn(j,i)/(x(j,i)*sum);
  end
end
%%%%%%%%%% FE-ANALYSIS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [U]=FE(nelx,nely,x,penal)
[KE] = lk;
K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
F = sparse(2*(nely+1)*(nelx+1),1); U=zeros(2*(nely+1)*(nelx+1),1);
for elx = 1:nelx
  for ely = 1:nely
    n1 = (nely+1)*(elx-1)+ely;
    n2 = (nely+1)* elx   +ely;
    edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2; 2*n2+1; 2*n2+2; 2*n1+1; 2*n1+2];
    K(edof,edof) = K(edof,edof) + x(ely,elx)^penal*KE;
  end
end
% DEFINE LOADS AND SUPPORTS (HALF MBB-BEAM)
F(2,1) = -1;
fixeddofs   = union([1:2:2*(nely+1)],[2*(nelx+1)*(nely+1)])
alldofs     = [1:2*(nely+1)*(nelx+1)];
freedofs    = setdiff(alldofs,fixeddofs);
% SOLVING
U(freedofs,:) = K(freedofs,freedofs) \ F(freedofs,:);
U(fixeddofs,:)= 0;
%%%%%%%%%% ELEMENT STIFFNESS MATRIX %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [KE]=lk
E = 1.;
nu = 0.3;
k=[ 1/2-nu/6   1/8+nu/8 -1/4-nu/12 -1/8+3*nu/8 ...
  -1/4+nu/12 -1/8-nu/8  nu/6       1/8-3*nu/8];
KE = E/(1-nu^2)*[ k(1) k(2) k(3) k(4) k(5) k(6) k(7) k(8)
                  k(2) k(1) k(8) k(7) k(6) k(5) k(4) k(3)
                  k(3) k(8) k(1) k(6) k(7) k(4) k(5) k(2)
                  k(4) k(7) k(6) k(1) k(8) k(3) k(2) k(5)
```

```
                     k(5) k(6) k(7) k(8) k(1) k(2) k(3) k(4)
                     k(6) k(5) k(4) k(3) k(2) k(1) k(8) k(7)
                     k(7) k(4) k(5) k(2) k(3) k(8) k(1) k(6)
                     k(8) k(3) k(2) k(5) k(4) k(7) k(6) k(1)];
```

B.2 88 Line Code

```
%%%% AN 88 LINE TOPOLOGY OPTIMIZATION CODE Nov, 2010 %%%%
function top88(nelx,nely,volfrac,penal,rmin,ft)
%% MATERIAL PROPERTIES
E0 = 1;
Emin = 1e-9;
nu = 0.3;
%% PREPARE FINITE ELEMENT ANALYSIS
A11 = [12  3 -6 -3;  3 12  3  0; -6  3 12 -3; -3  0 -3 12];
A12 = [-6 -3  0  3; -3 -6 -3 -6;  0 -3 -6  3;  3 -6  3 -6];
B11 = [-4  3 -2  9;  3 -4 -9  4; -2 -9 -4 -3;  9  4 -3 -4];
B12 = [ 2 -3  4 -9; -3  2  9 -2;  4  9  2  3; -9 -2  3  2];
KE = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]);
nodenrs = reshape(1:(1+nelx)*(1+nely),1+nely,1+nelx);
edofVec = reshape(2*nodenrs(1:end-1,1:end-1)+1,nelx*nely,1);
edofMat = repmat(edofVec,1,8)+repmat([0 1 2*nely+[2 3 0 1] -
   2 1],nelx*nely,1);
iK = reshape(kron(edofMat,ones(8,1))',64*nelx*nely,1);
jK = reshape(kron(edofMat,ones(1,8))',64*nelx*nely,1);
% DEFINE LOADS AND SUPPORTS (HALF MBB-BEAM)
F = sparse(2,1,-1,2*(nely+1)*(nelx+1),1);
U = zeros(2*(nely+1)*(nelx+1),1);
fixeddofs = union([1:2:2*(nely+1)],[2*(nelx+1)*(nely+1)]);
alldofs = [1:2*(nely+1)*(nelx+1)];
freedofs = setdiff(alldofs,fixeddofs);
%% PREPARE FILTER
iH = ones(nelx*nely*(2*(ceil(rmin)-1)+1)^2,1);
jH = ones(size(iH));
sH = zeros(size(iH));
k = 0;
for i1 = 1:nelx
  for j1 = 1:nely
    e1 = (i1-1)*nely+j1;
    for i2 = max(i1-(ceil(rmin)-1),1):min(i1+(ceil(rmin)-
          1),nelx)
      for j2 = max(j1-(ceil(rmin)-1),1):min(j1+(ceil(rmin)-
             1),nely)
        e2 = (i2-1)*nely+j2;
        k = k+1;
        iH(k) = e1;
        jH(k) = e2;
        sH(k) = max(0,rmin-sqrt((i1-i2)^2+(j1-j2)^2));
      end
    end
  end
end
H = sparse(iH,jH,sH);
Hs = sum(H,2);
```

```
%% INITIALIZE ITERATION
x = repmat(volfrac,nely,nelx);
xPhys = x;
loop = 0;
change = 1;
%% START ITERATION
while change > 0.01
  loop = loop + 1;
  %% FE-ANALYSIS
  sK = reshape(KE(:)*(Emin+xPhys(:)'.^penal*(E0-Emin)),64*nelx*nely,1);
K = sparse(iK,jK,sK); K = (K+K')/2;
U(freedofs) = K(freedofs,freedofs)\F(freedofs);
%% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
ce = reshape(sum((U(edofMat)*KE).*U(edofMat),2),nely,nelx);
c = sum(sum((Emin+xPhys.^penal*(E0-Emin)).*ce));
dc = -penal*(E0-Emin)*xPhys.^(penal-1).*ce;
dv = ones(nely,nelx);
%% FILTERING/MODIFICATION OF SENSITIVITIES
if ft == 1
  dc(:) = H*(x(:).*dc(:))./Hs./max(1e-3,x(:));
elseif ft == 2
  dc(:) = H*(dc(:)./Hs);
  dv(:) = H*(dv(:)./Hs);
end
%% OPTIMALITY CRITERIA UPDATE OF DESIGN VARIABLES AND PHYSICAL DENSITIES
l1 = 0; l2 = 1e9; move = 0.2;
while (l2-l1)/(l1+l2) > 1e-3
  lmid = 0.5*(l2+l1);
  xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-dc./dv/lmid)))));
  if ft == 1
    xPhys = xnew;
  elseif ft == 2
    xPhys(:) = (H*xnew(:))./Hs;
  end
  if sum(xPhys(:)) > volfrac*nelx*nely, l1 = lmid; else l2 = lmid; end
end
change = max(abs(xnew(:)-x(:)));
x = xnew;
%% PRINT RESULTS
fprintf(' It.:%5i Obj.:%11.4f Vol.:%7.3f ch.:%7.3f\n',loop,c, ...
