University of Southern Queensland

FACULTY OF ENGINEERING AND SURVEYING

**Bachelor of Engineering Thesis**

# PID TUNING CONTROLLER
USING
INTERNAL MODEL CONTROL METHOD

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In fulfilment of Course:
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## Abstract

There are three design methods commonly used nowadays for second order systems, the method of state space design, lag/lead filter compensation and finally the Internal Model Control (IMC) method [3, 8, 11]. The latter design is utilised in control of current in DC electrical motors (servo system) and many other second order systems. The internal model control design has capabilities of achieving high performance. The IMC technique, in theory, substitutes the closed loop system with a low-pass filter of the same order as the system itself. In this case the plant (process) under control is always second order, so if an exact representation of the plant (process model) is given, the step response for a change in the reference signal would look as a low-pass filter step response.

In practice, however, process-model mismatch is common [1,2,3], that is to say designers never achieve a 100% accurate model, and thus IMC method is usually chosen and implemented in the control design scheme. Even if the tracking of the reference input is adequate, most control systems are subjected to disturbances, and IMC is not only designed to optimally suppress these effects, it also tells us that its strategy and concepts that it embraces are clearly very powerful techniques.

The conventional IMC method often involves inversion of a process, which is often difficult or totally impossible [3, 4]. In other words the potential to achieve perfect control with model-based design is dependent on constraints with process operations. Therefore models which will contain some degree of error or to some extent cannot be invertible results in perfect control not being realised. However, for the purpose of control strategy specification, controller design and control system analysis, models that can replicate the dynamic trends of the target processes are usually sufficient. The internal model control scheme has been widely applied in the field of process control. This is due to its simple and straightforward controller design procedure as well as its good disturbance rejection capabilities and robustness properties.

In practice, the tuning of conventional PID controllers can be very time consuming [2, 16], as the effect of the controller coefficients on the control performance cannot be easily described. In addition, the optimization of controller parameters based on the predefined fitness functions is computationally expensive and the design of this fitness functions is difficult because of the involved tuning parameters. In IMC schemes a controller is designed in series with a low pass filter and utilization of this design method means that a controller design involves tuning only one parameter, namely the filter constant. From that, the corresponding parameters of the conventional controller can be computed and thus makes the design simple and reliable.

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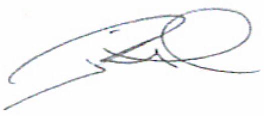

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# Certificate

I certify that the ideas, methods, simulations and planning in this case study are the work of my own. In this thesis, it is important to have specified relevant material in this form of documentation as has been done solely to note areas of related work and concepts. It has been specified that extensive work in this study has been done by quite a large number of disciplines and as acknowledged, I have to certify the importance of their work to help build the ideas of mine.

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*Date:* _____________

*Sign: Don Moh*

# Acknowledgements

It is a pleasure to acknowledge my supervisor Dr Paul Wen in coming to this far with this project. Great thanks also goes to all those I have put in reference section of this thesis. They have done tremendous amount of work in their research and development of ideas which has helped to accomplish the mission and philosophy of all the work done on this thesis. It will be odd to forget friends and acquaintances who have had been in this position before, gave me guidance after having discussed the subject with them at length and in detail. Thank you all, you deserve all.

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# 1.0 Chapter One: Introduction

## 1.1 Introduction

In theory, control systems can be condensed into a simple set of tasks- measure your system, make a decision based on the input, send a control signal to adjust your system to expected operation, and then repeat. In reality or so to say in practice, accomplishing each of these tasks can grow a lot more complex in many ways [12], once you take into account the types of measurement you need to make to get your inputs, the algorithms and the logic needed to make the decisions, the distributed nature of many control systems, the amount of I/O to manage, the speed of the control loop, and so forth. If your system is simple and digital, you may be able to find an off-the-shelf inexpensive hardware controller to do the job. But as the system grows and requires more functionality, you may want to re-evaluate such controllers and choose tools that may meet your existing system needs, and then scale to address future changes or technologies as they arise.

In the past, numerous organisations have been able to develop large and yet simple control systems with inexpensive hardware controllers. However, as the trends are pushing for more integrated systems and solutions, new control systems are being developed. It is the intent of this document to introduce concepts through development of some of the many attractive forms of control algorithms in today's exploding world of control systems. While there are abundant engineering software control tools to solve various simple control applications, few of them manage to solve all aspects of these new integrated control systems. This document is to introduce the theory of control systems, with particular emphasis on the applicability of the results to practical problems. Also, as for any theory of systems oriented towards practical applications, robustness is essential and will be the underlying concept throughout the development of the theory and relative results.

In today's practical control problems, it is highly indispensable to consider each phase of the control problem for robust identification so as to produce theoretical results that are closely related to the computational and experimental aspects of the control problem. The ever growing control system industry has indeed shown depth in accomplishing more complex tasks of the control problem, and the idea behind this document, is to keep the campaign alive by looking closely into two of the many important control schemes; namely:

- Proportional-Integral-Derivative (**PID**)
- Internal Model Control (**IMC**)

Proportional-Integral-Derivative (**PID**) control is certainly the most widely used control strategy today. It is estimated that over 90% of control loops employ PID control, quite often with the derivative gain set to zero (PI control). Over the last few years, a great deal of academic and industrial effort has been putting much attention on improving PID control, primarily in the area of tuning rules, identification schemes, and adaptation techniques. It is appropriate at this time to consider the stat of the art in PID control as well as new developments in this control approach.

In recent years, model-based control has lead to improved control loop performance. One of the clearest model based technique is Internal Model Control (**IMC**) and has proved to provide an effective framework for robust control of various classes of systems. Unlike many other developments of modern control theory, IMC was widely accepted by control engineering practitioners. It is therefore quite natural to attempt to extend IMC concepts to various classes of systems. It is thus in here where we utilize IMC concepts to servo system in order to explore the advantages it brings to their control.

### 1.2 Objective

The main objective of this document is to point out the development of two control methods, IMC and PID, and their application in the industry. Control systems are today pervasive, they appear practically everywhere in our homes, in industry, in communications, information technologies, etc. Process control continues to be a vital, important field with significant unresolved research problems and challenging industrial applications. The present trends in the process control design demand an increasing degree of integration. Furthermore, increasing problems with interactions, process non-linearity’s, operating constraints, time delay, uncertainties, and significant dead-times consequently lead to the necessity to develop more sophisticated control strategies capable to be incorporated into the software package following the present software engineering lines.

Control system design is currently undergoing an interesting phase of development and implementation in industrial plants. It is thus the intent of this document to further explore two control mechanisms in IMC and the convectional PID as a basis of control to a servo system. Hence, control performance of the two control schemes shall be explored by analysing in depth their methods of tuning, their adaptability to robust performance and their suitability to industrial applications. Algorithms for deriving control actions will be specified and tested in a **MATLAB/SIMULINK** environment. The objective is to specify the information, which will serve for process model derivation and parameter identification. Therefore, theoretical work on design of algorithms for control parameter tuning will then be coupled with implementing the model design techniques in software.

After a control system is installed in the plant, controller tuning is often required to determine suitable controller settings. Hence it is crucial to continuously re-tune the controller parameters if the process characteristics change in significant and unanticipated ways. Thus the development of simple, effective methods for updating controller setting to compensate for changing process conditions shall be established in this document and would be beneficial for both model-based IMC and convectional PID controllers.

### 1.3 Chapter Modules

The dissertation shall comprise eight core chapters. Each chapter has a specific relevant area to cover which include subtopics. All relevant materials to each chapte are subject change for improvement as this is only a partial draft of the thesis. The chapter contents are described below.

### 1.3.1 Chapter One

Chapter one is looking into introducing the project. This will go deeply into an overview of the project and why it is important. It is also set to give the objectives of the project and follows through to highlight important aspects of interest to the reader.

### 1.3.2 Chapter Two

This chapter is looking into detailing or giving all aspects involved with IMC. This means that we expect to have a clear detailed communication on the principles, properties of internal model control. The chapter shall also reveal the designing techniques involved and also brush through its area of applicability.

### 1.3.3Chapter Three

This chapter will introduce the servo system used as a plant for this project. It will reinforce on the mathematical modelling of the plant and give out the model as a transfer function for the servo system utilised in designing for IMC and PID implementations.

### 1.3.4 Chapter Four

This section's primary objective is to introduce the convectional PID and goes on to explain briefly the control actions provided by each of the PID parameters. It will also introduce parameter setting principles and show how control is provided to the servo system and follows through with some results and analysis based on the simulation outputs.

### 1.3.5 Chapter Five

This chapter's primary objective is to show how an IMC controller is designed based on the transfer function of the plant (servo system). It will elaborate extensively how we choose a filtering subsystem to run with the IMC controller. It also details the modelling and some implementation on a SIMULINK platform and concludes by summing up the results and analysis from the simulation outputs.

### 1.3.6 Chapter Six

This chapter will utilise the principles of IMC to set PID parameters. It will further explore the tuning of the parameters and their relevance to providing control to a servo system. Implementations in SIMULINK shall also provide assistance in results and analysis.

### 1.3.7 Chapter Seven

This chapter will provide a comparison in control performance between a model-based IMC and a convectional PID upon providing control to a servo system. It will also explore their differences when subject to external disturbances or uncertainties. Implementations shall be explored as a guide to further analyse the results.

### 1.3.8 Chapter Eight

This chapter compress all the material subject to discussion from all chapters into a form of a summary. It also looks into relating the future aspects of the control problem in to the ever-growing control system design community based on IMC and PID relativity.  It intends to extend the knowledge of IMC to other control framework structures and briefly outlines areas of utmost interest for future applications of the theory and related concepts.

# 2.0 Chapter Two: Internal Model Control

## 2.1 Literature Review

For a large number of single-input single-output (SISO) models typically used in process industries, the Internal Model Control (IMC) design procedure is shown to lead to PID controllers occasionally augmented with a first order lag. The IMC scheme has been widely applied in the field of process control. This is due to its simple and straight forward controller design procedure as well as its good disturbance rejection capabilities and robustness properties. In primary context IMC has been widely applicable to linear processes. This document will show how IMC have gained popularity in process control.

Internal Model control scheme has a lot much advantage in the design of control systems. The stability of IMC is only dependent upon that of the controller and the nominal plant. Even if the Internal Model Control system has control input saturation, stability of IMC is only dependent upon that of the controller and the plant. There are three control methods commonly used today for second order systems, the method of lead/lag filter compensation, the Internal Model Control method and the state space design. In practice, when a control systems designer is confronted with a system to control, one would choose the IMC method over lead/lag filter compensation and state space design. IMC has the main advantage in principle of its operation; in theory IMC substitutes the closed loop system with a low-pass filter of the same order as the system itself. In this case the plant under control is always second order such that if the designer has the plant model as an exact representation of the plant in the operating system, then a step response for a change in reference signal would look as a low pass filter step response

In many control systems particular emphasis has been put on the question of robustness and the design for robustness is always primary as it will be shown in the contents of this document. The IMC structure's conceptual usefulness lies in the fact that it allows the designer to concentrate on the controller design without having to be concerned with the control system's stability provided that the plant model is perfect.

## 2.2 Internal Model Control Principle

A control system comprise the process to be controlled and a control device chosen by a designer, which computes the control input so as to convey the desired control behaviour to the control system. Internal Model Control system are characterised by a control devise comprising the controller and of a simulation model of the process, the Internal Model. This idea leads us to state the Internal Model Control principle from the characterization above. It states that control can be achieved only if the control system encapsulates, either implicitly or explicitly, some representation of the process to be controlled. This means that the controlled output tracks a set of reference inpu without steady state error if the model which generates these references is included the stable closed loop system.

How did this idea come about? It is very important to first consider an open loop control theory from first principles.

An open-loop control system is controlled directly, and only, by an input signal, without the benefit of feedback. Open-loop control systems are not as commonly used as closed-loop control systems because of the issue of accuracy. We shall therefore develop its technique of control, and further extend the knowledge gathered to more advanced control loops the have the potential to attain accuracy. An open loop structure is shown in figure 1 below.

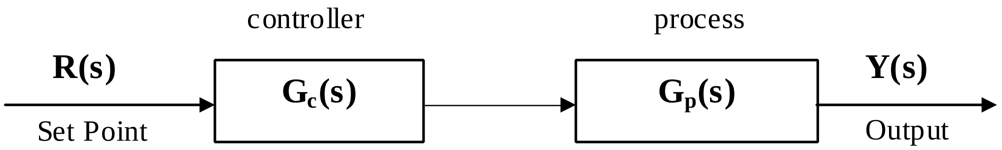



Figure 1: Open loop control scheme

With the controller $G_c(s)$, set to put control on the plant $G_p(s)$, then it is clear from basic linear system theory that the output Y(s) can be modelled as the product of the linear blocks as follows:

$$\mathbf{Y(s) = R(s)G_c(s)G_p(s)}$$

If we assume there exists a model of the plant with a transfer function modelled as $G_{pm}(s)$ such that $G_{pm}(s)$ is an exact representation of the process (plant), i.e. $G_{pm}(s)= G_p(s)$, then set point tracking can be achieved by designing a controller such that:

$$\mathbf{G_c(s) = G_{pm}(s)^{-1}}$$

This control performance characteristic is achieved without feedback and highlights two important characteristic features of this control modelling [3]. These features are as follows:

- perfect control can be theoretically achieved if complete characteristic features of the process are known or easily identifiable.
- feedback control is only necessary if knowledge about the process is inaccurate or incomplete.

This control performance as already said has been achieved without feedback and assumed that the process model represent the process exactly i.e. the process model has all features of the parent process. In real life applications, however, process models have capabilities of mismatch with the parent process; hence feedback control schemes are designed to counteract the effects of this mismatching. A control scheme that has gained high popularity in process control has been formulated and known as the Internal Model Control (IMC) scheme. This design is a simple build up from the ideas implemented in the open loop control strategy and has a general structure as depicted by **Figure 2** below:

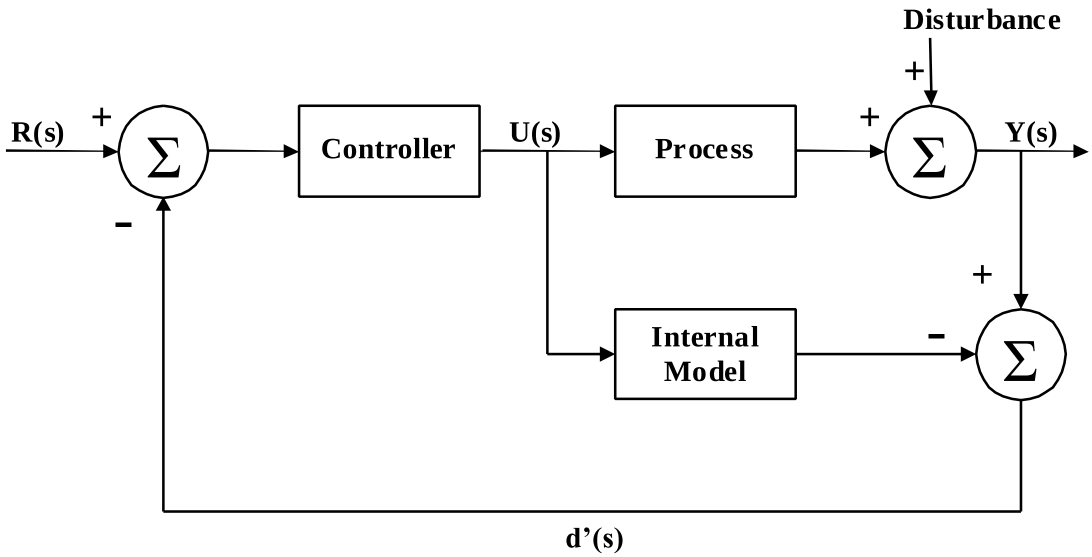



**Figure 2**: Internal model control scheme

From the figure above we shall use the following conventions to describe the blocks in the system:

- controller - $G_c(s)$
- process - $G_p(s)$
- internal model - $G_{pm}(s)$
- disturbance – d(s)
- disturbance transfer function – D(s)

The figure above shows the standard linear IMC scheme where the process model $G_{pm}(s)$ plays an explicit role in the control structure. This structure has some advantages over convectional feedback loop structures. For the nominal case $G_p(s) = G_{pm}(s)$, for instance, the feedback is only affected by the disturbance D(s) such that the system is effectively open loop and hence no stability problems can arise. This control structure also depicts that if the process $G_p(s)$ is stable, which is true for most industrial processes, the closed loop will be stable for any stable controller $G_c(s)$. Thus, the controller $G_c(s)$ can simply be designed as a feedforward controller in the IMC scheme.

From the IMC scheme depicted in Figure 2 above, the feedback signal is represented as follows:

$$\mathbf{d'(s) = [\, G_p(s) - G_{pm}(s)]U(s) + D(s)} \qquad \textit{Equation 2.1}$$

As said above, if the model is an exact representation of the process then d'(s) is simply a measure of the disturbance. If there exist no disturbance, then d'(s) is simp' a measure in difference in behaviour between the process and its model.
The closed loop transfer function of the IMC scheme can be seen modelled as below

$$Y(s) = \frac{R(s)G_c(s)G_p(s) + [1 - G_c(s)G_{pm}(s)]D(s)}{1 + [G_p(s) - G_{pm}(s)]G_c(s)} \quad \text{Equation 2.2}$$

The above transfer function will be shown to exist in the next section on transfer functions. From this closed loop analysis, we can see that if we design a controller such that $G_c(s) = G_{pm}(s)^{-1}$ where the process model is an exact representation of the process, then the design will yield good set point tracking and disturbance rejection. The controller is then detuned for robustness to account for a possible plant model mismatch. This is done by augmenting the controller with a low-pass filter to reduce the loop gain for high frequencies [3]. This idea also counteracts the effects of model inversion, as the pure inverse of the model is not physically realizable. The inversion of the process model may also lead to unstable controllers in case of unstable zeros in the model.

### 2.3 Internal Model Control Properties

In the IMC scheme shown by figure 2 above, the Internal Model loop calculates the difference between the outputs of the process and that of the Internal Model. This difference simply represents the effects of the disturbances and uncertainties as well as that of a mismatch of the model. Internal Model control devices have shown to have good robustness properties against disturbances and model mismatch in the case of the linear model of the process.

A control system is generally required to regulate the controlled variables to reference commands without steady state error against unknown and unmeasurable disturbance inputs. Control systems with this nature property are called servomechanisms or servo systems. In servomechanism system design, the internal model control principle plays an important role. Hence the design of a robust servomechanism system with plant uncertainty begins with three specifications as outline below:

- definition of the plant model and associated uncertainty
- specification of inputs
- desired closed loop performance

IMC theory provides a systematic approach in the synthesis of a robust controller for systems with specified uncertainties. This brings about the two important advantages of applying IMC control scheme in the synthesis of a servo controller.

1. the closed-loop stability can be assured by choosing a stable IMC controller
2. the closed-loop performances are related directly to the controller parameters, which makes on-line tuning of the IMC controller very convenient.