  mean(xPhys(:)),change);
%% PLOT DENSITIES
colormap(gray); imagesc(1-xPhys); caxis([0 1]); axis equal; axis off; drawnow;
end
```

B.3 104 Line Modified Code

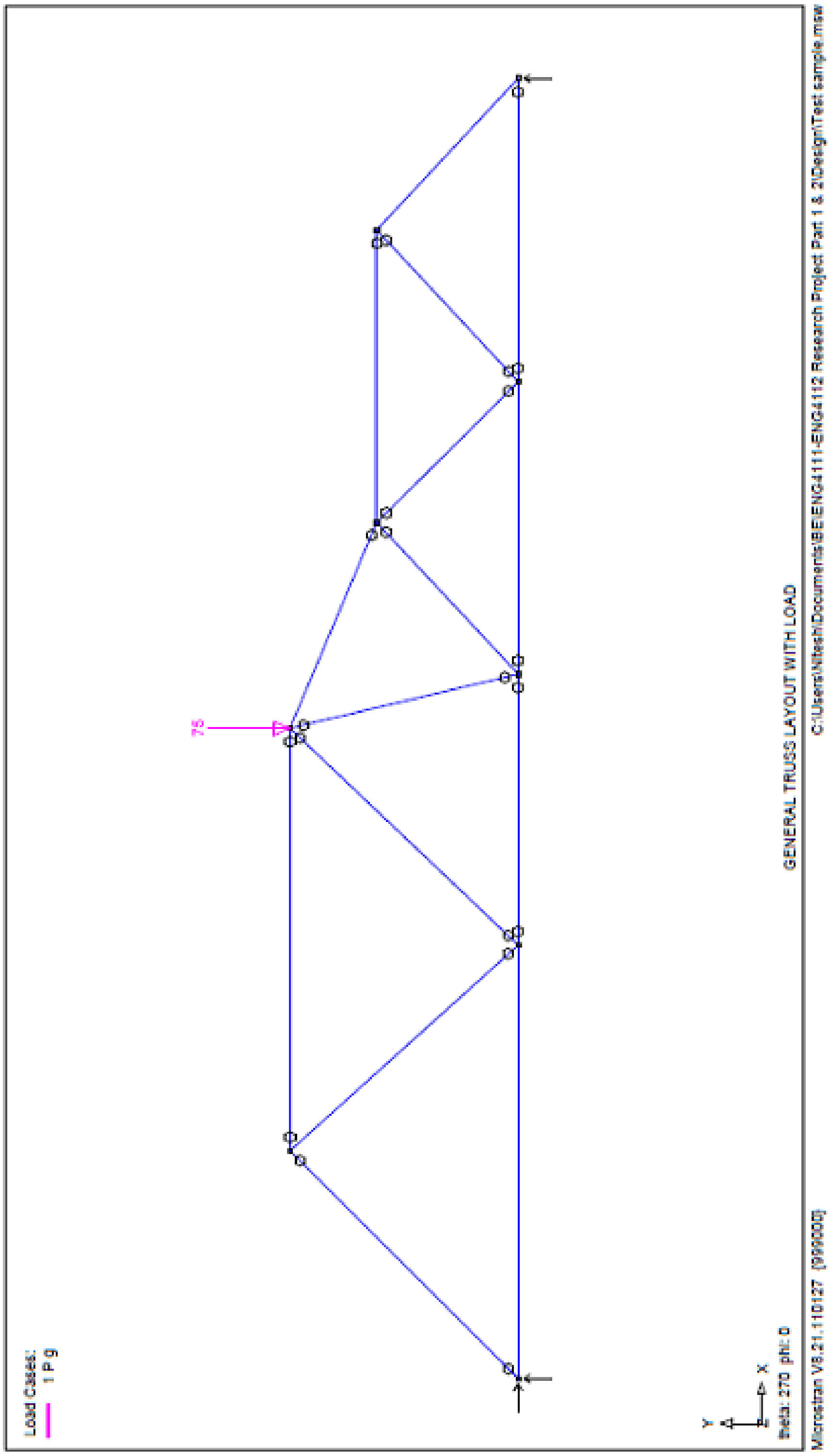
```
%%%% AN 105 LINE TOPOLOGY OPTIMIZATION CODE Aug, 2011 %%%%
function beam(nelx,nely,xv,yv,volfrac,penal,rmin)
%% MATERIAL PROPERTIES
E0 = 1;
Emin = 1e-9;
nu = 0.3;
%% DEFINE PASSIVE ELEMENTS
passive = zeros(nely,nelx);
passive(yv,xv)=1;
%% PREPARE FINITE ELEMENT ANALYSIS
A11 = [12  3 -6 -3;  3 12  3  0; -6  3 12 -3; -3  0 -3 12];
A12 = [-6 -3  0  3; -3 -6 -3 -6;  0 -3 -6  3;  3 -6  3 -6];
B11 = [-4  3 -2  9;  3 -4 -9  4; -2 -9 -4 -3;  9  4 -3 -4];
B12 = [ 2 -3  4 -9; -3  2  9 -2;  4  9  2  3; -9 -2  3  2];
KE = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]);
nodenrs = reshape(1:(1+nelx)*(1+nely),1+nely,1+nelx);
edofVec = reshape(2*nodenrs(1:end-1,1:end-1)+1,nelx*nely,1);
edofMat = repmat(edofVec,1,8)+repmat([0 1 2*nely+[2 3 0 1] -2 -
   1],nelx*nely,1);
iK = reshape(kron(edofMat,ones(8,1))',64*nelx*nely,1);
jK = reshape(kron(edofMat,ones(1,8))',64*nelx*nely,1);
% DEFINE LOADS AND SUPPORTS
F = sparse(nelx*(nely+1)+2,1,-1,2*(nely+1)*(nelx+1),1);
U = zeros(2*(nely+1)*(nelx+1),1);
fixeddofs =
   [2*(nely+1)*((nelx/14)+1),2*((13*nelx/14)+1)*(nely+1)-
   1,2*((13*nelx/14)+1)*(nely+1)];
alldofs = [1:2*(nely+1)*(nelx+1)];
freedofs = setdiff(alldofs,fixeddofs);
%% PREPARE FILTER
iH = ones(nelx*nely*(2*(ceil(rmin)-1)+1)^2,1);
jH = ones(size(iH));
sH = zeros(size(iH));
k = 0;
for i1 = 1:nelx
  for j1 = 1:nely
    e1 = (i1-1)*nely+j1;
    for i2 = max(i1-(ceil(rmin)-1),1):min(i1+(ceil(rmin)-
       1),nelx)
      for j2 = max(j1-(ceil(rmin)-1),1):min(j1+(ceil(rmin)-
         1),nely)
        e2 = (i2-1)*nely+j2;
        k = k+1;
        iH(k) = e1;
        jH(k) = e2;
        sH(k) = max(0,rmin-sqrt((i1-i2)^2+(j1-j2)^2));
      end
    end
  end
end
H = sparse(iH,jH,sH);
Hs = sum(H,2);
%% INITIALIZE ITERATION
```

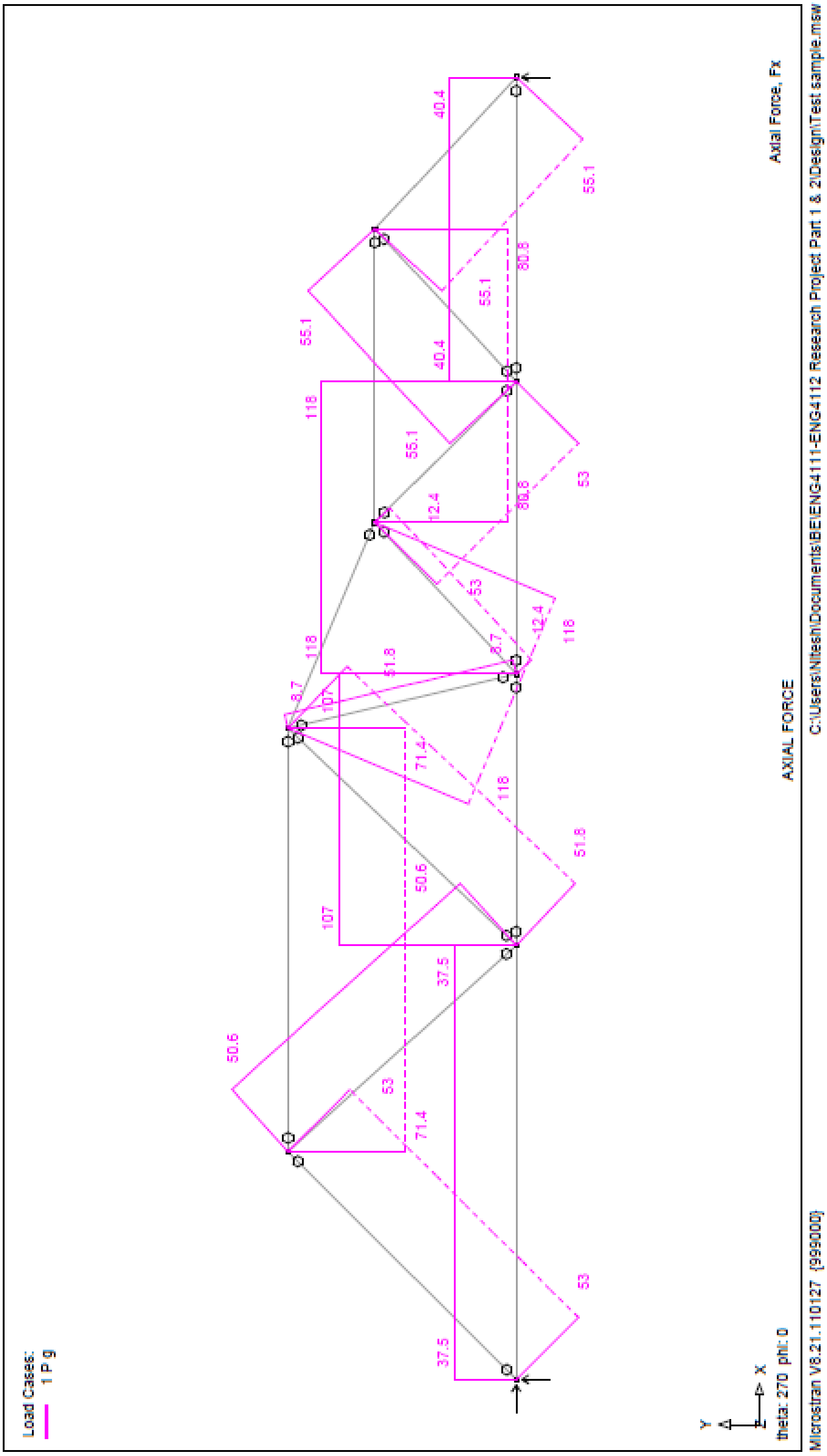
```
x = repmat(volfrac,nely,nelx);
xPhys = x;
loop = 0;
change = 1;
%% START ITERATION
while change > 0.01
  loop = loop + 1;
  %% FE-ANALYSIS
  sK = reshape(KE(:)*(Emin+xPhys(:)'.^penal*(E0-Emin)),64*nelx*nely,1);
  K = sparse(iK,jK,sK); K = (K+K')/2;
  U(freedofs) = K(freedofs,freedofs)\F(freedofs);
  %% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
  ce = reshape(sum((U(edofMat)*KE).*U(edofMat),2),nely,nelx);
  c = sum(sum((Emin+xPhys.^penal*(E0-Emin)).*ce));
  dc = -penal*(E0-Emin)*xPhys.^(penal-1).*ce;
  dv = ones(nely,nelx);
  %% FILTERING/MODIFICATION OF SENSITIVITIES
  dc(:) = H*(x(:).*dc(:))./Hs./max(1e-3,x(:));
  %% OPTIMALITY CRITERIA UPDATE OF DESIGN VARIABLES AND PHYSICAL DENSITIES
  l1 = 0; l2 = 1e9; move = 0.2;
  while (l2-l1)/(l1+l2) > 1e-3
    lmid = 0.5*(l2+l1);
    xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-dc./dv/lmid)))));
    xPhys = xnew;
    xPhys(passive==1)=0;
    xPhys(passive==2)=1;
    if sum(xPhys(:)) > volfrac*nelx*nely, l1 = lmid; else l2 = lmid; end
  end
  change = max(abs(xnew(:)-x(:)));
  x = xnew;
  %% PRINT RESULTS
  fprintf(' It.:%5i Obj.:%11.4f Vol.:%7.3f ch.:%7.3f\n',loop,c, ...