#### 2.3.1 Transfer Functions

It is always very important to develop transfer functions between the disturbance an the set point inputs by re-drawing figure 2 as follows:

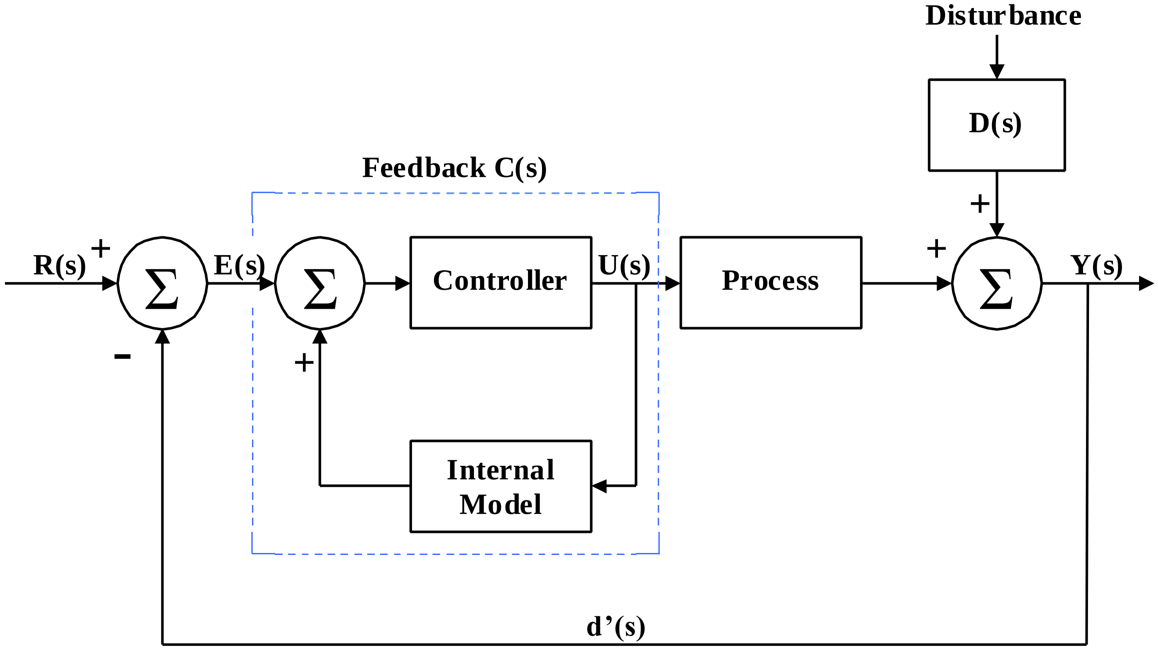



**Figure 3: Alternate design of IMC scheme**

From linear system theory in block reduction techniques, it has been shown that the transfer function between any input and the output of a single loop feedback system is the forward path transmission from the input to the output divided by one plus the loop transmission for negative feedback. Then we can establish the following equations:

$$C(s) = \frac{U(s)}{E(s)} = \frac{G_c(s)}{1 - G_c(s)G_{pm}(s)} \qquad \textit{Equation 2.3}$$

Therefore the input-output relationship of figure 3 is given by the following mathematical equations:

$$\frac{Y(s)}{R(s)} = \frac{G_p(s)C(s)}{1 + G_p(s)C(s)} \qquad \textit{Equation 2.4}$$

$$\frac{Y(s)}{d(s)} = \frac{D(s)}{1 + G_p(s)C(s)} \qquad \textit{Equation 2.5}$$

$$\frac{U(s)}{R(s)} Z \frac{C(s)}{1 H G_p(s)C(s)} Z \frac{Y(s)}{R(s)} \; G_p^{-1}(s) \quad \textit{Equation 2.6}$$

$$\frac{U(s)}{d(s)} Z \frac{\vartheta D(s)C(s)}{1 H G_p(s)C(s)} Z \left(\vartheta \frac{Y(s)}{d(s)}\right) C(s) \quad \textit{Equation 2.7}$$

Substituting equation 2.3 into equation 2.4 and 2.5 the overall result comes to be shown as below:

$$Y(s) Z \frac{G_p(s)G_c(s)R(s)}{1 H \left(G_p(s) \vartheta G_{pm}(s)\right) G_c(s)} \quad \textit{Equation 2.8a}$$

*OR*

$$Y(s) Z \frac{R(s)G_c(s)G_p(s) H \left(1 \vartheta G_{pm}(s)G_c(s)\right) D(s)}{1 H \left(G_p(s) \vartheta G_{pm}(s)\right) G_c(s)} \quad \textit{Equation 2.8b}$$

**2.3.2 No Offset Property of IMC**

The steady state gain of any stable transfer function can be obtained by replacing the Laplace variable *s* with zero. If equations 2.8a and 2.8b are stable and if we choose the steady state gain of the controller $G_c(0)$ to be the inverse of the process model gain such that $\mathbf{G_c(0)G_{pm}(0)= 1}$, then the gain on the denominators of equation 2.8a and equation 2.8b is effectively $\mathbf{G_c(0)G_p(0)}$. Therefore the gain between the set point R(s) and Y(s) is essentially equal to one and hence the gain between the disturbance d(s) and the output Y(s) is zero. These results prove that there is no steady state deviation of the process output from the setpoint.

**2.3.3 Perfect Control**

If we assume that the controller is equal to the model inverse, and that the closed loop system in figure 2 is stable, then Y(s)=R(s) for all disturbances affecting the system.

**2.4 Designing for IMC**

The design procedure for Internal Model Control comprises two major steps. In the first step, the IMC controller is designed to achieve so-called nominal performance without regard to plant uncertainty or equivalently with the assumption that the process model is an exact representation of the process i.e. $G_{pm}(s) = G_p(s)$.in the second step, the IMC controller is augmented with a robustness compensator $G_f(s)$ to meet the robustness specifications. This robustness compensator is usually a low pass filter of appropriate order as it is meant to counteract the increase in frequency of the plant uncertainties as well as the unmodelled dynamics of the overall system. Thus the IMC controller has the form below:

$$G_{IMC}(s) = G_c(s) G_f(s) \qquad \textit{Equation 2.9}$$

where $G_c(s)$ is an optimal controller obtained in the first step. If the reference nominal model, the IMC controller, and the robustness compensator (low-pass filter) are appropriately designed, the IMC control scheme will produce a servo controller with desired robustness performances.

**2.4.1 Practical Design of IMC**

The design procedure for designing an IMC controller involves two basic steps. The steps are summarised below;

**Step 1**: Given the transfer function of the process model, it is required that the transfer of the model be factorised into invertible and non-invertible components where we shall use the following conventions to relate to the relevant terms:

- $G_{pm}^{H}(s) - \textit{invertible components}$
- $G_{pm}^{\vartheta}(s) - \textit{non-invertible components}$

With this transition, it implies that with steps one, we only need to show the model in the following representation if the process to be controlled contains both the invertible and non-invertible components:

$$G_{pm}(s) = G_{pm}^{+}(s) G_{pm}^{-}(s) \qquad \textit{Equation 2.10}$$

**Step 2**: With the transition from step one, the designer's task is to select or model the controller as the inverse of the invertible components i.e. it is required that the controller be given the following representation:

$$G_c(s) Z G_{pm}^{H}(s)^{\vartheta 1} \qquad \textit{Equation 2.11}$$

If the process model contains only components which cannot be factorised but is does show stability with no right half poles (RHP) on the *s-plane* then the model is considered invertible and the controller takes the form shown by equation 2.11. If th process model contains only the non-invertible components and with instability, the other improved methods can be used because the IMC controller depends on the stability and the invertibility of the process model. The non-invertibility of components may lead instability and realisability problems when inverted.

### 2.4.2 IMC Filter Design

The principal objective in the utilization of the IMC scheme into controlling a plant or process is to design an IMC controller $G_{IMC}(s)$ which comprise an optimal controller augmented with a low pass filter as shown in the figure below:

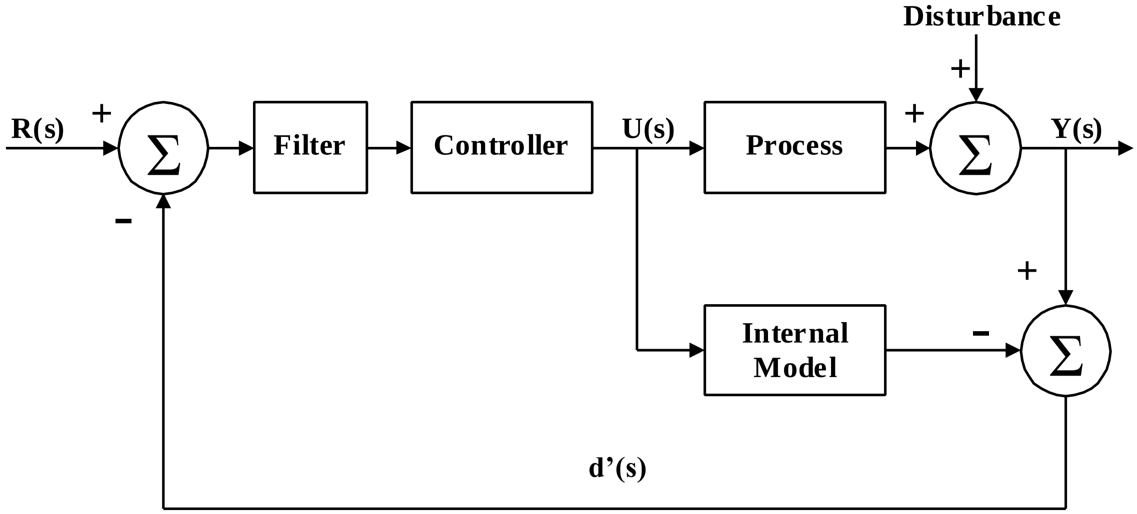



**Figure 4: IMC Scheme with a Filter**

The transfer function of the filter shall be represented in this document as $G_f(s)$, with the filter in cascade with the controller. Hence, we have the IMC controller as formulated below:

$$G_{IMC}(s) Z G_f(s) G_c(s) \qquad \textit{Equation 2.12}$$

The filter is modelled as below:

$$G_f(s) = \frac{1}{(1+\tau_f s)^n} \qquad \textit{Equation 2.13}$$

where $\tau_f$ is the filter parameter and n is the order of the filter.

The order of the filter is chosen such that $G_{IMC}(s)$ is proper to prevent excessive differential control action. In this case the order of the filter is chosen as being the same as the order of the process as will be shown in the next chapter on process modelling and IMC design.

**2.5 IMC Applications**

There are a lot many areas where the principle of IMC can be utilised. In essence the use of control is extremely broad and it encompasses a number of different applications, thus this include control of electromechanical systems, where computer controlled actuators and sensors regulate the behaviour of the system, control of electronic systems, where feedback is used to compensate for component variation and provide reliable, repeatable performance and control of information and decision systems where limited resources are dynamically allocated based on estimates for future needs.

In recent year more advancements in control systems has led to more technical and reliable control schemes such as the Internal Model Control. IMC control technology has spread far beyond its initial applications. Visible success from the past investments in IMC control includes the following:

- Control systems in the manufacturing industries, from automotive to integrated circuits. Computer controlled machines provide the precise positioning and assembly required for high quality, high yield fabrication of components and products.
- Industrial process control systems, particularly in the hydrocarbon and chemical processing industries. These maintain high product quality by monitoring thousands of sensor signals and making corresponding adjustments to hundreds of valves, heaters, pumps, and other actuators.
- Guidance and control systems for aerospace vehicles, including commercial aircraft, guided missiles, advanced fighter aircraft, launch vehicles, and satellites. These control systems provide stability and tracking in the presence of large environmental and system uncertainties.
- Control of communications systems, including the telephone system, cell phones, and the Internet. Control systems regulate the signal power levels in transmitters and repeaters, manage packet buffers in network routing equipment, and provide adaptive noise cancellation to respond to varying transmission line characteristics.

These applications have had an enormous impact on the productivity of modern society. In addition to its impact on engineering applications, IMC control has also made significant intellectual contributions. Control theorists and engineers have made rigorous use of and contributions to mathematics, motivated by the need to develop provably correct techniques for design of feedback systems. They have been consistent advocates of the "systems perspective," and have developed reliable techniques for modelling, analysis, design, and testing that enable development and implementation of the wide variety of very complex engineering systems in use today.

## 3.0 Chapter Three: Servo System and Modelling

This section of the project is looking primarily into introducing how the plant is modelled as an extension for collaborative use in the discussion of the simulation in internal model control (IMC) and PID implementations for this project. The section discusses two features of the project as listed below.

- Plant Modelling in mathematical terms
- Servo System controllers (IMC and PID)

### 3.1 Servo Background

Servo control, which is also referred to as "motion control" or "robotics" is used in industrial processes to move a specific load in a controlled fashion. These systems can use either pneumatic, hydraulic, or electromechanical actuation technology. The choice of the actuator type (i.e. the device that provides the energy to move the load) is based on power, speed, precision, and cost requirements. Electromechanical systems are typically used in high precision, low to medium power, and high-speed applications. These systems are flexible, efficient, and cost-effective. Motors are the actuators used in electromechanical systems. Through the interaction of electromagnetic fields, they generate power. These motors provide either rotary or linear motion.
Servo drives and amplifiers are used extensively in motion control systems where precise control of position and/or velocity is required. The drive/amplifier simply translates the low-energy reference signals from the controller into high-energy signals to provide motor voltage and current. In some cases the use of a digital drive replaces the controller/drive or controller/amplifier control system. The reference signals represent either a motor torque or a velocity command and can be either analogue or digital in nature.

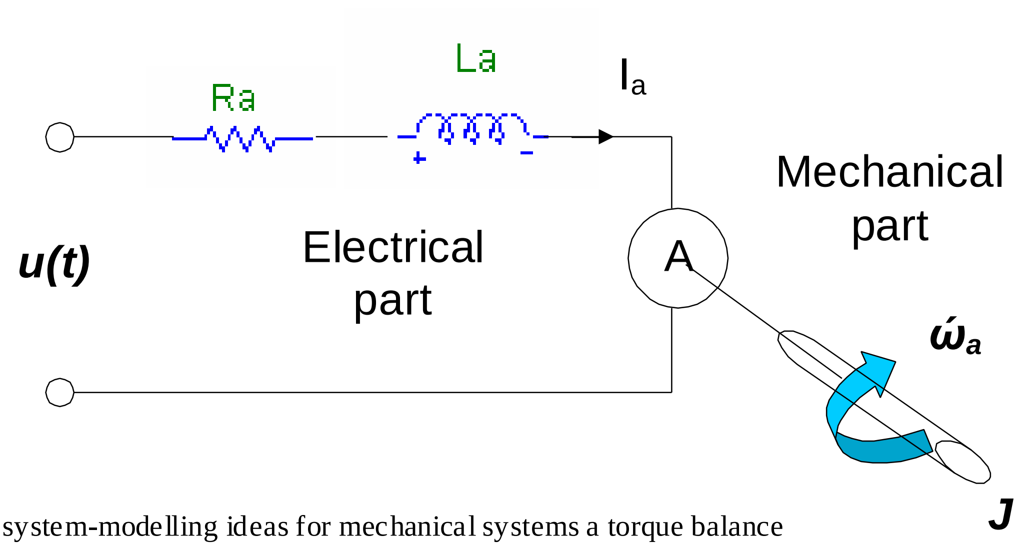
#### 3.1.1 Overview

The extension of modelling the plant for this project emanates from a build up of electric motors which are almost universally used in modern commercial and industrial occupancies to furnish the required mechanical motive power to drive mechanical machinery and control various industrial processes. Such machinery or other mechanical devices (valves, mechanical linkages, etc) connected to motor shafts (either directly or coupled through gears, belts or pulleys) are called motor loads or in simpler terms just loads. In many cases, a load must be driven at a variety of speeds in either direction, in accordance with some desired preset sequence (for example; an elevator in a high rise building). Frequently, several motors are required in combinations in more complex sequences to control interrelated loads (as happens in chemical plants and still mills).

The energy supplied to the motor, depending on the nature of the load requirements, usually programmed and controlled to obtain the desired load torque, speed and direction of rotation, at any given time, by a device called a controller. The principle utilized in Internal Model Control theory come into play from this point of view to design a controller used in this context. A controller may be manually operated by maybe an experienced human operator or can be made to run in an automated fashion. The degree of automation is dictated by the requirements of the process to be controlled. Where more precise control of the process or load speed and torque is indicated, closed loop control is utilized, and the controller and its associated control devices are somewhat more complex and complicated. In part, the selection of the type of motor used is dictated by the nature of the load requirements, the type of energy available and the types of controller commercially manufactured to adequately meet the load requirements.

**3.1.2 Mathematical Model**

The goal in the development of the mathematical model is to understand in mathematical terms the behaviour of the system without control before we can put control over it. Therefore, modelling simple servo systems can be developed by considering the electrical and mechanical characteristics of the system. In this context we shall consider a simple servo-DC position control servomechanism.
The basic form of a DC servo system is made of an electric motor with an output shaft that has an inertial load ***J*** on it, and friction in the bearings of the motor and load. There exist an electric drive circuit where an input voltage ***u(t)*** is transformed into a torque ***T(t)*** in the motor output shaft. The general view of this interpretation is shown by the schematic below:




Using system-modelling ideas for mechanical systems a torque balance can be written between the input torque from the motor and the torque required to accelerate the load and overcome friction. This can be modelled by the following equation from Newton's second law of motion, ***F=ma:***

$$J\ddot{\theta}+b\dot{\theta}=T(t) \qquad \textit{Equation 3.1}$$

Where $\theta$ is the angular position of the servo output shaft; and b is a constant representing the friction in the bearings of the motor and the load. The principle involved in the control objective is simply to control the angular position $\theta$ or the shaft velocity to be some desired value. The input voltage ***u(t)*** is related to the torque ***T(t)*** by a gain ***K***. The system model reduces to the following:

$$\tau\ddot{\theta} + \dot{\theta} = Ku(t) \quad \textit{Equation 3.2}$$

where $\tau$ is the system time constant defined by ***J/b*** while ***K*** is the system gain defined by ***1/b***.

In practical servomechanisms there are additional components of the system which are obviously important. Many of these will relate to nonlinearities in the drive amplifier and friction in the mechanical components of the system, hence a good control system must incorporate these features to overcome the nonlinear characteristics.
In this section and for the project we concentrate our modelling on the linear parts of the servo system for simplicity. The linear part of the servo system model for this project can be put in transfer function form as follows:

$$y(s) = \frac{K}{s(\tau s + 1)}u(s) \quad \textit{Equation 3.3}$$

where ***y(s)*** is the system output and ***u(s)*** is the system input. The parameters for the project are therefore defined as: (i) system gain ***K*** = 109
(ii) system time constant $\tau$ = 1
(iii) inertial load ***J*** = 1/109
(iv) friction constant ***b*** = 1/109

With the growing interest on high performance of a mechanical positioning system, more accurate and robust control algorithms are required. The control technique set in this document is looking into providing control to a servo system modelled as below:

$$G_p(s) = \frac{109}{s(s+1)}$$

*Equation 3.4*

The aim of the project is now to utilize the techniques of IMC and PID algorithms to provide control to the process modelled above. This idea will of the design will follow in the next chapter, first we shall establish the principles of PID implementations an design in the next chapter and follow with the techniques associated with the design for Internal Model Control.

### 3.2 Servo System Controllers

There are many alternative control design theories that can be used to control a servomechanism. In this project we are looking into using two of the many forms and document analysis on their control performance with respect to the modelled servomechanism above. These two control mechanism include the convectional PID used widely in commercial and industrial applications and secondly we look into a model based approach in Internal Model Control (IMC) to see what sort of control results we achieve with these two control schemes on their control performance actions when providing control to a servo system.

While Internal Model Control implementations are becoming more popular, the standard industrial controllers remain the proportional (**P**), proportional plus integral (**PI**), and the proportional plus integral and derivative (**PID**) controllers. Morari and Zafiriou (1989) and Rivera et al. (1986) show how to approximate the IMC controller for a limited class of processes with PI and PID controllers. To obtain a PID controller for the industrially important first-order lag and dead time process model, they approximate the dead time with a low-order Padé approximation. Because their approximations are surprisingly good enough, they conclude that there is relatively little to be gained in dynamic response by implementing the IMC controller rather than the PI or PID approximations.

The controller is the "brains" of a servo system. It is responsible for generating the motion paths and for reacting to changes in the outside environment. Controllers can be something as simple as an ON/OFF switch or a dial controlled by an operator. They can also be as complex as a multi-axis controller that actively servos several drives as well as monitors I/O and maintains all of the programming for the machine.

In practical servo control, the controller sends a signal to the drive; the drive provides power to the motor; and the feedback from the motor is sent back to the controller and drive. Feedback from the load is also routed to the controller. The controller analyses the feedback signal and sends a new signal to the amplifier to correct for errors. The controller is considered to be the intelligent part of the servo, closing the velocity and/or position loops while the amplifier closes the current loop. However, many amplifiers will close the velocity and/or position loops allowing less demand from the controller.

# 4.0 Chapter Four: PID Control for the Servo System

## 4.1 Introduction to PID controllers

Proportional-Integral-Derivative controllers are the most common controllers in electric drives and many other applications of the process industry. PID control is widely used in servo systems as it has a simple structure, safety and reliability. In the superposition design approach PID control can be viewed as combining proportional, derivative and integrating elements or some system signal weighted by a factor. The popularity of these controllers has led to research on tuning methods resulting in numerous methods published over the years. The most popular and acceptable methods relate to work done in recent publications by Ziegler-Nichols.

However, the tuning of the PID control systems is not always easy, because of its simple control structure for wide class of process characteristics. The PID controller has three tuning parameters which can be tuned by trial and error or by using tuning rules available in literature such as the Ziegler-Nichols. These rules are based on the open-loop stable first order or second order plus dead time process models or critical point information for stable processes. However, under certain circumstances published tuning rules or methods may not provide satisfactory closed loop performances. This document is looking forward to extend the knowledge that may have already been existence by exploring the principles involved in PID parameter representation as well as tuning techniques.