    mean(xPhys(:)),change);
  objhis(loop) = c;
  %% PLOT DENSITIES
  colormap(gray); imagesc(1-xPhys); caxis([0 1]); axis equal; axis off; drawnow;
  if mod(loop,10)==0
    tfname = sprintf('T%04i.pdf',loop);
    print('-dpdf',tfname);
  end
end
%% FINAL OUTPUTS
% print the final topology
tfname = sprintf('T%04i.pdf',loop);
print('-dpdf',tfname);
% write the history of the objectve function values to the file his.csv
fh=fopen('his.csv','w');
for i=1:loop
  fprintf(fh,'%i , %f \n',i,objhis(i));
```

```
end
fclose(fh);
% plot the evolution of the objective function and print it to his.pdf
figure;
plot(1:loop,objhis,'r','LineWidth',2);
ylabel('objective function'); xlabel('Iteration');
print('-dpdf','his.pdf');
```

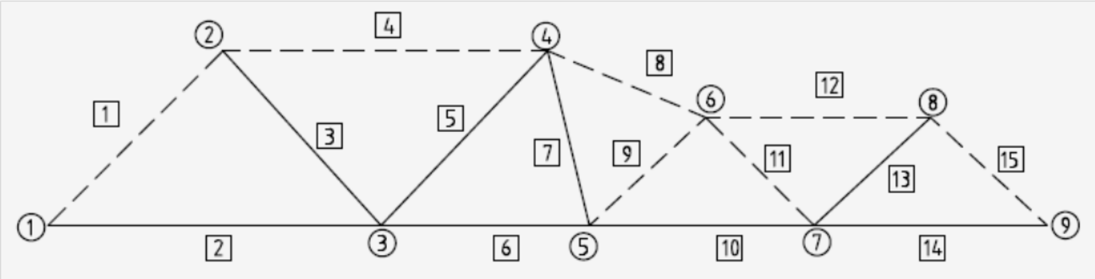
Appendix C – Beam Analysis & Calculations

C.1 Microstran Output




Load Cases:
1 P g
Axial Force, Fx
AXIAL FORCE
theta: 270 phi: 0
Microstran V8.21.110127 (999000)
C:\Users\Nitesh\Documents\BE\ENG4111-ENG4112 Research Project Part 1 & 2\Design\Test sample.msw

C.2 STM design calculations



Member	Force kN
1	53 (C)
2	37.5 (T)
3	50.6 (T)
4	71.43 (C)
5	51.8 (C)
6	407.4 (T)
7	8.7 (T)
8	118.44 (C)
9	12.4 (C)
10	118.27 (T)
11	53 (C)
12	80.77 (C)
13	55.1 (T)
14	40.4 (T)
15	55.1 (C)

STRUT 1 – 2 $\boxed{1}$

w = 40

Ω = 40, based on 20mm cover

$d_c = \sqrt{40^2 + 40^2} = 56.6$

$l_b = \sqrt{z^2 + a^2} - d_c$

$= \sqrt{210 + 210^2} - 56.6 = 240.4$

$\Theta = \tan^{-1} 210/210 = 45°$

$\propto = \tan^{-1} 1/2 = 26.57°$

$\tau_{b/2} = 26.5 \frac{\sin 26.57}{\cos 26.57} = 13.28$ kN

$\tau_b^* = 2 * 13.285 = 26.57$ kN

$\tau_{bcr} = 0.7 (100) 240.4 * 2.276 * 10^3 38.31$ kN

Since $\tau_b^* > 0.5 \tau_{bcr}$ transverse reo required

$B_s = 1/ 1 + 0.66 \cot^2 95 = 0.602$

Strength of Strut

$\text{Ø}_s t = 0.6$

$A_c = 56.6 * 100 = 5660\text{mm}^2$

$= (0.6)(0.602)(0.9)(40)(5660)(10^{-3})$

$= 73.6$ kN > 53 kN **OK**

<u>Tie 1 – 3</u> $\boxed{2}$

$\frac{37.5\ \text{kN}}{0.8*5.00} = 93.75\text{mm}^2 = \text{Ast}$

Use 2/N10

<u>Tie 2 – 3</u> $\boxed{3}$

$\frac{50.6}{0.8*25} *10^3 = 253\text{mm}^2$

Use 5/R8 or 2/N10

<u>Strut 2 – 4</u> $\boxed{4}$

$\Theta = \tan^{-1}(210/190) = 47.86$

$B_b = 1/ 1 + 0.66 \cot^2 47.86$

$= 0.65$

Design Strength = $((0.6)(0.65)(0.9)(40)(5660)(10^{-3})$

$= 79.47$ kN > 71.43 kN **OK**

$l_b = \sqrt{56.6^2 + 390^2} - 56.6 = 337.49$

$\tau_{b/2} = 35.715 \frac{\sin 26.57}{\cos 26.57} = 17.8575$ kN

$\tau_b^* = 35.715$ kN

$\tau_{bcr} = (0.7)(100)(337.49)(2.276)(10^{3)\ =}\ 38.31$ kN

Since $\tau_b^* > 0.5\ \tau_{bcr}$ transverse reo required

<u>Strut 3- 4</u> $\boxed{5}$

$\text{Ø}_1 = \tan^{-1}(210/200) = 47.4°$

$\text{Ø}_2 = 180- 46.4- 47.26 = 85.74$

$B_s = 1/ 1 + 0.66 \cot^2 46.4°$

$= 0.626$

Design Strength = $(0.6)(0.626)(0.9)(40)(5660)(10^{-3})$

$= 76.48$ kN > 51.8 kN OK

<u>Tie 3 – 5</u> $\boxed{6}$

$107.1/ (0.8)(500) = 267.75\text{mm}^2 = \text{Ast}$

Use 4/N10

Tie 4 – 5 $\boxed{7}$

$\frac{8.7}{0.8*25}$ *10^3

Use 1/R8

Strut 4 – 6 $\boxed{8}$

$Ø_1$ = $\tan^{-1}$ 80/190 = 22.83°

$Ø_2$ = $\tan^{-1}$ 210/140 = 56.31°

= 56.31 – 22.83 = 34.48°

B_s = 1/ 1+ 0.66 $\cot^2$ 34.48

= 0.399

Design Strength = (0.6)(0.399)(0.9)(40)(5660)(10^{-3})

= 48.72 kN < 107.1 kN **OK**

107.1 – 48.77 = 58.38 kN

$\frac{58.38}{0.8*500}$ *10^3 = 145.95

Use 4/N10

Strut 6 – 5 $\boxed{9}$

$Ø_1$ = $\tan^{-1}$ 130/140 = 42.88°

$Ø_2$ = $\tan^{-1}$ 130/150 = 68.96°

$Ø_3$ = 180 – 42.88 – 68.96 = 68.16 °

B_s = 1/ 1+ 0.66 $\cot^2$ 42.88°

= 0.566

Design Strength = (0.6)(0.566)(0.9)(40)(5660)(10^{-3})

= 69.72 kN > 12.4 kN **OK**

Tie 5 – 7 $\boxed{10}$

$\frac{118.27}{0.8*500}$ *10^3 = 295.675mm^2

Use 2/ N10

Strut 6 – 7 $\boxed{11}$

$Ø_1$ = $\tan^{-1}$ 130/130 = 45°

$Ø_2$ = $\tan^{-1}$ 130/140 = 42.87°

$Ø_3$ = 180 – 45 – 42.87 = 92.12 °

B_s = 1/ 1+ 0.66 $\cot^2$ 45°

= 0.602

Design Strength = (0.6)(0.602)(0.9)(40)(5660)(10^{-3})

= 73.65 kN > 53 kN **OK**

Strut 6 – 8 $\boxed{12}$

B_s = 1/ 1+ 0.66 $\cot^2$ 42.87°

= 0.566

Design Strength = $(0.6)(0.566)(0.9)(40)(5660)(10^{-3})$

$= 69.25$ kN < 82.77 kN **OK**

80.71 – 69.25 = 11.52kN

$\frac{11.52}{0.8*250} *10^3 = 57.6$

Use 2/ R8

Tie 7 – 8 $\boxed{\textbf{13}}$

$\frac{55.1}{0.8*500} *10^3 = 137.75mm^2$

Use 2/ N10

Tie 7 – 9 $\boxed{\textbf{14}}$

$\frac{40.4}{0.8*500} *10^3 = 101mm^2$

Use 2/ N10

Strut 8 – 9 $\boxed{\textbf{15}}$

$Ø = \tan^{-1} 130/140 = 42.88°$

$B_s = 1/\ 1+ 0.66 \cot^2 42.88°$

$= 0.566$

Design Strength = $(0.6)(0.566)(0.9)(40)(5660)(10^{-3})$

$= 69.25$ kN > 55.1 kN **OK**

C.3 Conventional design calculations

The following figures are from Excel spreadsheet that was set-up to design the beam using the provisions of section 8, AS3600:2009.

Table C.1 Beam reinforcement for bending.