## 4.2 PID Mathematical representation

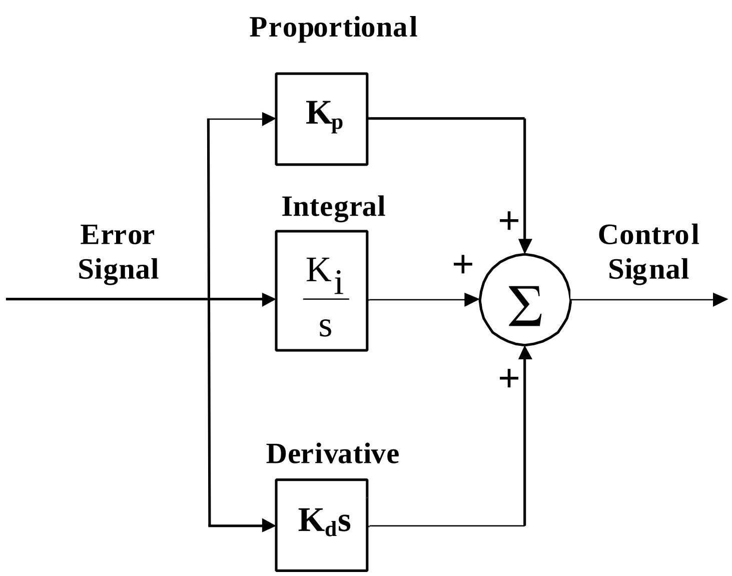



**Figure 4.1: Proportional-Integral-Derivative (PID) controller**

Figure 4.1 above shows how the PID parameters are represented in a control architecture. Whatever the nature of the process, there will be some desired point of operation, and some difference from that desired point, usually called the error. The technique in control is to minimize such an error to a value as close to zero as possible. By employing feedback to a control scheme we desire to replicate the error to a control command so that we conceive appropriate control action for better performance of the overall system. The PID achieves this principle with its parameters as it imposes the following commands to an incoming error:

1. it ensures that its response has a related proportion to the magnitude of the error
2. it has the ability to take stronger action should the error persist in time
3. it has the ability to act quickly at the onset of error in an attempt to get ahead of it

The time domain representation of the PID control output is given as PID(t) below:

$$PID(t) = K_c\left(e(t) + \frac{1}{T_i}\int_0^t e(t)dt + T_D\frac{de(t)}{dt}\right) \quad \textit{Equation 4.1}$$

where the following conventions apply:

- $K_c$ – *controller gain*
- $e(t)$ – *error signal*
- $T_i$ - *integral time*
- $T_D$ – *derivative time*

**4.3 PID Parameter Characterisation**

This section is set to define the action attributes of each of the PID parameters, the proportional, the integral and the derivative.

**4.3.1 Proportional Command**

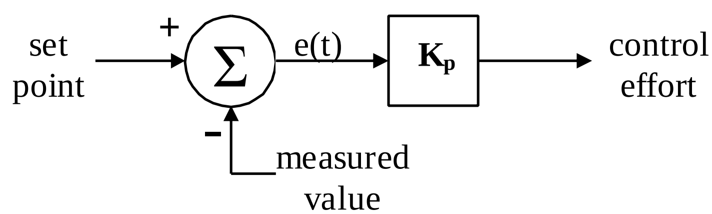



**Figure 4.2: Proportional Control**

The figure above shows the proportional control. The proportional control implies tl if you have a reference you are trying to control to, you simply provide a control output proportional to the error from your reference. Hence from the above we shall get the output represented mathematically as;

$$\text{Control effort} = K_p * (\text{Set Point} - \text{Measured Value}) \quad \textit{Equation 4.2}$$

where the error $e(t)$ = Set Point – Measured Value

### 4.3.2 Integral Command

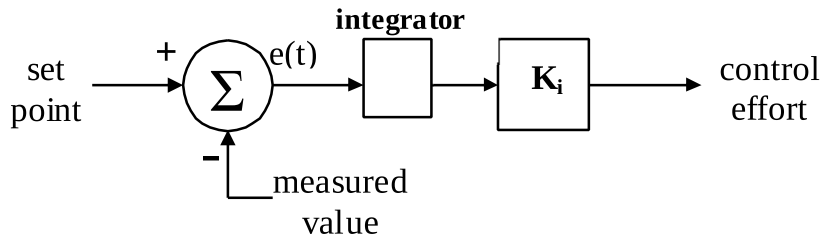



**Figure 4.3: Integral Control**

Figure 4.3 above shows the integral control action. The integral is needed mainly for the off-sets and biases in the system. The integral term yields zero steady-state error in tracking a constant set point. Integral control enables also enables the complete rejection of constant disturbances. While integral control filters higher frequency sensor noise, it is slow in response to the current error.

The integral exists in the system to calculate the integral of the input over time. It uses the input to create an output which will continue to grow until the input is reduced to zero. Thus the control effort is summarised below:

$$\text{Control effort} = K_i * (\text{Integrator Output}) \quad \textit{Equation 4.3}$$

### 4.3.3 Derivative Command

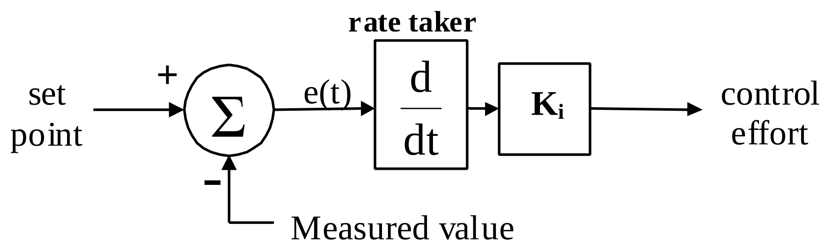



**Figure 4.4: Derivative Control**

Figure 4.4 shows the derivative control action from the PID algorithm. For process with significant dead time, the effects of the proportional and the integral actions are poorly represented in the current error. This situation may lead to large transient errors when PI control is used. The derivative control action combats this problem by basing a portion of the control on a prediction of the future error. Unfortunately, the derivative amplifies higher frequency sensor noise; thus, a filtering of the differentiated signal is typically employed, introducing an additional tuning parameter.

The derivative tells how fast a signal is approaching or departing from a set point. The important signal in the above configuration is the rate signal that is output of the rate taker and multiplied by the derivative constant to provide the control effort. Hence

$$\text{Control effort} = K_d * \text{Rate} \qquad \textit{Equation 4.4}$$

The overall control effort becomes the sum of the efforts to represent the standard PID algorithm. We therefore represent the overall control effort as ***u(t)*** as follows:

$$u(t)=K_p e(t)+K_i \int e(t)dt+K_d \frac{de(t)}{dt} \qquad \textit{Equation 4.5}$$

**4.4 PID Design for the Servo System**

There are typical steps chosen as rule of thumb to design a PID controller. The following steps works in many cases:

1. determine what characteristics of the system need to be improved.
2. use the proportional gain $K_p$ to reduce the rise time.
3. use the derivative gain $K_d$ to reduce the overshoot and settling time
4. use the integral gain $K_i$ to eliminate the steady state error

There are design methods though that has been put into literature which simplifies the need for beating about the bush. We shall explore one of these ideas here and utilise in the design of our PID for the servo system. This method is famously known as the Ziegler-Nichols method and is chosen in this context because it is simple and a little straight forward.

Ziegler and Nichols conducted numerous experiments and proposed rules for determining the values of the PID parameters ($K_p$, $K_d$, *and* $K_i$) based on the transient response of a plant or process. The two, Ziegler and Nichols proposed a few methods but we shall limit ourselves to what they referred as the first method of Ziegler and Nichols. In this method an open loop response of the system is obtained as shown by the figure below. A tangent line is then drawn as the point of inflection to characterise the curve into two other parameters L and T as shown inscribed within the time axis. Parameter K is also shown as the final value or the steady-state value of the open loop system. These parameters are then used to set PID parameters as in Table 4.1.

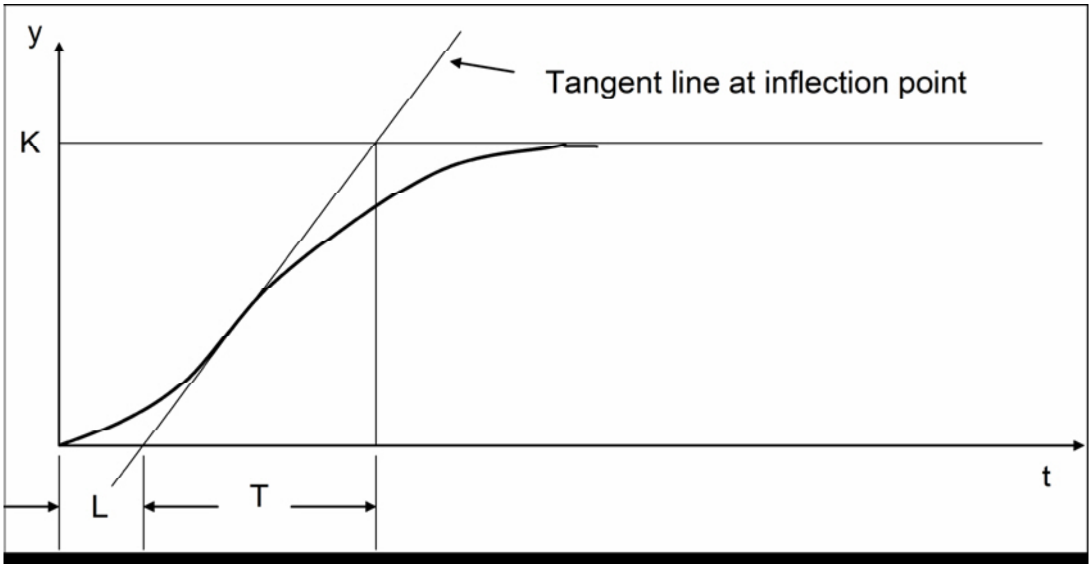



**Figure 4.5: An S-shaped open loop response of a system**

| Controller | Proportional Kp | Integral $K_i$ | Derivative $K_d$ |
|---|---|---|---|
| PID | $0.5\frac{T}{L}$ | $0.6\frac{T}{L^2}$ | $0.6T$ |

**Table 4.1**

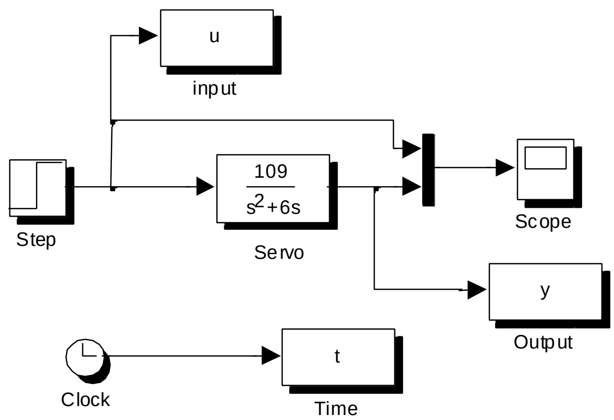
It is estimated that with these parameters set as above, a response with an overshoot of 25% and good settling time should be obtained. Fine tuning will then be necessary if the performance deviates from optimal. This can be done using the basic rules that relate each parameter to the response characteristics.
However, in the ZN method, tuning is based on the critical gain and the period, which are determined by increasing the proportional gain until the stability limit is reached. In practice, this method may cause the risk of instability and it is difficult to automate, but forms a development of a tuning mechanism that has the potential to achieve optimal control. The ZN tuning rule uses the ultimate information of the process. Therefore, it cannot systematically consider the concrete control performance to tune the PID parameters.

From the open loop response we approximate the values of the two parameters L and T as derived from **figure 4.6** below:

- L= 1.2
- T= 9.8

These values are then used as per table 4.1 to calculate the settings of the PID parameters which can be deduced from table 4.2. Using this values the PID parameters are set as direct in simulink to produce a plot of a response to a step input. These implementations and results can be seen in the section that follows.

See: **Model M4.1 below: Simulink Implementations**




**Model M4.1**: **Open loop response of a servo**

In the above model, an open loop response was to be observed. A step input as reference was applied to the plant without a controller. It was therefore important to link the matlab workspace with the simulink environment by linking the two environments with the simout block. In this way the matlab can reference all outputs from simulink to some memory address, and thus can be used further in the simulations.

The response curve in open loop is shown in figure 4.6 below. With all concepts of the ZN tuning method put in mind, the response curve can be used to verify the parameters as shown by figure 4.5. These parameters can then be further explored to evaluate the PID parameters as per table 4.1.

Hence the values of the PID controller can be set as below in Table 4.2. (ZN First Trail), with the help of the values obtained from figure 4.6 below. i.e.

- **L= 1.2**
- **T= 9.8**

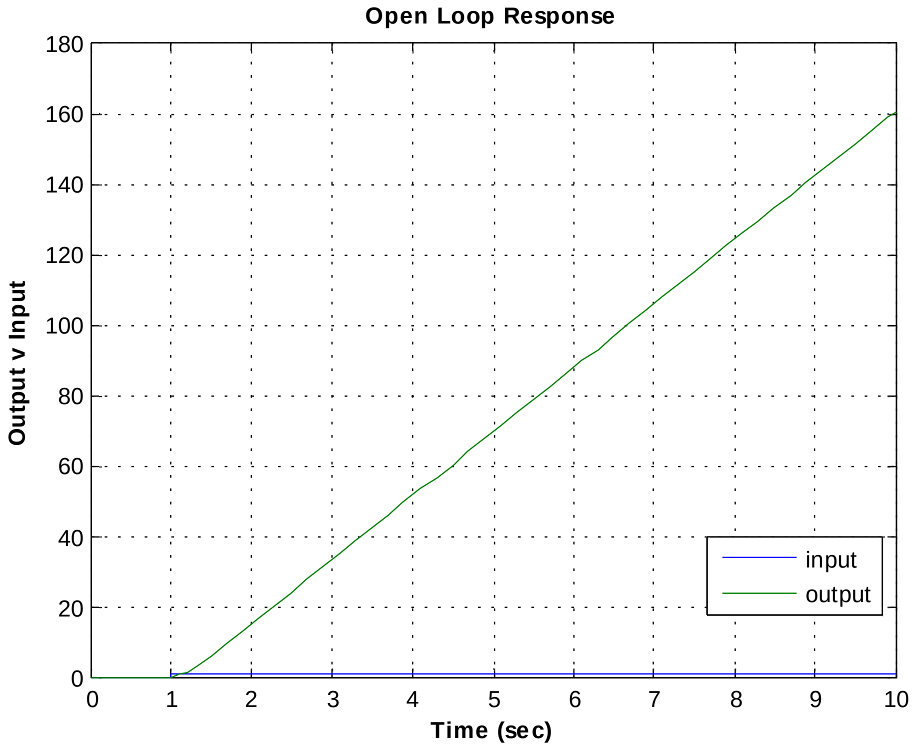
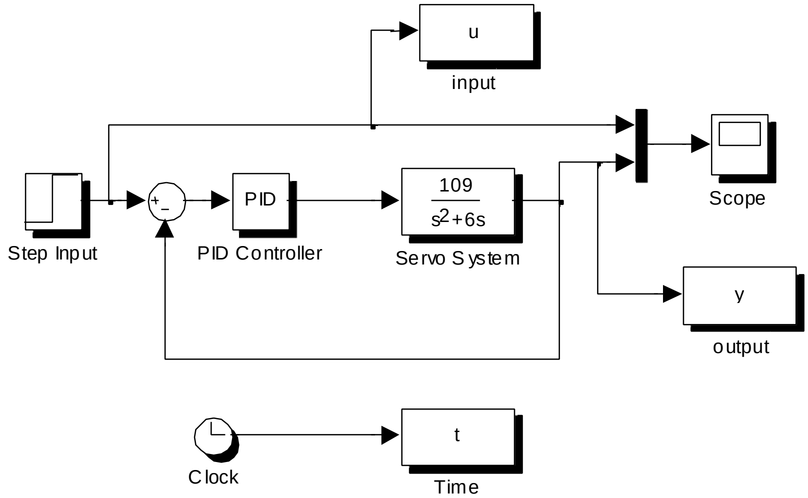



**Figure 4.6: Open Loop response to a step Input for Ziegler-Nichols First Trial**

**Table 4.2**

| Controller | Proportional Kp | Integral $K_i$ | Derivative $K_d$ |
|---|---|---|---|
| PID | 4.083 | 3.7 | 5.88 |

See: **Model M4.2:**




**Model M4.2: PID Control- ZN First Trial 1**

On simulation of the above system having set the parameters of the PID as per table 4.2 we get the response as shown on **figure 4.7** below.

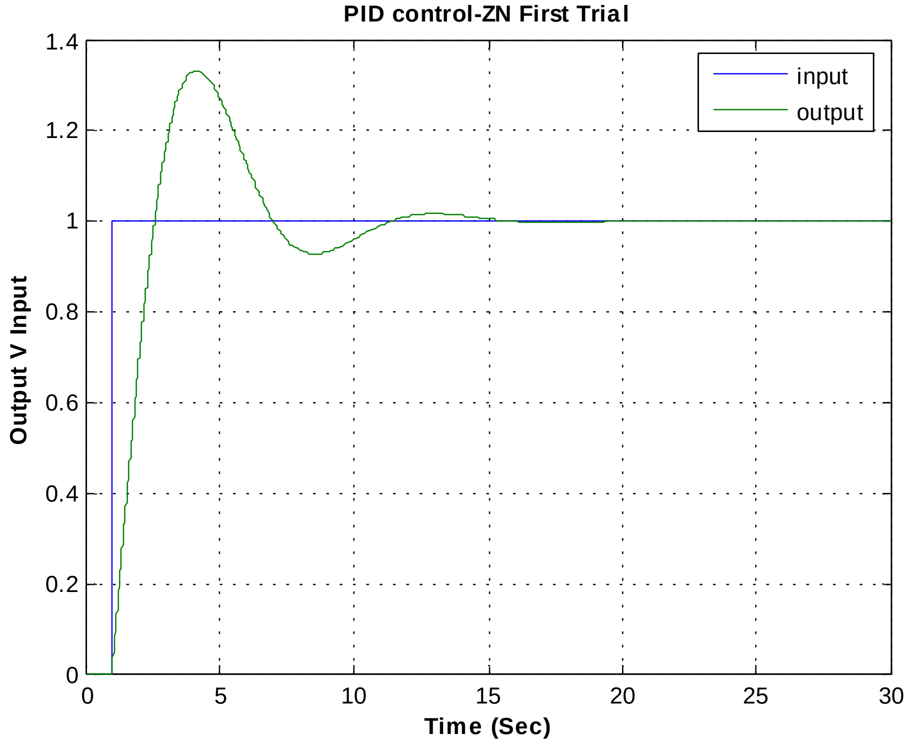



**Figure 4.7: PID control-ZN first Trial**

With the first trial on application of the Ziegler-Nichols PID parameter setting, it is evident that the response is offset in set point tracking with an overshoot and the next trial involves fine tuning the parameters to get the best response. This can be done with the help of the following Table 4.3 which shows the effects of increasing each of the PID parameters on the response characteristics.

| Response | Rise Time | **Overshoot** | **Settling Time** | **Steady-State Error** |
|---|---|---|---|---|
| $K_p$ | Decrease | Increase | Inconclusive | Decrease |
| $K_i$ | Decrease | Increase | Increase | Eliminates |
| $K_d$ | Inconclusive | Decrease | Decrease | Inconclusive |

**Table 4.3**

Based on Model M4.2 from Appendix, PID parameters were tuned by trial and error to obtain the best response with the following settings on the second trial:

**Table 4.2**

| Controller | Proportional Kp | Integral $K_i$ | Derivative $K_d$ |
|---|---|---|---|
| PID | 9.5 | 5.5 | 1 |

Using these new values after a tremendous effort of refining the response, the new version of the response is observed as per figure 4.8 below. There is little overshoot in the response and setpoint tracking can be easily achieved. The response is shown below:

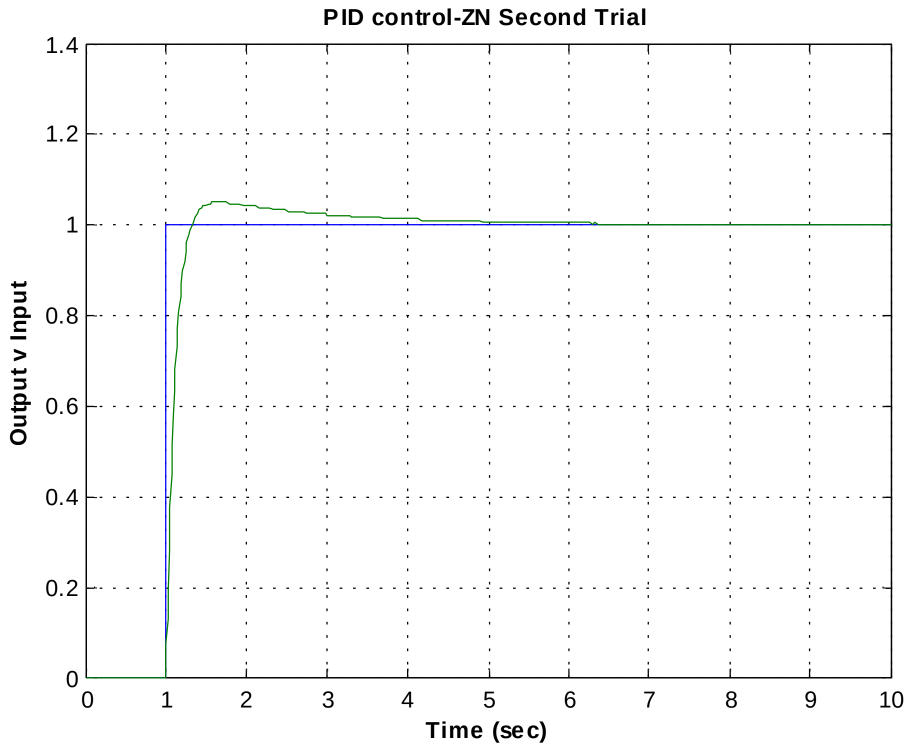


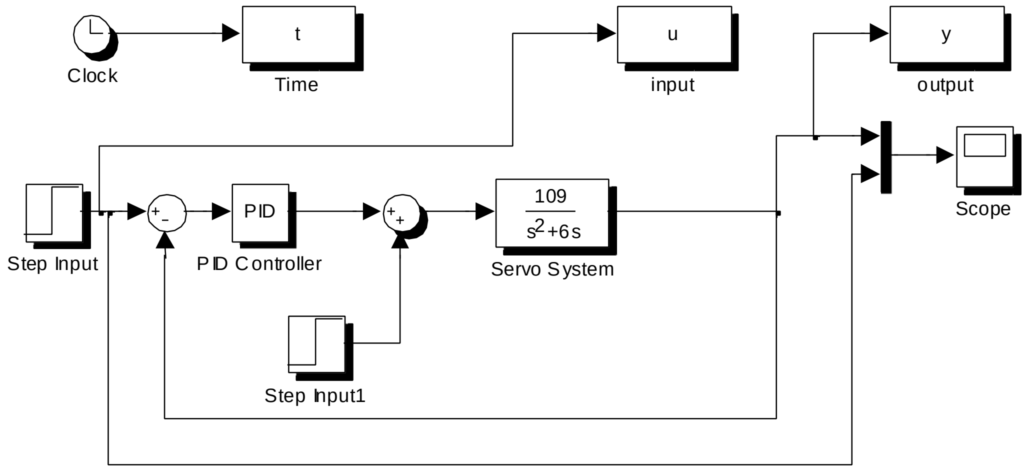

**Figure 4.8: PID control-ZN Second Trail**

### 4.5 Disturbance effects with PID design

In this section we are looking into establishing the effects of disturbances at both the input and the output of the plant with the best PID parameters as per table 4.2. For simplicity we shall model disturbances as a simple step input and a sinusoidal wave and see how the response deviates from the optimal response.

#### 4.5.1 Step Input Disturbance at the input of the Plant

**Simulink Implementations:** See below




**Model M4.3: Step Input Disturbance at input of Plant**

The output plots for the above simulation in simulink can be seen below in figure 4.9.

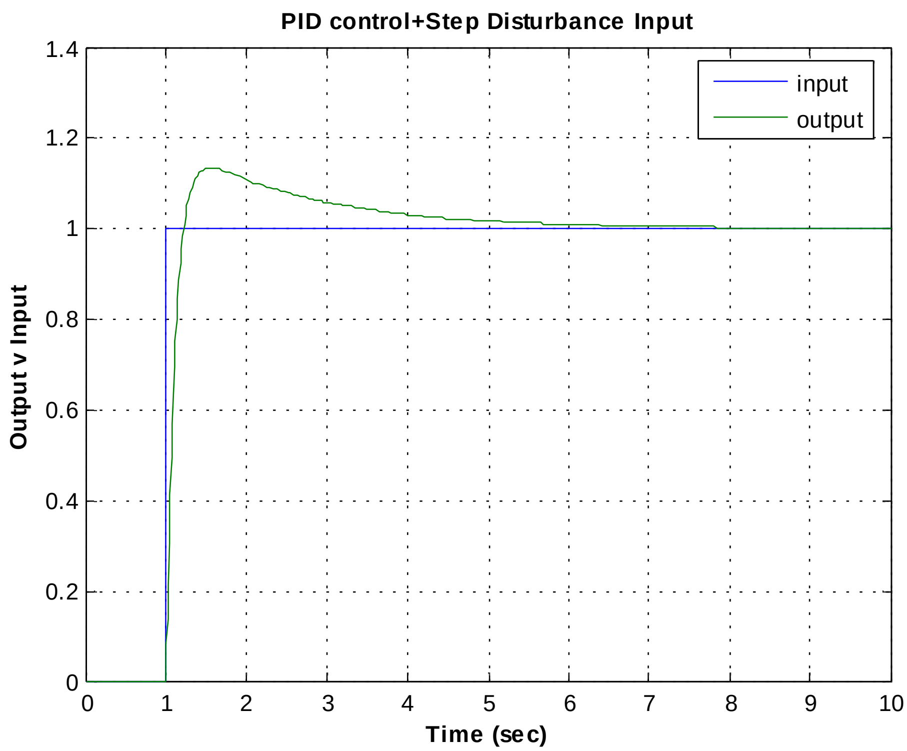
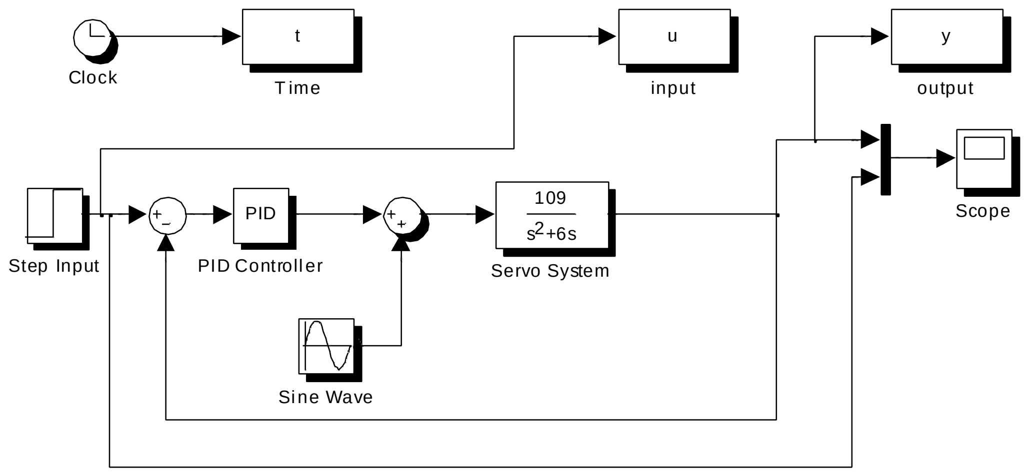



**Figure 4.9: PID control under a step disturbance at input of plant**

It can be seen from the output results that if a step disturbance is introduced at the input of the plant, the response deviates form the optimal response with a little offset of an overshoot but then sets a little later to follow the set point.

### 4.5.2 Sinusoidal Disturbance at the input of the plant

**Simulink Implementations:** See below




**Model M4.4: Sinusoidal Disturbance at input of Plant**

On simulation of the above system with a sinusoidal disturbance at the input of the plant, the response to a step input can be shown as in **figure 4.10** below.

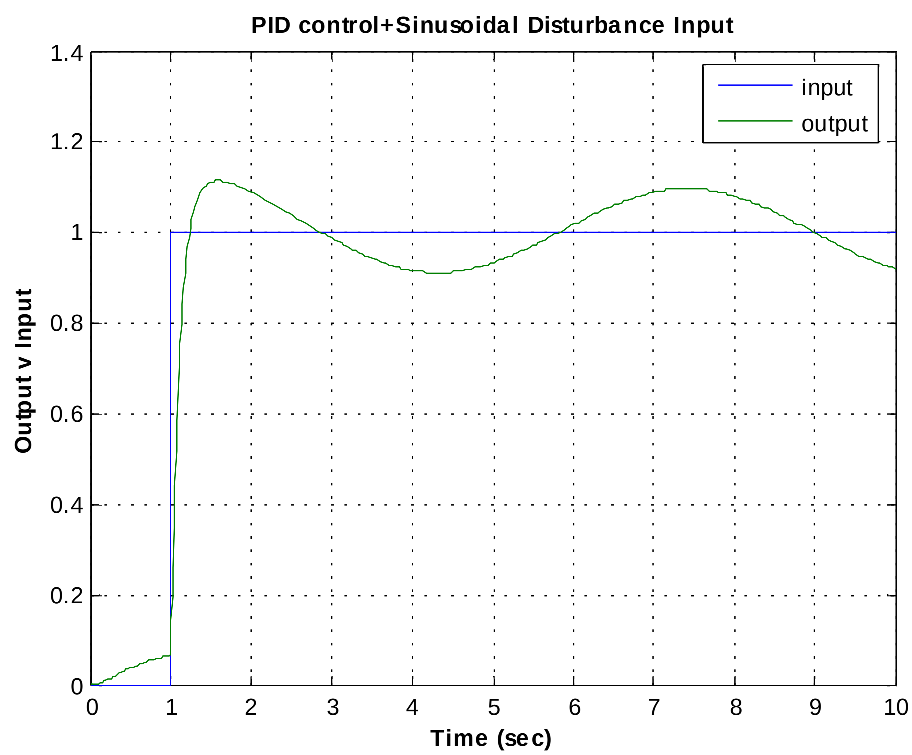
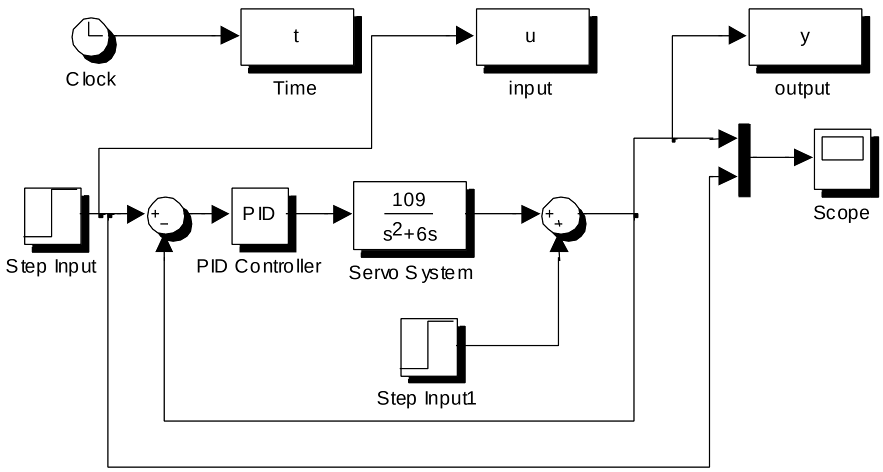



**Figure 4.10: PID control under a Sinusoidal disturbance at input of plant**

It can be seen here that with a sinusoidal disturbance at the input of the plant, contro action is lost and the response becomes oscillatory around the setpoint. This shows poor performance as far as manipulation of the error signal is concerned. Further tuning of the PID parameters by trial and error is therefore necessary to attain the optimal response.

### 4.5.3 Step Input Disturbance at the output of the Plant

**Simulink Implementations:** See below




**Model M4.5: Step Input Disturbance at output of Plant**

On simulation of the above system the response is plotted as per **figure 4.11** below:

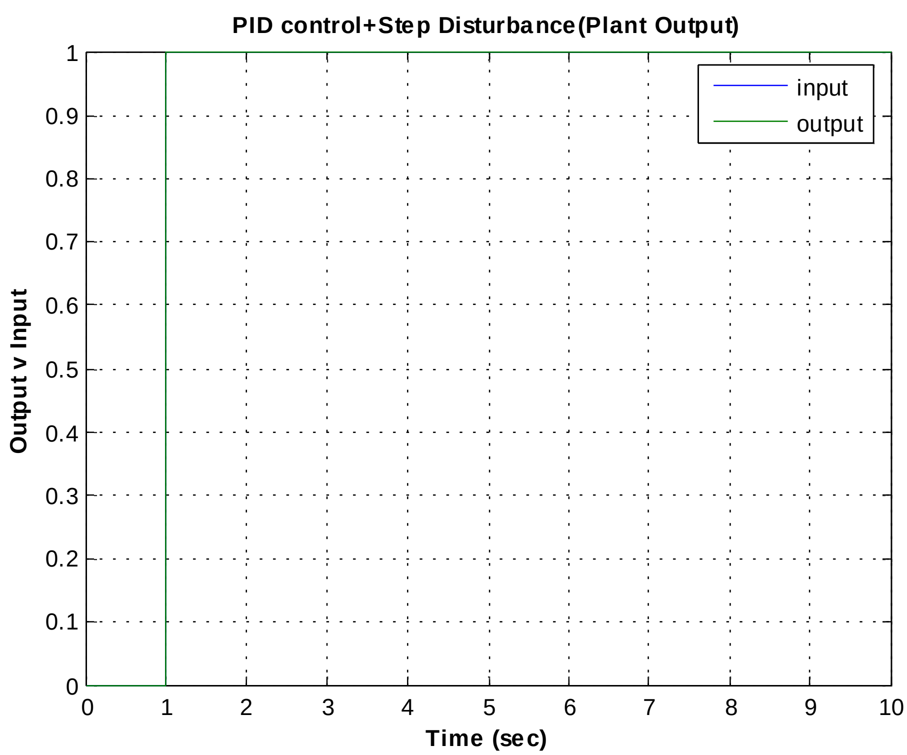
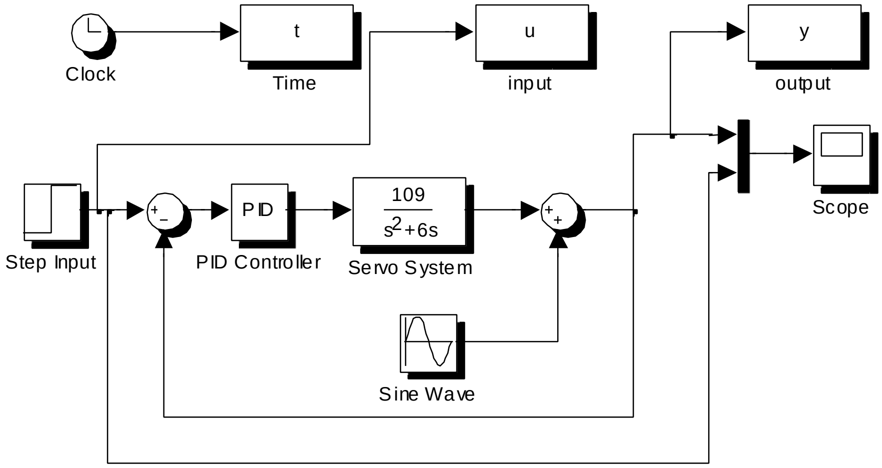



**Figure 4.11: PID control with step input disturbance at output of plant**

With a step input at the output of the plant, set point tracking and disturbance rejection is achieved as can be seen from the output of the scope and as per **figure 4.11** above. The output is perfectly embedded on setpoint, hence perfect setpoint tracking and disturbance rejection.

### 4.5.4 Sinusoidal Disturbance at the output of the Plant

**Simulink Implementations:** See below:




**Model M4.6: Sinusoidal Disturbance at output of Plant**

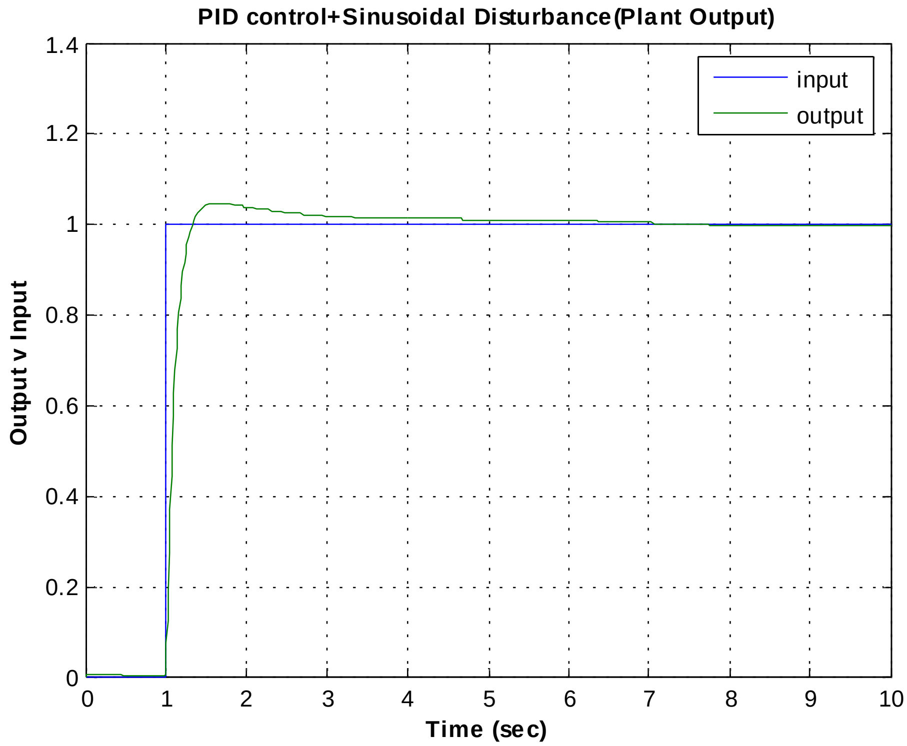
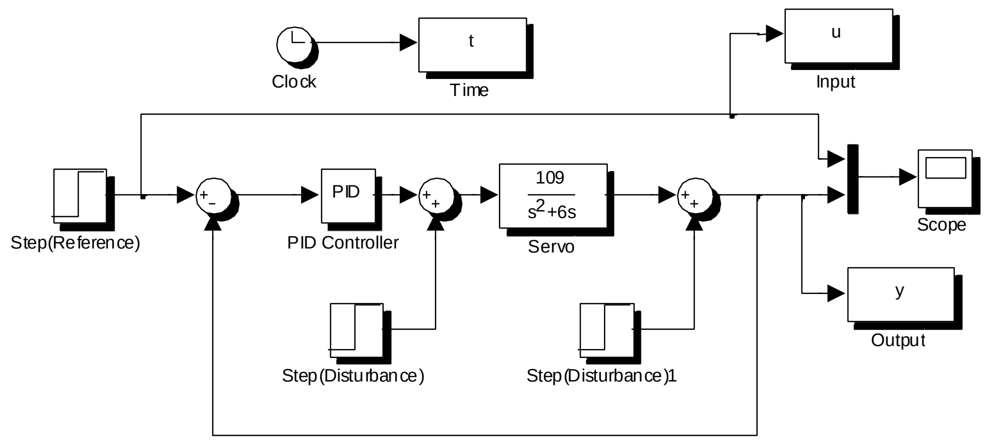



**Figure 4.12: PID control with a sinusoidal disturbance at the output of plant**

On simulation of the above system (Model M4.6) the response is as in figure 4.12 above. The response cannot be seen as oscillatory anymore and it simulates that of a step disturbance at the input of the plant.

It can also be seen that the offset from set point tracking is not too big as was seen with the sinusoidal disturbance at the output of the plant. Perfect control is theoretically possible in this instance as there is a little overshoot in the response of which settles quickly to attain setpoint tracking.