Beam Reinforcement Calculations

M^* =	22.5	kNm	
ϕ =	0.8		Table 2.3
D =	250	mm	
cover =	15	mm	Include for Ligs
d =	230	mm	
B =	100	mm	
f_{sy} =	500	MPa	
f'_c =	40	MPa	
f'_{cf} =	3.79	MPa	Flexural Tensile strength of Concrete (Cl 6.1.1.2)
γ =	0.766		
Ec =	31975	MPa	

Estimate of Area of Steel

Ast =	268	mm^2	from equation $\frac{c2 - (c2^2 - 4{*}c1{*}c3)^{0.5}}{2{*}c1'}$

Minimum Area of Steel

Ast.min =	41	mm^2	Minimum Strength requirements (Cl 8.1.4.1)
Ast =	268	mm^2	
ku =	0.224		
γ.ku.d =	39	mm	
Bar size:	N	12	
Minimum Bars Required:	3		

Actual Area of Steel to be used:

Size	N 10		
Number =	4		
Ast =	314	mm^2	**Reo OK**
Ignoring Compression Steel			
ku =	0.26		**OK, Under-reinforced**
γ.ku.d =	46.20		
ϕ Mu =	26.0	kNm	
	OK		

Table C.2 Shear reinforcement at right end of beam (250mm depth)

Shear Reinforcement for concrete Beam (or slab) AS3600 Clause 8.2

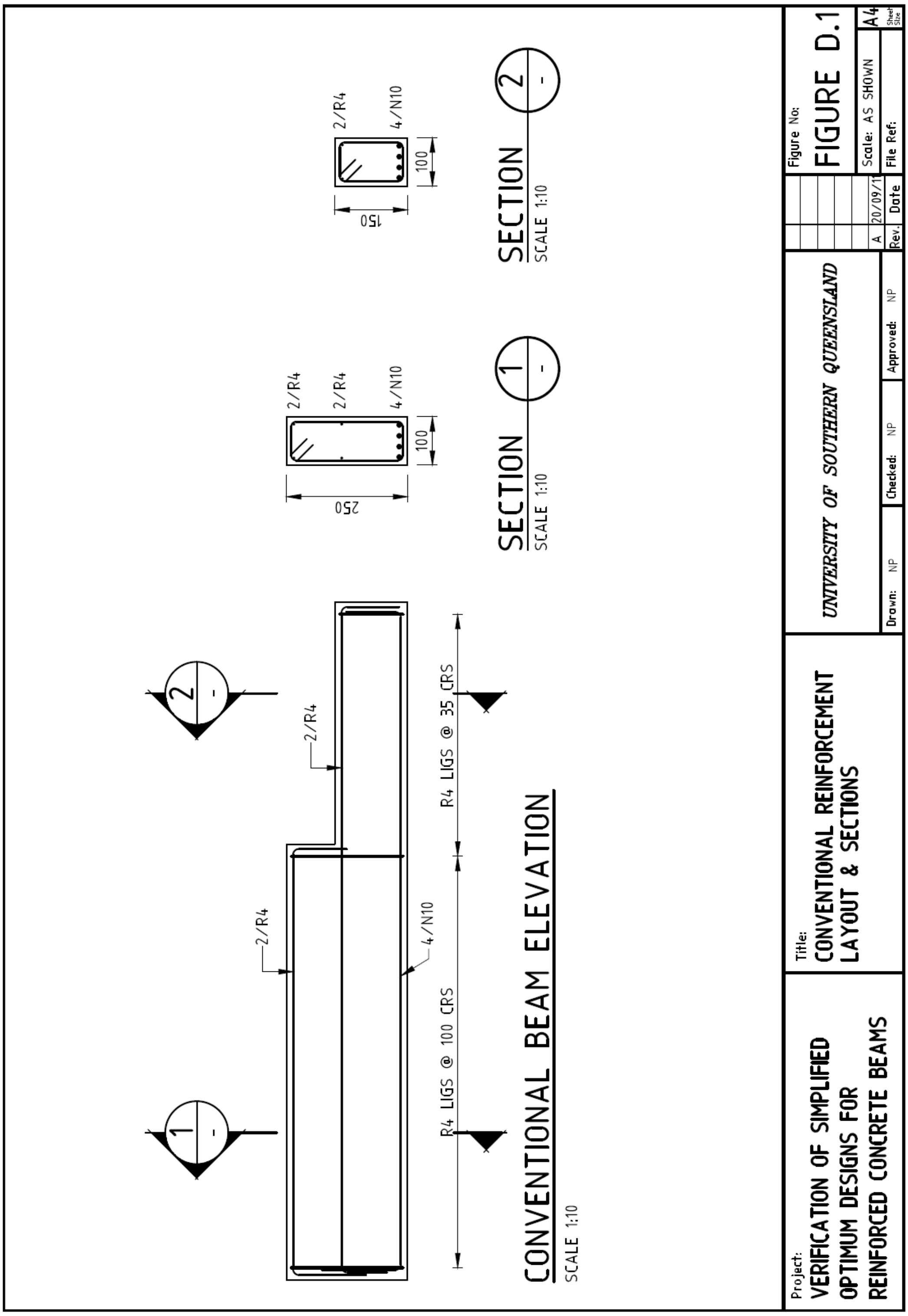
Description :

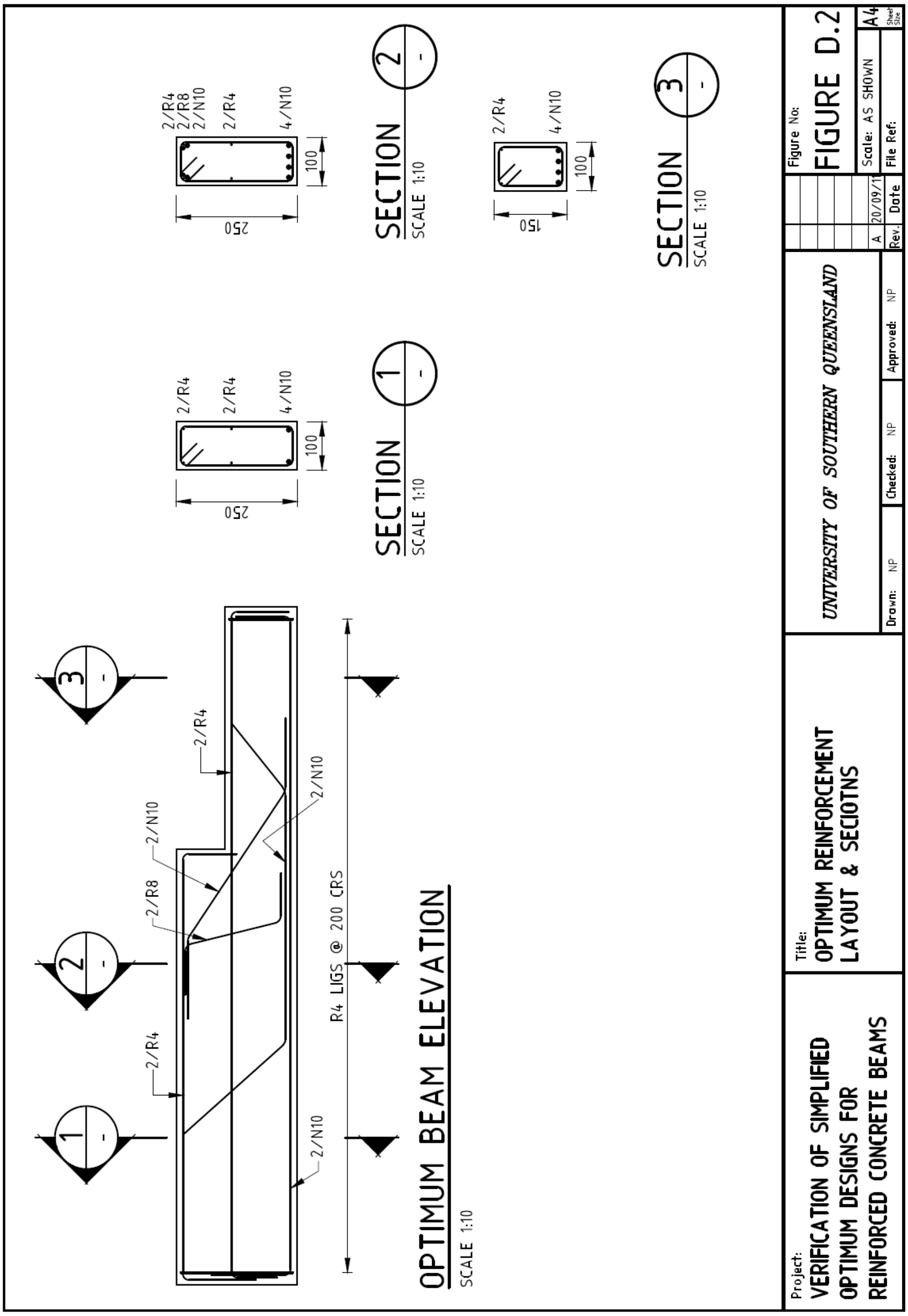
ϕ	=	0.7	
V*	=	37.5 kN	
d_o	=	250 mm	Distance from extreme fibre to centre of outer layer of Tensile steel
D	=	230 mm	Overall depth of section
b_v	=	100 mm	Shear width of section (use 1000mm for a slab)
A_{st}	=	314 mm^2	
f 'c	=	40 MPa	
f_{sy}.f	=	250 MPa	For 10mm use 250MPa, For >=N12 use 500MPa
Shear Reo Required?			**Yes, Shear Reo is required**
β_1	=	1.485	(Cl 8.2.7.1)
β_2	=	1	
β_3	=	1	
ϕ Vuc	=	20.7 kN	(Cl 8.2.7.1) **Provide Max Shear Reo if designing a SLAB**
0.5 ϕ Vuc	=	10.3 kN	
Minimum Shear Reo Required?			**No, need to provide more than minimum Reo**
$\phi V_{u\,min}$	=	31.2 kN	(Cl 8.2.9)
Asv.min/s	=	0.14 mm^2/mm	
Stirrups	=	4	
No. of legs	=	2	
req Spacing	=	180 mm	
Max spacing	=	**See Below mm**	(Cl 8.2.12.2)
Amount of Shear Reo Required			**This section governs the design (Cl 8.2.10)**
$\phi V_{u.max}$	=	140.0 kN	(Cl 8.2.6)
θ v	=	31 degrees	Generally 30° and conservatively up to 45° (Cl 8.2.10)
Asv/s	=	0.23 mm^2/mm	
Stirrups	=	4	
No. of legs	=	2	
req Spacing	=	109 mm	
Max spacing	=	**115 mm**	(CL8.2.12.2)

Table C.3 Shear reinforcement at left end of beam (150mm depth)

Shear Reinforcement for concrete Beam (or slab)			**AS3600 Clause 8.2**
Description :			
ϕ	=	0.7	
V*	=	37.5 kN	
d_o	=	130 mm	Distance from extreme fibre to centre of outer layer of Tensile steel
D	=	150 mm	Overall depth of section
b_v	=	100 mm	Shear width of section (use 1000mm for a slab)
A_{st}	=	314 mm^2	
f 'c	=	40 MPa	
$f_{sy}.f$	=	250 MPa	For 10mm use 250MPa, For >=N12 use 500MPa
Shear Reo Required?			**Yes, Shear Reo is required**
β_1	=	1.617	(Cl 8.2.7.1)
β_2	=	1	
β_3	=	1	
ϕ Vuc	=	14.5 kN	(Cl 8.2.7.1) **Provide Max Shear Reo if designing a SLAB**
0.5 ϕ Vuc	=	7.3 kN	
Minimum Shear Reo Required?			**No, need to provide more than minimum Reo**
$\phi V_{u\ min}$	=	20.0 kN	(Cl 8.2.9)
Asv.min/s	=	0.14 mm^2/mm	
Stirrups	=	4	
No. of legs	=	2	
req Spacing	=	180 mm	
Max spacing	=	**See Below mm**	(Cl 8.2.12.2)