**Model M4.7: Step Disturbance at both output & input of Plant**

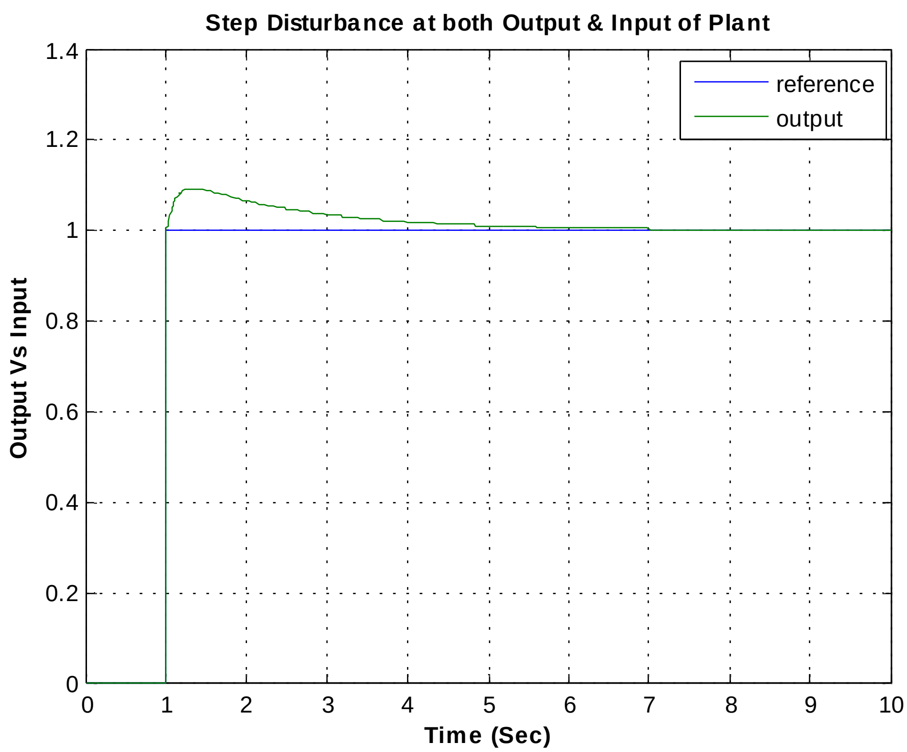
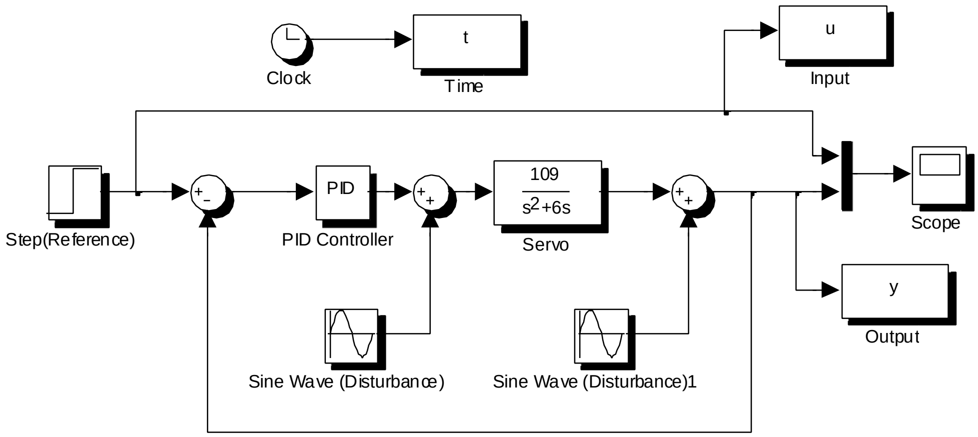



**Figure 4.13: PID Control (Step Disturbance at both Input and Output)**

It was interesting to observe the output as the process is made subject to a step disturbance at both the input and output as shown by model M4.7. The response curve as shown in figure 4.13 shows the output. The response curve overshoots and settles to setpoint at a period of about seven seconds. Not a bad setpoint tracking though! Tuning the parameters may be needed to adjust tracking time to minimal.




**Model M4.8: Sinusoidal Disturbance at both output & input of Plant**

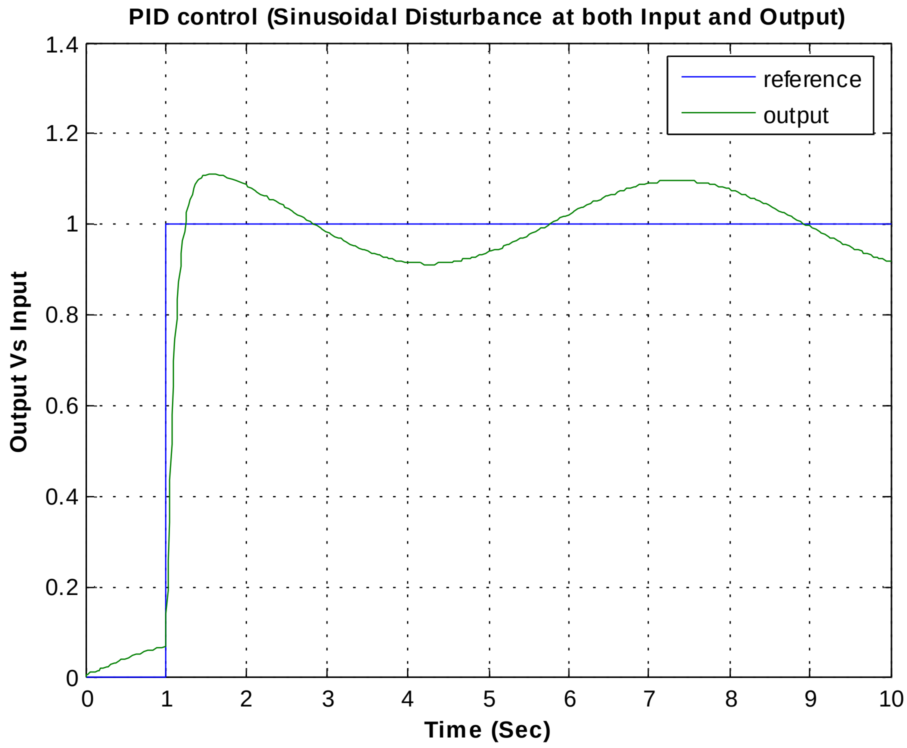



**Figure 4.14: PID Control (Sinusoidal Disturbance at both Input and Output)**

Model M4.8 shows a system where a process is subject to a sinusoidal disturbance at both the input and output. In practice such disturbances are difficult to put under control because they add new dynamics to the system. The response curve to the model above is shown in **figure 4.14.** The response is oscillatory which shows poor performance as far as the PID control effort is concerned. Good regulatory behaviour will be attained by proper tuning of the gains.

### 4.6 SIMULINK Implementations

All SIMULINK implementations are as shown in each section described above. Relative discussion is also taken briefly in each step of the system simulations.

### 4.7 Results and Analysis

Results are modelled in the form of comparing the response to a step input in relation to tracking the setpoint as already discussed above. PID control requires a parameter tuning which of course rounds of to a process of trial and error. It requires a lot of patience because when one parameter is altered, it causes a deviation in one or more of the stable parameters.

## 5.0 Chapter Five: IMC design for the Servo System

### 5.1 IMC Controller

The principal objective in the utilization of the IMC scheme into controlling $G_p(s)$ is to design an IMC controller $G_{IMC}(s)$ which comprise a series combination of $G_c(s)$ and $G_f(s)$ where $G_f(s)$ is a low pass filter of appropriate order. Hence:

$$\mathbf{G_{IMC}(s) = G_f(s)G_c(s)} \quad \textit{Equation 5.1}$$

Therefore, from the steps in designing the controller, we already established that the controller will replicate the inverse of the process model as per the Equation below:

$$\mathbf{G_c(s) = G_{pm}(s)^{-1}} \quad \textit{Equation 5.2}$$

If we assume the process model exactly matches the parent process, then the principle in designing the controller $G_c(s)$ is to set $G_c(s)$ as below:

$$G_c(s) = \frac{s^2 + 6s}{109} \quad \textit{Equation 5.3}$$

### 5.2 Filter Design for the Servo System

In Internal Model Control a filter is normally augmented with the optimal controller to attenuate the effects of process-model mismatching and remove the higher frequency part of the noise in the system in order to meet robust specifications. The robust compensator (filter) plays a pivotal role in the system as it combats plant uncertainties in the system design so that the designed control system can achieve the design objectives of robust stability and robust performance.

The filter is modelled as below from basic control system theory
:

$$G_f(s) = \frac{1}{(1 + \tau_f s)^n} \quad \textit{Equation 5.4}$$

where $\tau_f$ is the filter parameter and n is the order of the filter. The order of the filter is chosen such that $G_{IMC}(s)$ is proper to prevent excessive differential control action. The filter parameter in the design can be chosen as a rule of thumb; hence the filter parameter values are often dictated by modelling errors, as has already stated that in the design, it remains the only tuneable parameter. In this application we would like simulate the system with n=2 to make $G_{IMC}(s)$ proper. We would also like to tune th filter parameter such that it is to be at least twice as fast as the open loop response. Therefore the filter of interest is now modelled as below with n=2;

$$G_f(s) = \frac{1}{(1+\tau_f s)^2} \qquad \text{Equation 5.5}$$

The filter parameter $\tau_f$ remains the only tuneable parameter. Therefore the IMC controller becomes a cascade combination of the optimal controller $G_c(s)$ and the robustness compensator $G_f(s)$. This is mathematically represented below:

$$G_{IMC}(s) = \frac{s^2+6s}{109(1+\tau_f s)^2} \qquad \text{Equation 5.6}$$

or

$$G_{IMC}(s) Z \frac{s^2 H 6s}{109\tau_f{}^2 s^2 H 218\tau_f s H 109} \qquad \text{Equation 5.7}$$

As a result of the above formulations, we can summarise the design with the following descriptions:

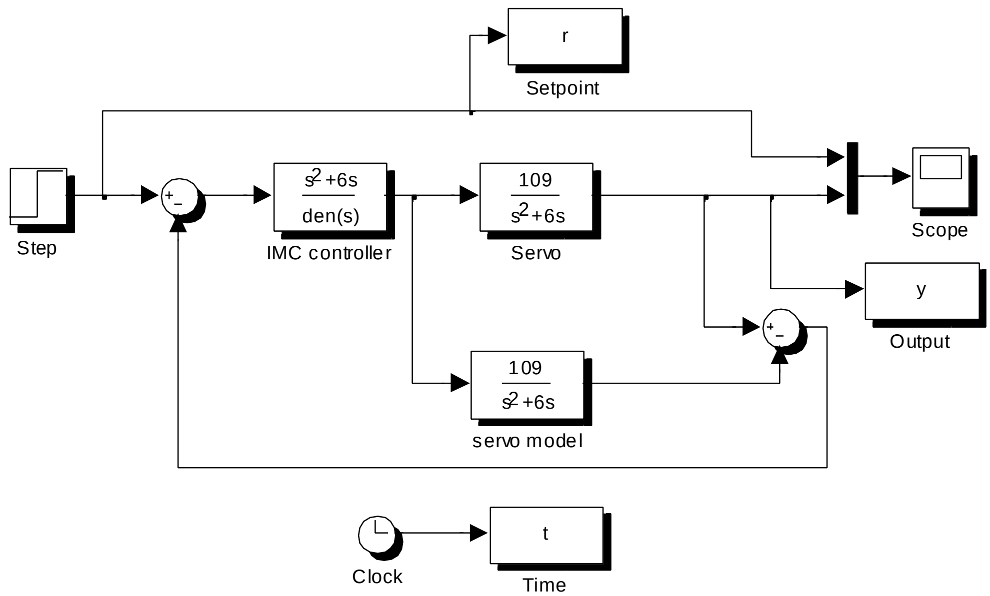
- If the process and the controller (input-output) are stable, and if the process model is perfect, then the control system is stable.
- If the process and the controller are stable, if the process model is perfect, if the controller is the inverse of the process model and if there is no disturbance, then perfect control is realizable.
- If the controller's steady state gain is equal to the inverse of the process model's steady state gain, and if the control system is stable with this controller, then offset-free control is obtained for constant set points and output disturbances.

As a consequence of the latter, if the controller is made of the inverse of the process model cascaded with a low pass filter, and if the control system is stable, then offset-free control is achieved for constant inputs, i.e. set point and output disturbances. Moreover, the filter introduces robustness against a possible mismatch of the process model, and though the gain of the control device without the filter is not infinite as i the continuous-time case, its interest is to smooth out rapidly changing inputs.

Modelling the control, we start with a reasonable number to represent the filter time constant $\tau_f$ as it is the only tuneable parameter in IMC design. We simply let $\tau_f = 1$ so that the controller $G_{IMC}(s)$ is modelled as follows:

$$G_{IMC}(s) = \frac{s^2 + 6s}{109s^2 + 218s + 109} \quad \textit{Equation 5.8}$$

The IMC scheme can then be modelled in SIMULINK as shown by **Model M5.1** below:




**Model M5.1: IMC Design for the Servo System**

The above system has been simulated with a filter parameter of $\tau_f = 1$ and the results of the simulation can be shown in **figure 5.1** below:

The IMC controller in the simulation is modelled as per **equation 5.8** above. If we propose an online tuning a system run with Matlab with define the characteristics of the system and let the filter parameter be input for control action. The next step will of course be continual tuning to obtain the optimal results or response.

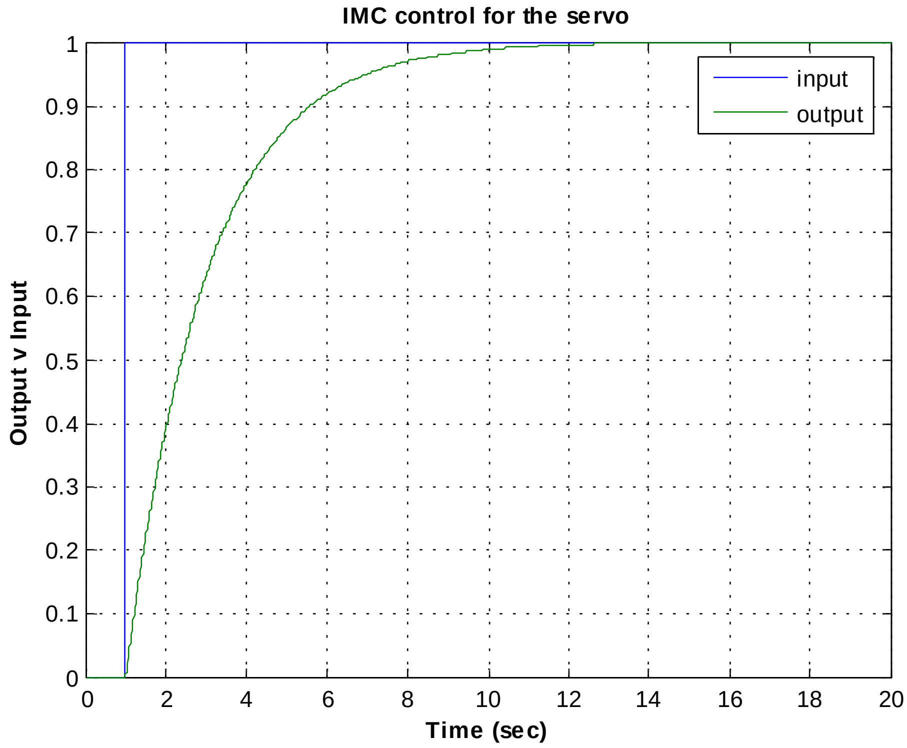



**Figure 5.1: Response to a step input with IMC controller**

Based on the model, the filter parameter was set to a value of 1 and the response tracked the set point after a period of time (12 seconds) as can be seen on the response curve as per **figure 5.1** above. But then when the filter parameter was changed by reducing it to smaller values the response quickly matches the setpoint. For example when the value of $\tau_f$ =0.1 the response looks as shown in figure 5.2 below: In process control good regulatory behaviour is highly indispensable. The need for high precision, fast setpoint tracking and disturbance rejection is in total demand. With this IMC control structure, it is easy to observe that robust control has been imposed to minimize the effects due to disturbances or parametric uncertainties over the dynamic behaviour of the controlled system, particularly over its stability characteristics. The idea of introducing an Internal Model Controller (IMC) for regulation of a servo system is developed here, as a first approach in order to find suitable conditions for the implementation of this technique for servo systems. Simulation results are also shown for the system when subject to disturbances.

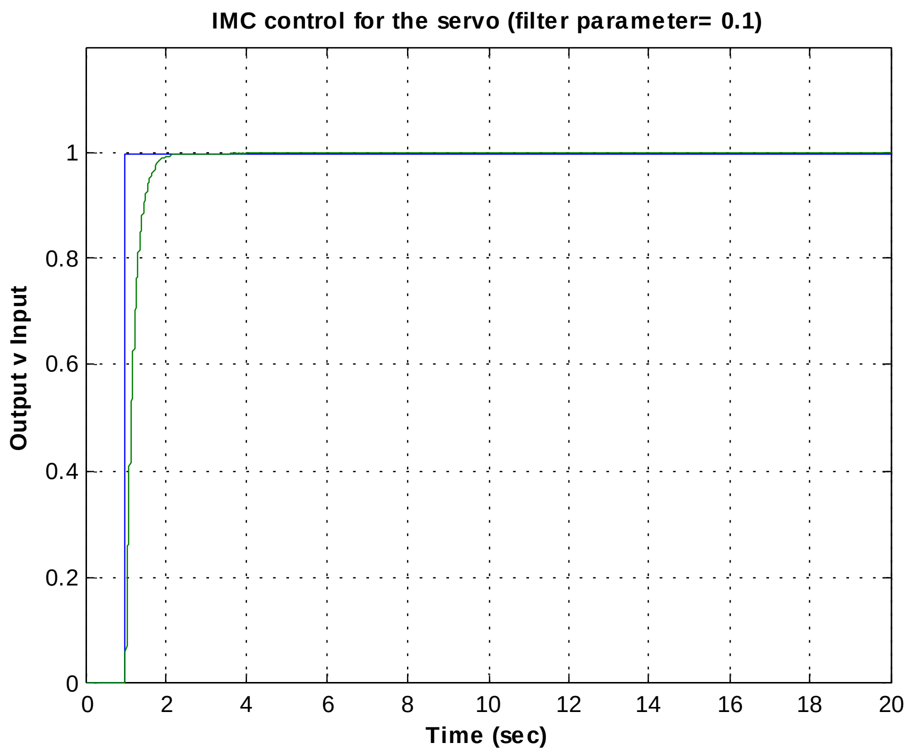



**Figure 5.2: IMC control-Filter Parameter=0.1**

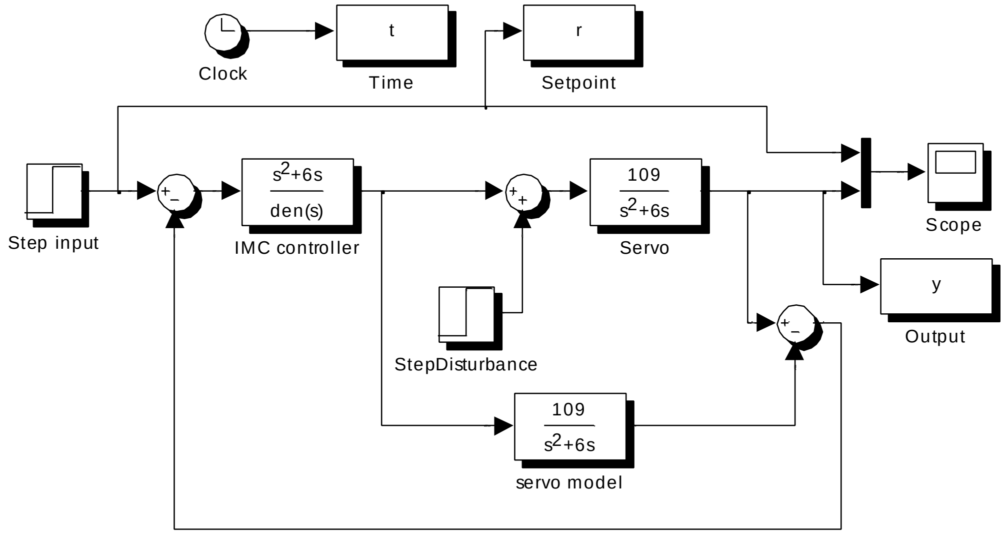
We shall now use this figure $\tau_f$ =0.1 to investigate the effect of disturbances at both the input and the output of the plant as was done with the PID implementations.

### 5.3 Disturbance effects on the Servo System

In this section we are looking into establishing the effects of disturbances at both the input and the output of the plant with the best filter parameters chosen for the IMC controller. For simplicity we shall model disturbances as a simple step input and a sinusoidal wave and see how the response deviates from the optimal response.