Amount of Shear Reo Required			**This section governs the design (Cl 8.2.10)**
$\phi V_{u.max}$	=	72.8 kN	(Cl 8.2.6)
θ v	=	35 degrees	Generally 30° and conservatively up to 45° (Cl 8.2.10)
Asv/s	=	0.71 mm^2/mm	
Stirrups	=	4	
No. of legs	=	2	
req Spacing	=	36 mm	
Max spacing	=	**75 mm**	(CL8.2.12.2)

Appendix D – Beam Reinforcement





OPTIMUM BEAM ELEVATION
SCALE 1:10
R4 LIGS @ 200 CRS
2/R4
2/R8
2/N10
2/R4
2/N10
2/N10
SECTION 1
SCALE 1:10
250
100
2/R4
2/R4
4/N10
SECTION 2
SCALE 1:10
250
100
2/R4
2/R8
2/N10
2/R4
4/N10
SECTION 3
SCALE 1:10
150
100
2/R4
4/N10
Project:
VERIFICATION OF SIMPLIFIED
OPTIMUM DESIGNS FOR
REINFORCED CONCRETE BEAMS
Title:
OPTIMUM REINFORCEMENT
LAYOUT & SECIOTNS
UNIVERSITY OF SOUTHERN QUEENSLAND
Drawn: NP
Checked: NP
Approved: NP
A 20/09/11
Rev. Date
Figure No:
FIGURE D.2
Scale: AS SHOWN
File Ref:
A4
Sheet Size

Appendix E – Testing Photos

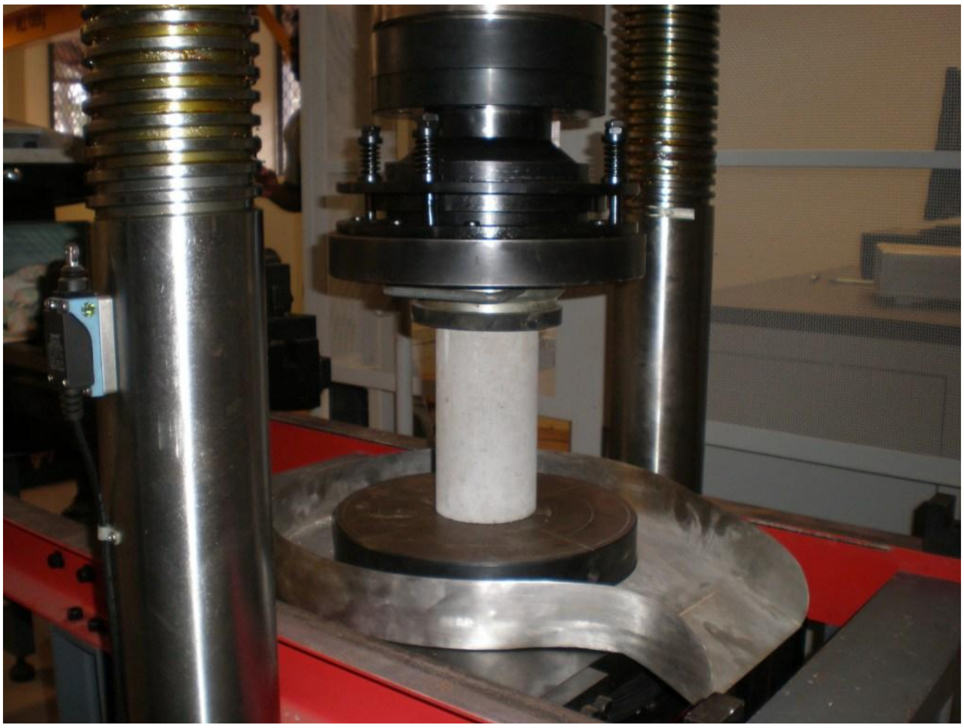


Photo 1 – Specimen 1 (100Øx200) loaded in the machine

Photo 2 – Specimen 1 at failure

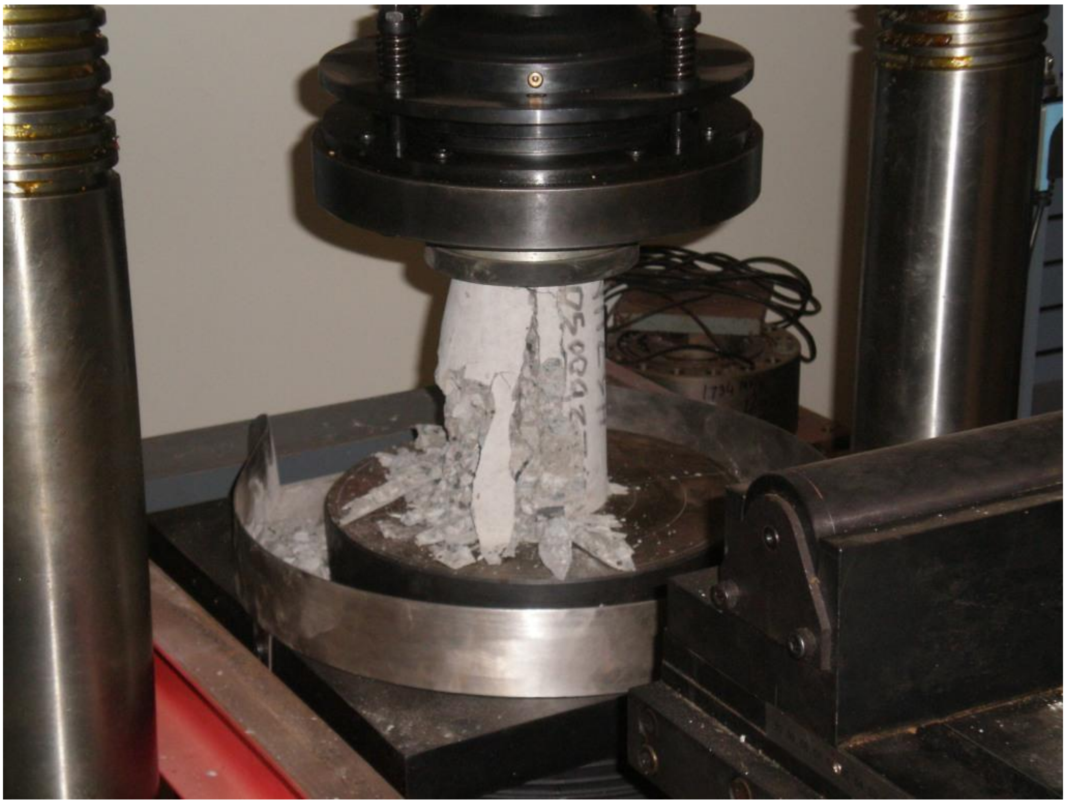


Photo 3 – Specimen 2 at failure

Photo 4 – Specimen 3 at failure

Photo 5 – Initial cracking in the optimum beam

Photo 6 – Further cracking in the optimum beam


Photo 7 – Failure of optimum beam


Photo 8 – Optimum beam at failure


Photo 9 – Initial cracking in conventional beam


Photo 10 – Initial cracking in conventional beam at depth discontinuity

Photo 11 – Failure of conventional beam

Photo 12 – Conventional beam at failure