#### 5.3.1 Step Input disturbance at the input of the plant

The simulink implementations for a step disturbance can be analysed from the model below:




**Model M5.2: IMC Design: Step Input Disturbance at the input of Plant**

On simulation of the above system, the response plotted with the reference is shown on **figure 5.3** below:

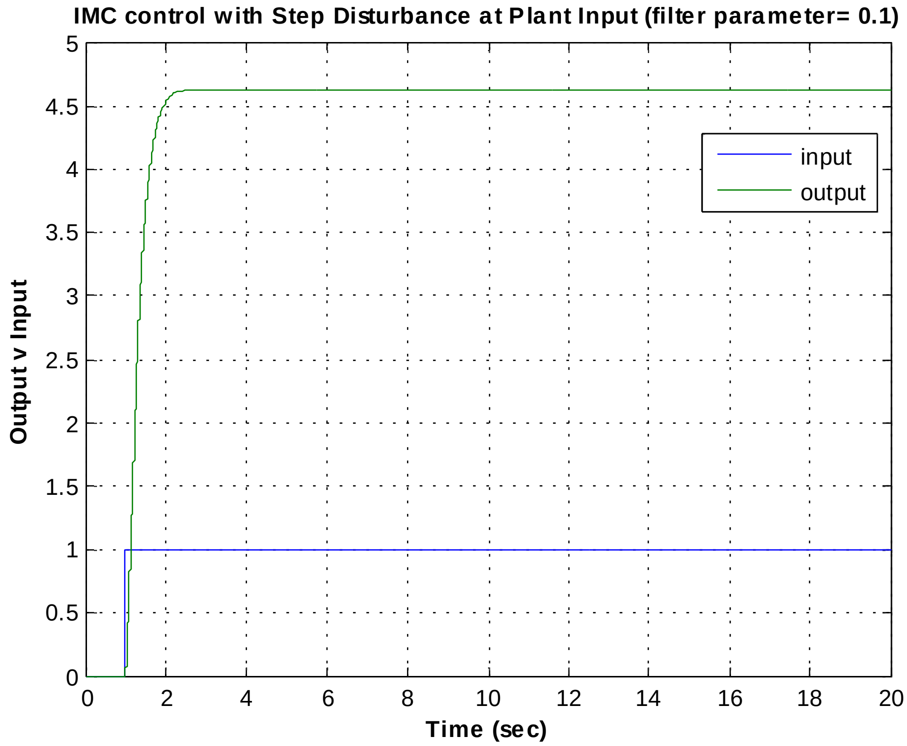


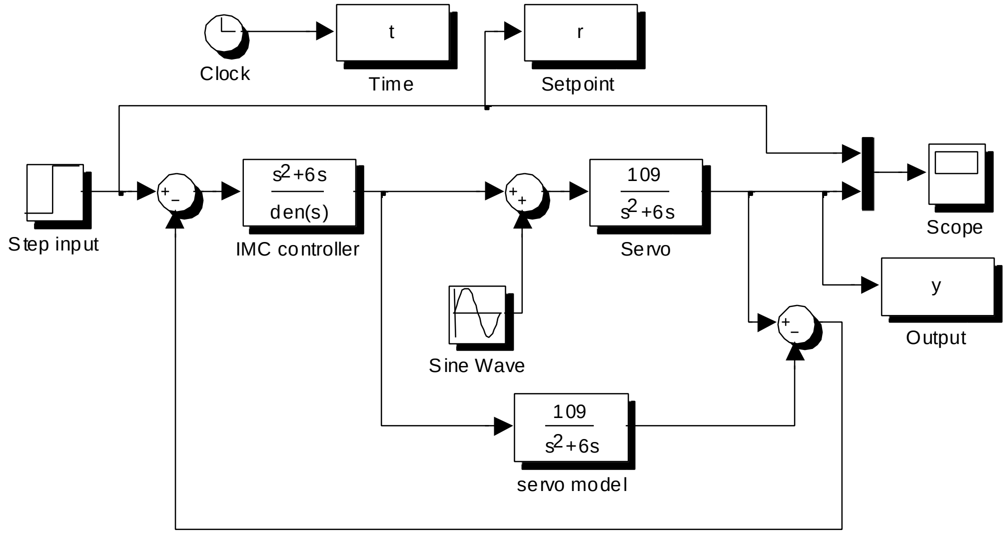

**Figure 5.3: IMC control with Step Disturbance at plant input**

The response as shown by figure three above deviates from the optimal response shown by figure 5.2 when the system is subject to a step disturbance at the input of the plant. This disturbance adds new dynamics to the dynamics of the original plant.

The response though reaches steady state after a period of about two seconds but does not show oscillations which can easily be corrected by tuning the filter parameter.

### 5.3.2 Sinusoidal Input disturbance at the input of the plant

it is also important in this section to consider the effect of a sinusoidal disturbance at the input of the plant since disturbances of this nature are almost everywhere in the daily life of control system. Implementation in simulink is shown by **model 5.3** below:




**Model M5.3: IMC Design: Sinusoidal Disturbance at the input of Plant**

The response to this system above can be shown below on **figure 5.4.** In this system simulation, it is evident that the response with a sinusoidal disturbance is sinusoidal but tuning the filter parameter can diminish the offset to take the response to set point tracking unlike the PID where several PID parameters, the three parameters, will need to be tuned in an alternate fashion to take the response to optimal. In this exercise when decreasing the filter parameter to smaller values, it was evident that setpoint tracking can be achieved as can be shown on **figure 5.5** with filter parameter value set to $\tau_f$ =0.001. It is now a certainty that further reducing the filter parameter disturbance rejection and setpoint tracking can easily be achieved. Proper tuning is always necessary to ensure perfect control and disturbance rejection anyway. The filter parameter plays an important role in the design of an IMC controller as it cab be seen that only the filter parameter experience changes to drive the output to fast setpoint tracking and disturbance rejection. These features describes above are demonstrated with the plot outputs as shown in the figures that follow.

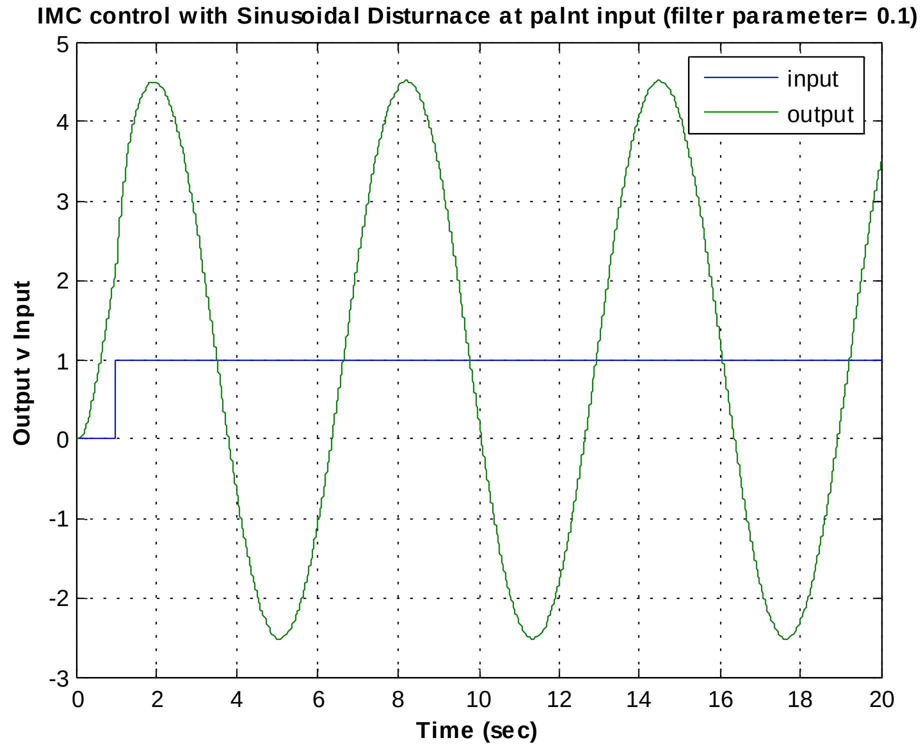



**Figure 5.4: IMC control with Sinusoidal Disturbance at plant input ( $\tau_f$ =0.01)**

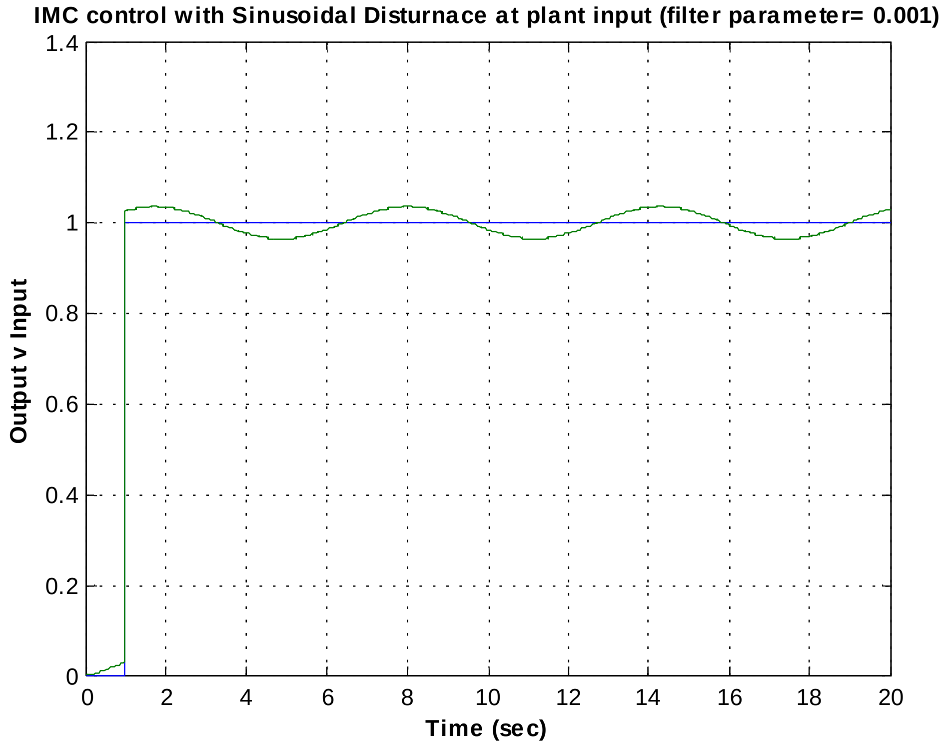
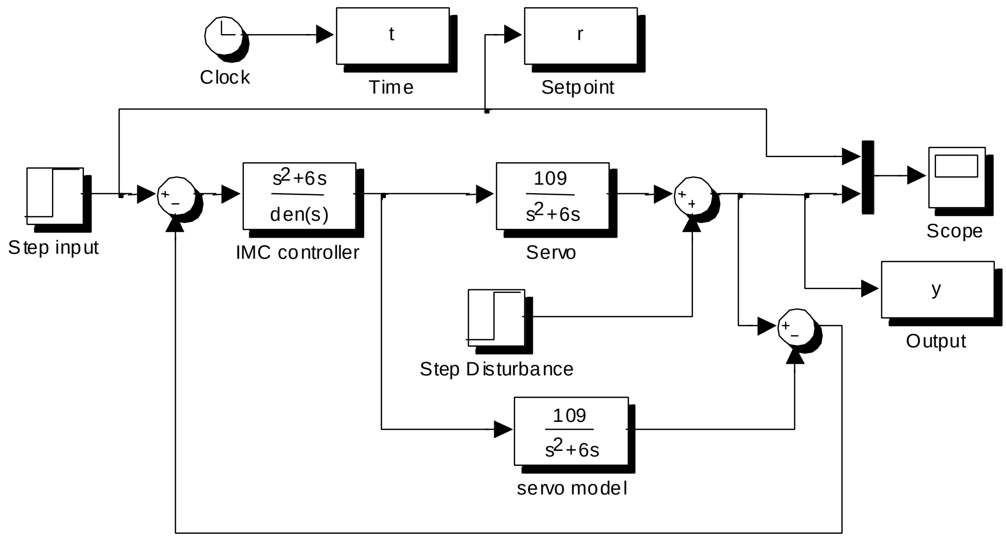



**Figure 5.5: IMC control with Sinusoidal Disturbance at plant input ( $\tau_f$ =0.001**

### 5.3.3 Step Input disturbance at the output of the plant

It is also important to view the effects of disturbances at the output of the plant. First we consider a step input as a disturbance at the output of the plant as can be shown by the simulink model below:




**Model M5.4: Step Input Disturbance at output of Plant**

The response to this system above can be viewed as from figure 5.6 below.

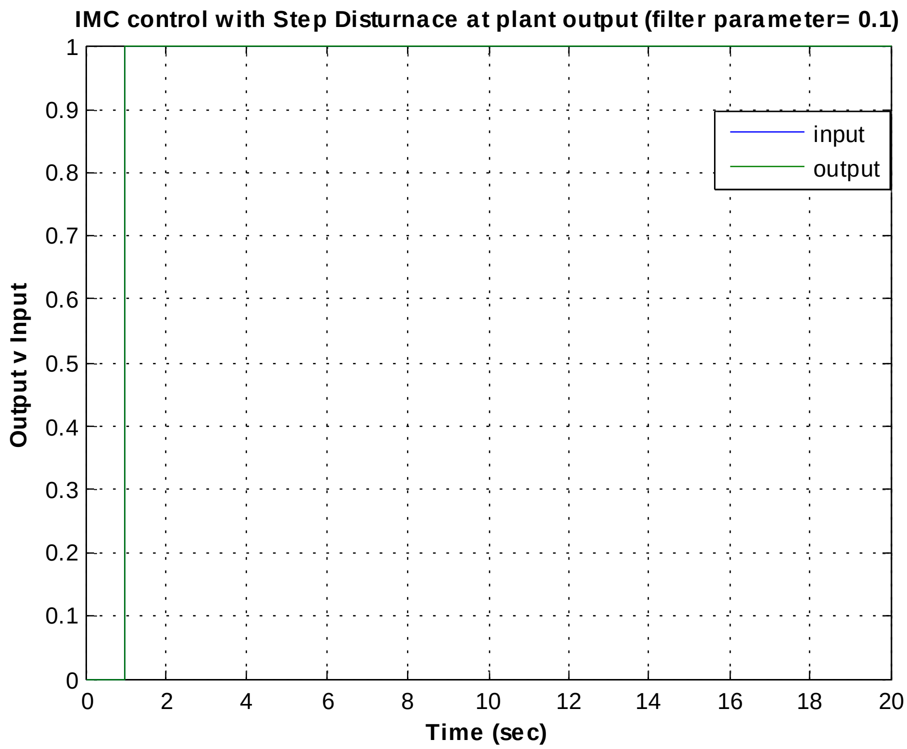
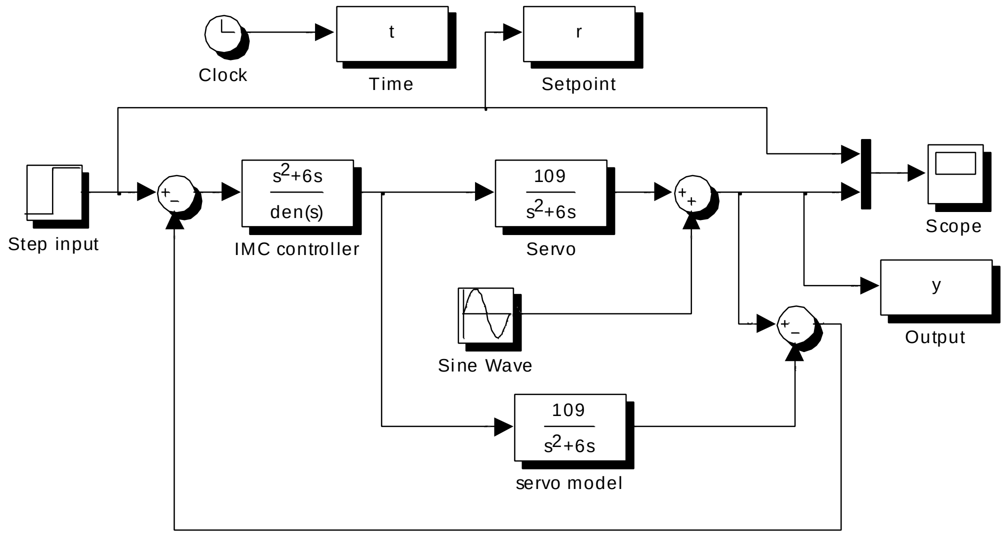



**Figure 5.6: IMC control with a Step Disturbance at plant input ($\tau_f$ =0.1)**

It can be seen from the response above that perfect setpoint tracking and disturbance rejection are easily attained with a step disturbance at the output of the plant.

### 5.3.2 Sinusoidal disturbance at the input of the plant

Simulink Implementations, see the model below:




**Model M5.5: IMC Design: Sinusoidal Disturbance at the output of Plant**

The resulting response as per simulation of the above system can be seen below:

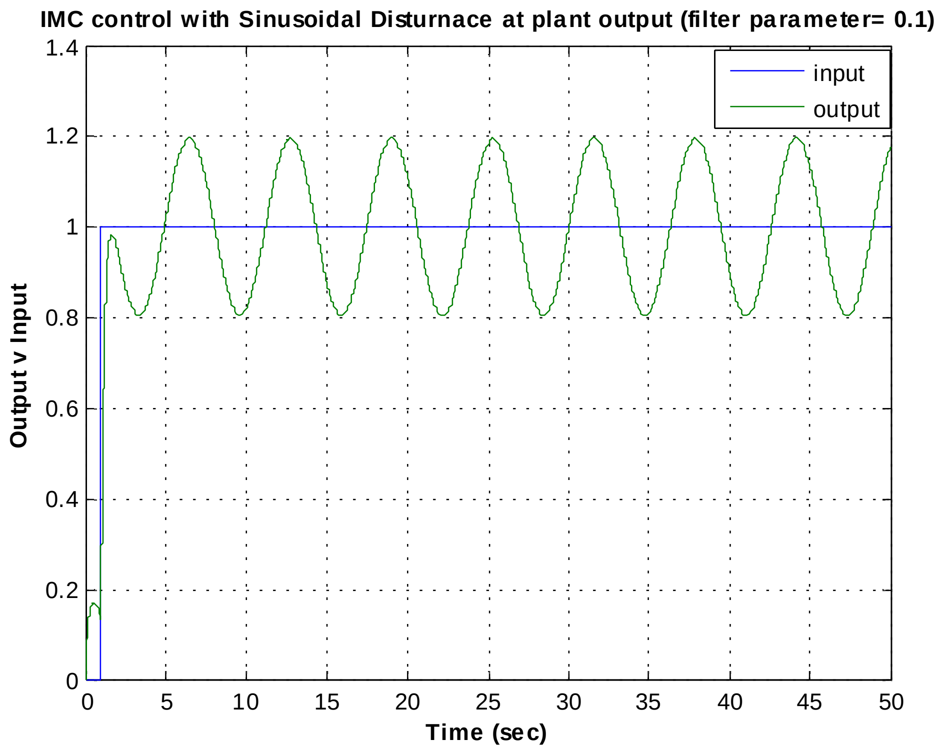



**Figure 5.6: IMC control with a Sinusoidal Disturbance at plant output ($\tau_f$ =0.1)**

With a sinusoidal disturbance at the output of the plant, set point tracking is not entirely lost, the response becomes oscillatory around the reference hence the adjustment of the filter parameter can re- track the optimal desired response as can be viewed from figure 5.7 below by setting the filter parameter to a smaller value, in this case $\tau_f$ **=0.001** . With a sinusoidal disturbance at the output of the plant for this instance, perfect set point tracking and disturbance rejection is achieved.

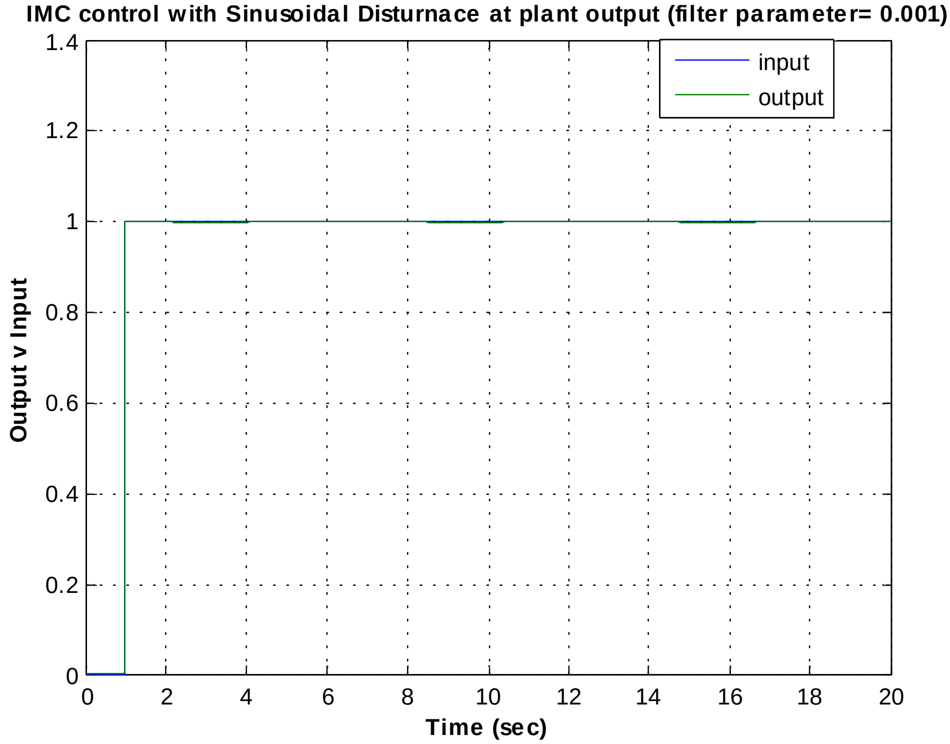



**Figure 5.7: IMC control with Sinusoidal Disturbance at plant output (**$\tau_f$ **=0.001)**

### 5.4 PLANT- MODEL MISMATCHING

Model-based control is a control technique in which a model is used to forecast the future behaviour of the plant. In practice, however, models are not perfect and hence plant-model mismatching is common. Steady state offset is obtained in the presence of plant-model mismatch.
It is therefore important in this study to observe the effects of plant-model mismatch as per the designed IMC controller. Hence we shall consider a few cases in relation to a change in gain, change in damping parameter and a change in pole position.

#### 5.4.1 GAIN VARIATION

This section intends to intercept the response as the model gain is varied. Trial and error procedure has been done and it has been shown that the system and hence the design is sensitive to gain variation. In the process of gain variation it was notices th increasing the gain over a certain value deteriorates the overall performance and hence the stability of the system.

**5.4.1a Gain Reduction**

When the model gain is reduced the system attempts to reach the steady state of the setpoint but fails at a lower value as can be seen from the response characteristics in Appendix A.

**See: APPENDIX B**
**Model 5A.1**
**Figure 5A.2**

**5.4.1b Increase in Gain**

When the model gain is increased, the overall performance attempts to copy the reference actions but settles at a much higher value as compared to the reference.

**See: APPENDIX B**
**Model 5A.1**
**Figure 5A.3**

**5.4.2 Damping Factor Variation**

The effect of damping is another component we need to observe as process-model mismatch encounters this area. In this section, it is quite common to observe the effects of increasing or decreasing the damping factor with respect to the original design.

**5.4.2a Reduction in Damping**

In our simulation study, damping parameter was reduced and the effects can be seen from appendix A. In the response the output follows the setpoint from the start and eventually loses stability.

**See: APPENDIX B**
**Model 5A.3**
**Figure 5A.4**

**5.4.2b Increase in Damping**

When the damping factor is increased but not beyond the point of instability, the response curve overshoots and reaches steady state below the reference value.

**See: APPENDIX B**
**Model 5A.4**
**Figure 5A.5**

**5.4.3 Change in Pole Position**

Process-Model mismatch can incorporate poles which may be complex and non-complex which of course may tell the stability of the system. The concept in this section is to monitor the effect of change in poles between the Right Hand Plane (RHP), the Left Hand Plane and the imaginary axis of the s-plane.

#### 5.4.3a Model with Stable Complex Poles

Incorporating a second order model into the system with complex RHP poles, the response curve is shown to overshoots and begins to settle at a lower position with respect to the reference. The settling is not steady though as small oscillatory motion is evident from the response curve.

**See: APPENDIX B**
**Model 5A.5**
**Figure 5A.6**

#### 5.4.3b Model with Unstable Complex Poles

Though the effect of an unstable model may lead to instability, it was important to see the vulnerability of IMC design with these aspects. A model with complex LHP poles was introduced into the system, simulated and plotted in comparison with the reference. The response curve looks to initially follow the setpoint but eventually loses stability.

**See: APPENDIX B**
**Model 5A.6**
**Figure 5A.7**

#### 5.4.3c Model with Imaginary Axis Poles

A second order model with poles sitting on the imaginary axis was introduced into the system, simulated and plotted with respect to the setpoint. The response curve is shown to overshoot, and later begins to settle at a lower value with respect to the reference.

**See: APPENDIX B**
**Model 5A.7**
**Model 5A.8**

### 5.5 SIMULINK Implementations

All SIMULINK implementations described above can be seen as models from each respective section. In each system modelling by trial and error was for the filter parameter and has been under practice with respect to achieving the optimal response. IMC design involves one parameter tuned every time an optimal response is needed. Hence it saves time as has been shown how easy the response is made to track setpoints while disturbance rejection has been attained as has been discussed in each section.

### 5.6 Results and Analysis

Results for the simulations has been plotted in each section and discussed in details. It is evident from the word go that the responses in each section has been satisfactory. Parameter tuning has been done where necessary to attain some desired results. The importance of this technique (Internal Model Control) has been shown as to how easy it is to attain some optimal results based on parameter setting and tuning with respect to the design of the controller itself. Effects of process-model mismatching have been discussed and relative simulations done in conjunction with possible options of mismatch. The effects of gain variations, pole position and stability components have well been studied as discussed in each section.

Internal model control (IMC) tuning rules have proven to yield acceptable performance and robustness properties when used in the control of typical processes. In general, analytical IMC tuning rules are derived for proportional-integral (Proportional Integral)/Proportional-Integral-Derivative compensators by matching an approximate process model to a low-dimensional reference model. This idea will be explored in detail in the next chapter.

The next chapter studies the optimality of PID-IMC tuning rules to match the prescribed closed-loop behaviour (i.e., the reference model response). To this end, optimal PID settings are computed by means of an IMC design technique. However, significant deviations are displayed for large time delays, which motivate the use of tuning techniques based on numerical optimization to refine IMC settings.

# 6.0 Chapter Six: IMC-PID Framework

## 6.1 IMC-PID parameter setting

The Internal Model Control philosophy can also be used to generate settings for conventional PID controllers [4, 5]. Tuning PID position loops can be challenging because there are three servo gains namely: $K_p$ (proportional), $K_i$ (integral) and $K_d$ (derivative) to be tuned for desired performance of the system under control. Each of the gains plays a different role in terms of the control action for the servo system but once the roles are well understood tuning becomes easier where each gain can be tuned independently, saving time and ensuring consistency. A convectional PID loop structure is depicted below for a position servo system in **figure 6.1**

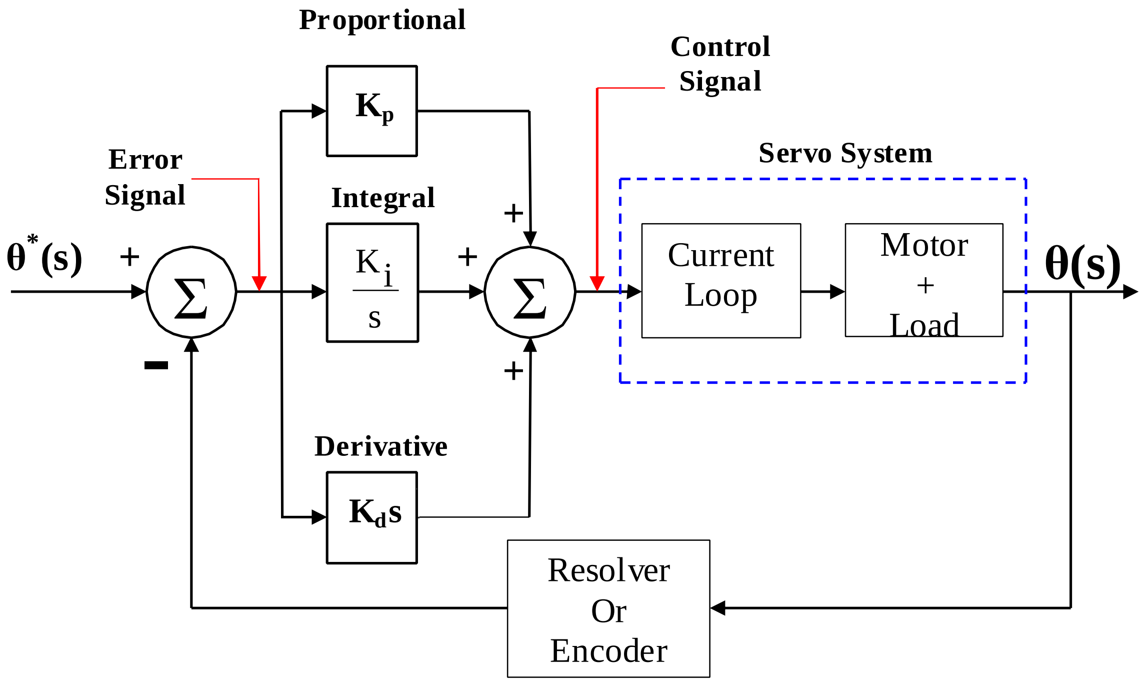



**Figure 6.1: Servo Control Topology**

One of the methods for Proportional Derivative and Integral controller (PID) parameter tuning is the internal model control and PID (IMC-PID) tuning method, which is based on keeping the controlled variable response close to the desired closed-loop response [3, 5]. An important advantage of this method is that the closed-loop time constant, which is the same as the internal model control (IMC) filter time constant, provides convenient tuning parameter to adjust the speed and robustness of the closed-loop system. However, this method gives derivative and integral time constants which do not depend on the closed-loop system time constant. Also, this method can not be used for every process model. Research have made tremendous amount of work in this field and came up with a lot of ideas and structures which best fit the problem at hand. In [13], a new approach is established to IMC-PID tuning method and gained the PID parameters for general models by approximating the ide controller with a Maclaurin series in s domain. With this method, controller parameters become dependent on the closed-loop time constant and the closed-loop response becomes better if not optimal.

Tuning of these conventional controllers involves several parameter tuning but with IMC implementation it is easy to set PID parameters based on the controller design methodology [5]. The implementation of a conventional PID in an IMC structure can be seen as a block below where the block is simply rearranging the IMC structure discussed earlier from **figure 2**:

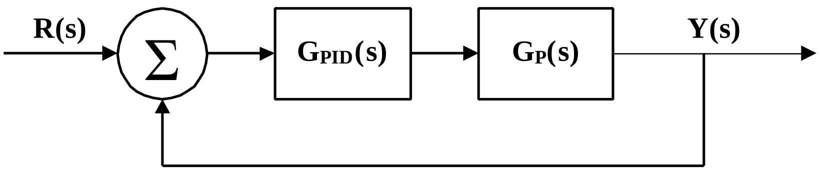



**Figure 4: IMC-PID Framework [4,5]**

This design philosophy extends from the knowledge discussed earlier on IMC design techniques. Thus modelling the PID in IMC means that we structure the PID as follows:

$$G_{PID}(s) = \frac{G_{IMC}(s)}{1 - G_{IMC}(s)G_{pm}(s)} \quad \textit{Equation 6.1}$$

where $G_{pm}(s)$ and $G_{IMC}(s)$ are as discussed earlier. i.e.

$$G_{pm}(s) = G^{+}_{pm}(s)G^{-}_{pm}(s) \quad \textit{Equation 6.2}$$

and

$$G_{IMC}(s) = G_{f}(s)G_{c}(s) \quad \textit{Equation 6.3}$$

where

$$G_{c}(s) = G^{+}_{pm}(s)^{-1} \quad \textit{Equation 6.4}$$

Therefore $G_{PID}(s)$ is finally modelled as:

$$G_{PID}(s) = \frac{s+6}{218\tau_{f}(1+0.5\tau_{f}s)} \quad \textit{Equation 6.5}$$

See **Appendix C**

Using the table as per **Appendix E** for IMC based PID controller proposed by [14] with the model used in this dissertation, it is easy to compare relevant terms with the following conventions:

From the Table: **Filter Parameter = $\varepsilon = \tau_f$**

Hence following through **Case K**, it becomes convenient and in context very easy to represent the model used in this study in the given form.

$$G_{pm}(s) = G_p(s) = \frac{109}{s(s+6)} \quad \text{Equation 6.6}$$

$$G_{pm}(s) = \frac{109}{s(s+6)} = \frac{109/6}{s(1/6\, s+1)} \quad \text{Equation 6.7}$$

As per the table from the **Appendix E,** it thus follows that the parameter setting would look like below:

Table parameters:

| k | τ |
|---|---|
| $\frac{109}{6}$ | $\frac{1}{6}$ |

**Table 6.1**

These two parameters can therefore be used to calculate parameters for the PID as defined in the table below:

| Proportional | Integral Time | Derivative Time |
|---|---|---|
| $k_c k$ | $T_I$ | $T_D$ |
| $\frac{2\varepsilon+\tau}{\varepsilon^2}$ | $2\varepsilon+\tau$ | $\frac{2\varepsilon\tau}{2\varepsilon+\tau}$ |

**Table 6.2**

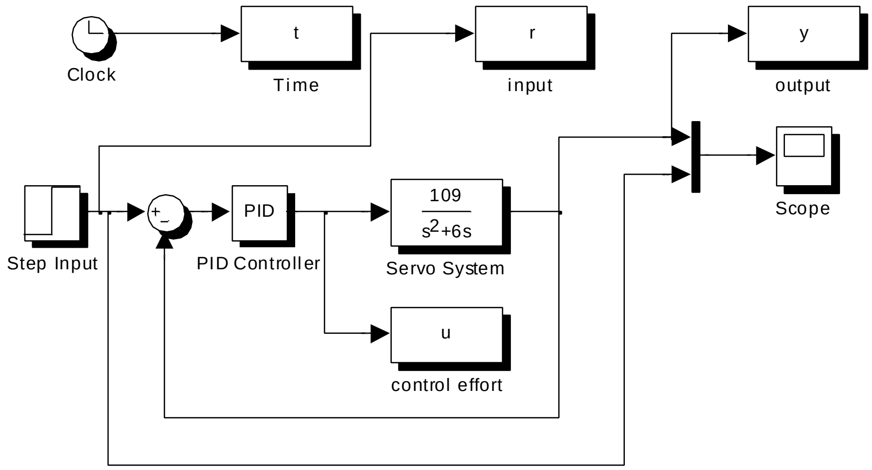
As has been shown in the development of convention for parameters of the same class, it is worthwhile to retouch these parameters once more. In this convention, a filter parameter has been defined from previous sections as $\geq_f$ and has already been emphasised that $\geq_f = \varepsilon$

It is now easy to see that we can set the gain parameters of the conventional PID as per table 6.2 above. By setting these parameters we can now tune only one parameter, the filter parameter $\geq_f = \varepsilon$, for desirable responses. Simulation study can be viewed with the following lines of code from Matlab used to control parameters in the PID settings:

```
% ***This M-File is used to generate settings for the PID parameters ***
% **Proportional Gain (KP) ****
% ****Integral Gain (KI) *****
% ****Derivative Gain (KD) *******

k=109/6; % Model Gain
tau=1/6; % time constant
eps=input ('Enter the Filter Parameter and Press Enter   ')
KP= (2*eps+tau)/ (eps^2*k);
KI=1/ (2*eps+tau);
KD= (2*eps*tau)/ (2*eps+tau);
```

These lines of code where used to provide an online tuning of the PID parameters for the model shown below:




**Model M6.1: IMC-PID Framework ($\geq_f = \varepsilon = 0.5$)**

The value of the filter parameter has been first chosen arbitrarily as 0.5 and the output response is shown below:

The response is shown to overshoot from the reference and reaches steady state after a period of ten seconds. The choice of the filter can thus further improve performance of the system. The objective in every control system design is fast and accurate setpoint tracking, implying that the effect of external disturbances should be corrected as efficiently as possible. We therefore further alternate the filter parameter to find the best if not optimal response. This is again a trial and error approach to diminish the overshoot and to minimize the time the system takes to reach steady state.

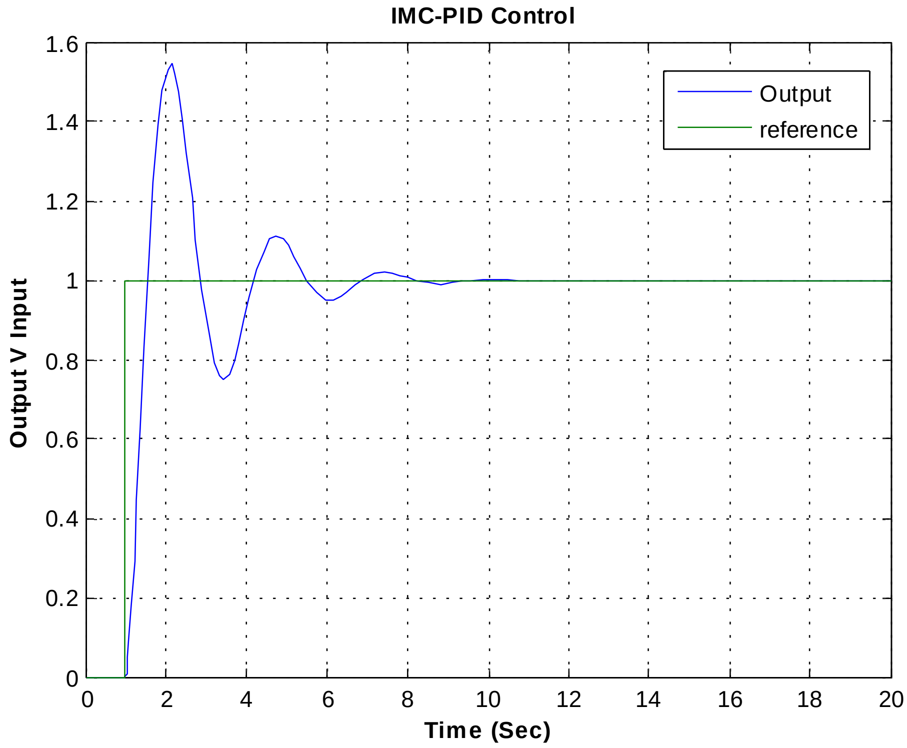
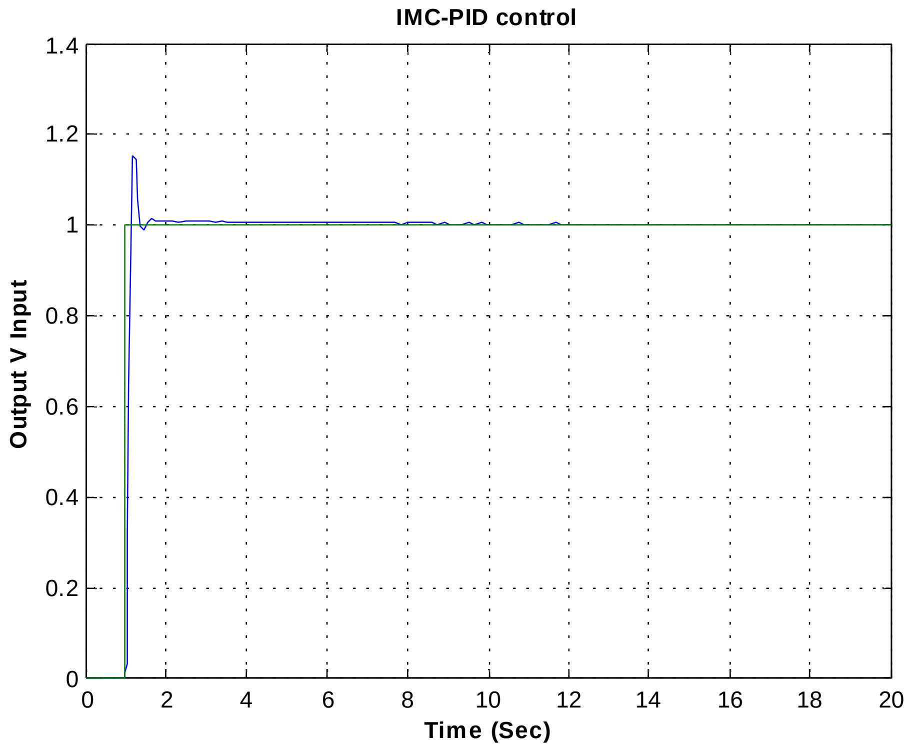



**Figure 6.1: IMC-PID Control**

With the filter parameter being set to 0.09 we model the system in SIMULINK as shown by Model M6.1, and the overall response look like as in **figure 6.2** below:



IMC-PID framework has the capability to achieve perfect set point tracking and disturbance rejection by setting up the PID parameters as shown above. Not only do the IMC-PID parameters lead to better performance and robustness than the traditional methods, the IMC design procedure also makes the search for appropriate parameters simpler.

### 6.2 IMC-PID parameter tuning

The IMC-PID framework allows one parameter to be tuned for the PID, unlike convectional PID where three parameters need constant lookout when one parameter is offset by some trial number. This makes life easier for online tuning as the parameters are relative to one another as are independent on the filter parameter in the case of the system employed in this dissertation. It is important, however, to observe the effects of disturbances with respect to the chosen filter parameter and the settings as per **table 6.2** above

### 6.3 External Disturbances

In practice, there are quite a lot of systems to control and each system may be subject to various external inputs such as noise, interferences etc, which may deteriorate performance capabilities if not monitored with care and absolute efficiency. In this section it will be normal to consider only a step and a sinusoidal input as a form of disturbance affecting the system, both at the input and the output of the plant.

#### 6.3.1 Step Input at the Input of the Plant

In the nature of control it is very hard to deal with disturbances that affect the system at the input of the process to be controlled. The reason being that the disturbance will add new dynamics to the process as can be shown by Model 6C.1. The output response is shown in the figure that follows (Figure 6C.1). All SIMULINK implementations discussed in this section can be seen in Appendix D.

See (Appendix D): **Model M6C.1**
**Figure M6C.1**

#### 6.3.2 Step Input at the Output of the Plant

It is also crucial to monitor control action as another disturbance exists at the output of the plant. In this case, new system dynamics are added to the overall output and hence the error signal will impose new fundamental dynamics to the controller.

See (Appendix D): **Model M6C.2**
**Figure M6C.2**

#### 6.3.3 Step Input at both the Output and Input of the Plant

The system is made to be subject to a step disturbance at both the input and the outp of the plant.

See (Appendix D): **Model M6C.3**
**Figure M6C.3**

**6.3.4 Sinusoidal Disturbance at Input of the Plant**

The system is also made subject to a sinusoidal disturbance at the input of the plant. Sinusoidal disturbances are in nature very common, because sinusoidal harmonic distortions are a true representation of disturbances sines and cosines as the principles of Fourier has proposed in literature. It is important to observe the nature of these signals and their impact on influence of control systems in general.

See (Appendix D): **Model M6C.4**
**Figure M6C.4**

**6.3.5 Sinusoidal Disturbance at Output of the Plant**

The system is influenced by the effect of a sinusoidal disturbance at the output of the plant. The dynamics of the overall output are altered in general theory and the response is shown with a corresponding output.

See (Appendix D): **Model M6C.5**
**Figure M6C.5**

**6.3.6 Sinusoidal Disturbance at both Input and Output of Plant**

In this section, numerous attempts are validated with the system being subjected to a sinusoidal disturbance at both the input and output of the plant.

See (Appendix D): **Model M6C.6**
**Figure M6C.6**

**6.4 Results and Analysis**

Results can be seen in Appendix D. The output response was plotted and compared with the reference signal (setpoint). In comparison with the optimal response as plotted in **figure 6.2** above, when the process is subject to a step disturbance at the input, it produce an output which overshoots and settles steadily and perform setpoint tracking as desired. With this in mind tuning the filter parameter can enhance the quality of the response thereby reducing the overshoot and reaching steady state with setpoint tracking very quickly.
When the system encounters a step at the output plant, the response is shown to exhibit the setpoint perfectly without any overshoot. However, when the step is introduced at both the input and output of the plant, the response is shown to overshoot and follows the setpoint at a later time.

The effects of sinusoidal disturbances show similar results. In general when the disturbance is affecting the system at the output side of the process, proper control can be easily maintained, but when the system is subject to a disturbance at the input, control action is lost to some extent as the disturbance adds new dynamics to the overall control effort.

# 7.0 Chapter Seven: IMC and PID Comparison

The manner in which a measured process variable responds over time to changes in the controller output signal is fundamental to the design and tuning of a PID controller. The best way to learn about the dynamic behaviour of a process is to perform experiments which may involve a simulation study based on trial and error. Critical to success is that the process data generated by such experiments be descriptive of actual process behaviour. Discussed above are the qualities required for “good” dynamic data and methods for modelling the dynamic data for controller design. Parameters from the dynamic model are not only used in correlations to compute tuning values, but also provide insight into controller design parameters such as loop sample time and whether dead time presents a performance challenge. It is becoming increasingly common for dynamic studies to be performed with the controller in automatic (closed loop). For closed loop studies, the dynamic data is generated by bumping the set point. The method for using closed loop data has been illustrated and demonstrated fully.

## 7.1 Effects of disturbances

It is essential that the test data contain process variable dynamics that have been clearly (and in the ideal world exclusively) forced by changes in the controller output. Dynamics caused by unmeasured disturbances can seriously degrade the accuracy of an analysis because the modelling tool will model those behaviours as if they were the result of changes in the controller output signal. In fact, a model-based control, a model can look perfect, yet a disturbance that occurred during data analysis and observation can cause the model to be useless in the controller design. In every aspect used for control in this dissertation, effects of disturbances were well monitored, modelled and simulated incorporated in each section of the control aspect. It is as well important to consider such disturbances because in practice, control systems are often subject to unknown disturbances, non-linearities and non-uniformities which may degrade performance objectives of the system at hand.

All simulation results have been showing that with the disturbances the IMC control scheme takes the upper hand in set point tracking and disturbance rejection. It is also shown that with numerous attempts to put control to the servo system, IMC prove to be capable of being a robust control system as it has the capability to combat uncertainties affecting the system.

## 7.2 Parameter Tuning

"Tuning" a control loop is the adjustment of its control parameters to the optimum values for the desired control response. The optimum behavior on a process change or setpoint change varies depending on the application. Some processes must not allow an overshoot of the process variable from the setpoint. Other processes must minimize the energy expended in reaching a new setpoint. Generally stability of response is required and the process must not oscillate for any combination of process conditior and setpoints. Tuning of loops is made more complicated by the response time of th process; it may take minutes or several hours for a setpoint change to produce a stab effect. Some processes have a degree of non-linearity and so parameters that work well at full-load conditions don't work when the process is starting up from no-load.

Convectional PID has shown that tuning the parameters can be exhaustive. One adjustment of one parameter may offset one or many of the parameters which were may be set to optimal degree of performance. This then require continual tuning and hence tiring and time consuming. Unlike the PID, IMC design architecture require only one parameter tuning and resolve the aspect of time consuming as the method is effective and efficient. A PID designed with theory and concepts of IMC as has already been established has shown that the PID parameters may depend on the filter parameter which is adjustable. One adjustment of the filter parameter causes all the PID parameter to offset to optimal at the same time.

# 8.0 Chapter Eight: Future Work

Future work in this study may relate to several aspects of the IMC design. First, it would be wise to consider this method on non-linear system to observe its adaptability to a variety of systems and what sort of extension may need to be added to make the principle more advanced in the area of control and design.

Proportional-integral-derivative (PID) controller tuning method based on the internal model control is one of the simplest tuning rules and provides excellent performances for various processes. It has one design parameter which determines the speed of closed-loop response. As the design parameter changes, the controller gain changes while integral and derivative times usually remain constant (if independent of the filter parameter). Hence, the method can be extended easily to the design of multiloop control systems because there are simple methods to find stable multiloop proportional controller gains. However, integral and derivative times independent of the design parameter can cause poor closed-loop responses for some processes such as those with small dead times. For such processes, 2 degree of freedom control systems are usually required, and a two-step method where a proportional controller is designed first and then a PID controller is designed for the compensated system can be used to design the 2 degree of freedom control systems. This two-step method is applied to the design of multiloop control systems for interacting multivariable processes

In process control, the internal model control (IMC) scheme [3] has gained high popularity. This is due to the good disturbance rejection capabilities and the robustness properties of the IMC structure. Furthermore, the controller design is simple and straightforward such that the controller can easily be tuned by the process engineer. The IMC controller design is theoretically well explored for linear processes. In practice, however, almost every process displays nonlinear behaviour especially if it is driven in a wide operating range. Hence, the need emerges to extend the linear design procedure to nonlinear systems.

Neural networks as well as fuzzy systems have been widely employed for the representation of nonlinear systems and the idea of internal model control can be combined with these types of models [15].Thus, it will be a tremendous effort to extend the knowledge of IMC design technique to other control architectures such as the neural networks and fuzzy logic control as discussed above.

Nevertheless, it would be important to device a software control tool that can be parameter tuned online and used in industry to experience the performance objective of the whole material at large as theory has to be developed into practice.

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# APPENDIX A

University of Southern Queensland

FACULTY OF ENGINEERING AND SURVEYING

## ENG 4111/4112 Research Project
## PROJECT SPECIFICATION

**PREPARED BY**: Donald A. Mohutsiwa

**PROJECT TOPIC**: PID controller tuning using Internal Model Control (IMC) Method

**SUPERVISOR**: Dr Paul Wen

**PROJECT AIM**: The aim of the project is primarily to design a PID controller with parameter tuning based on the theory and concepts of Internal Model Control. This simply implies that the parameter setting of the PID are modelled in the design of the IMC controller. The performance objective of the design is then tested and simulated in control of a servo system. Hence in this project, a PID tuning control system using Internal Model Control will be designed, simulated, implemented and tested.

**Research Structure:**

1. Research and study the principle and the use of IMC.
2. Study PID tuning principles with its modeling and simulation
3. Provide a model of the PID control system and formulate its inverse
4. Build up an IMC tuning control system for the PID
5. Provide a simulation Model using Matlab/Simulink
6. IMC control system tests and improvement
7. Results, analysis and conclusions

**As time permits:**

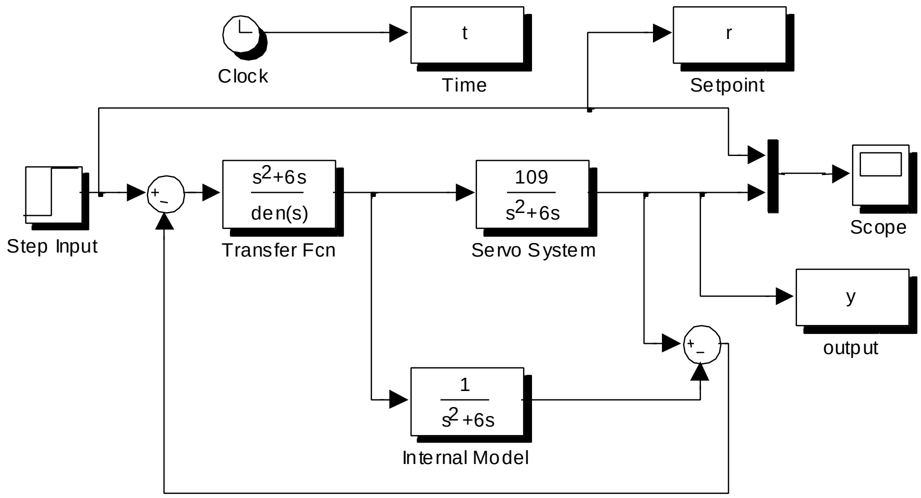
1. Develop a software tool based on a Matlab/Simulink platform that can be used industrially as a PID tuner using the Internal Model Control

Supervisor Approval

Donald A. Mohutsiwa (Student) _________________ , ___/11/06

Dr Paul Wen (Supervisor) _________________, ___/11/06

# APPENDIX B




**Model M5A.1: Process-Model Mismatch (Gain Variation)**

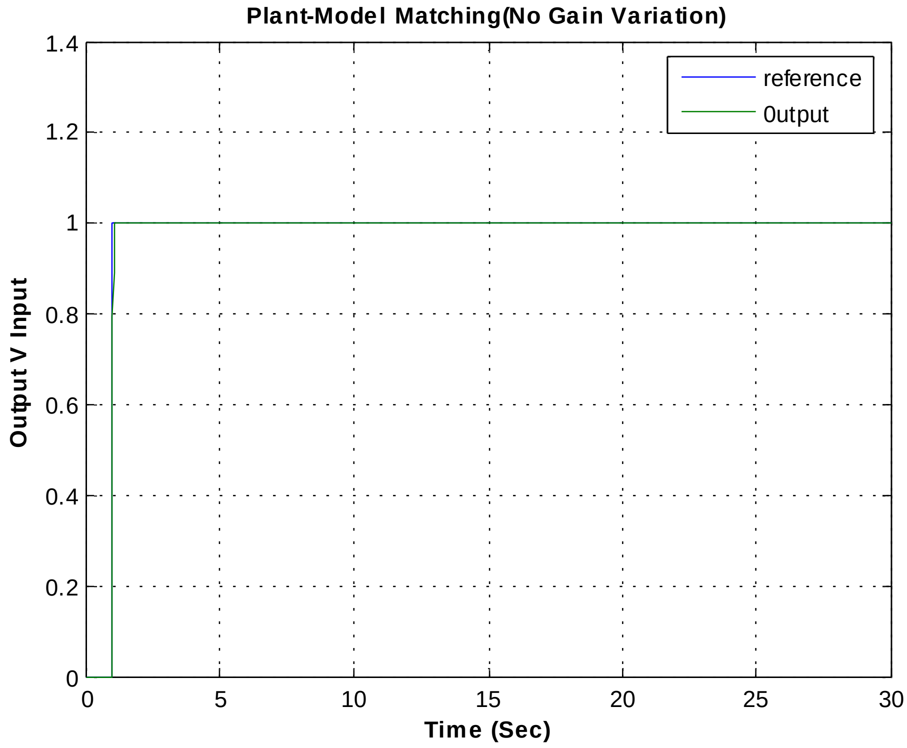



**Figure 5A.1: Process Model Mismatch (Gain=109)**

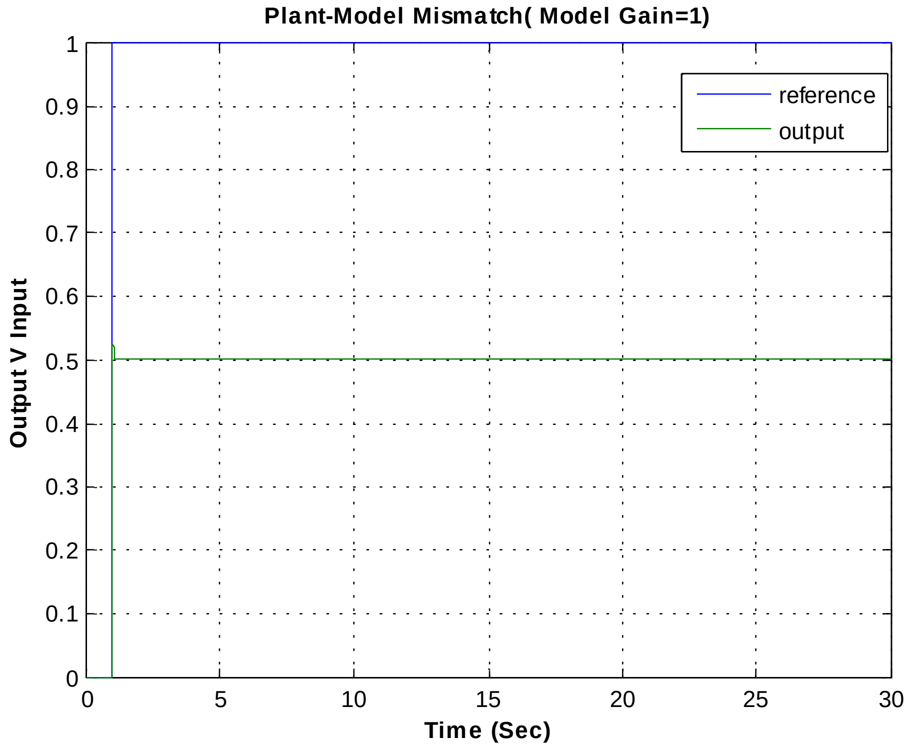
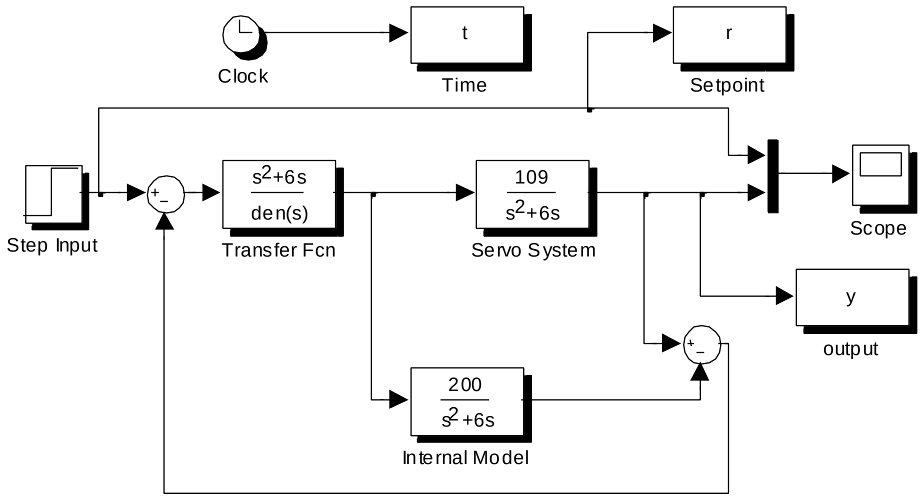



**Figure 5A.2: Process- Model Mismatch (Gain=1)**




**Model M5A.2: Process-Model Mismatch (Gain = 200)**

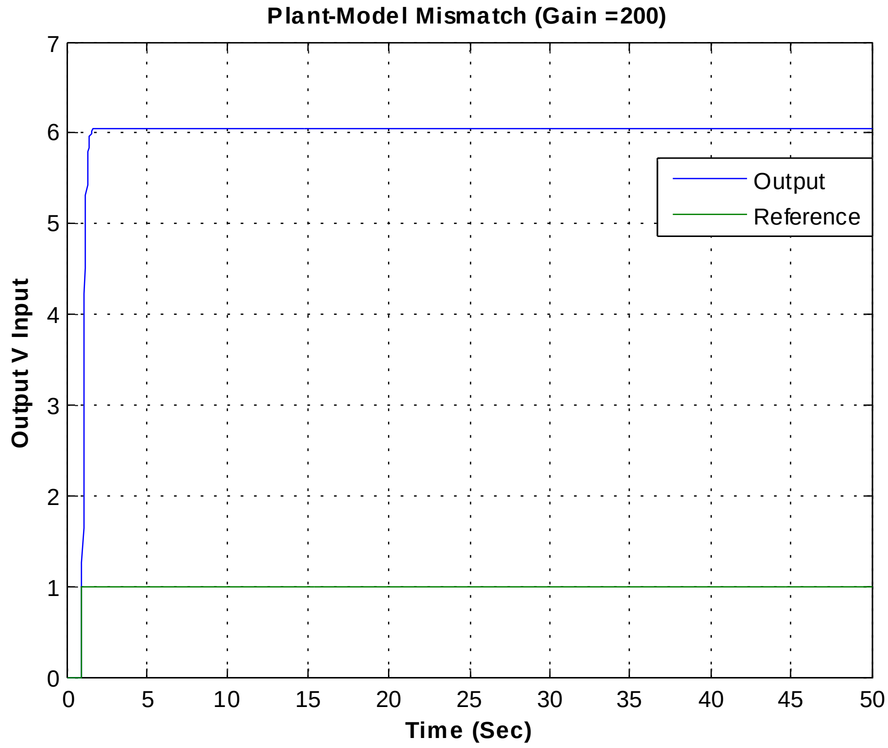
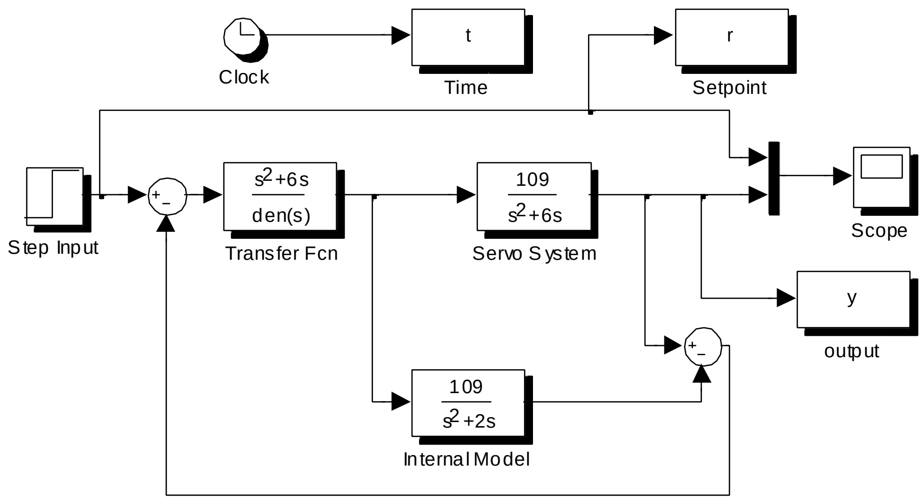



**Figure 5A.3: Process-Model Mismatch (Gain=200)**




**Model M5A.3: Process-Model Mismatch (Damping Reduced)**

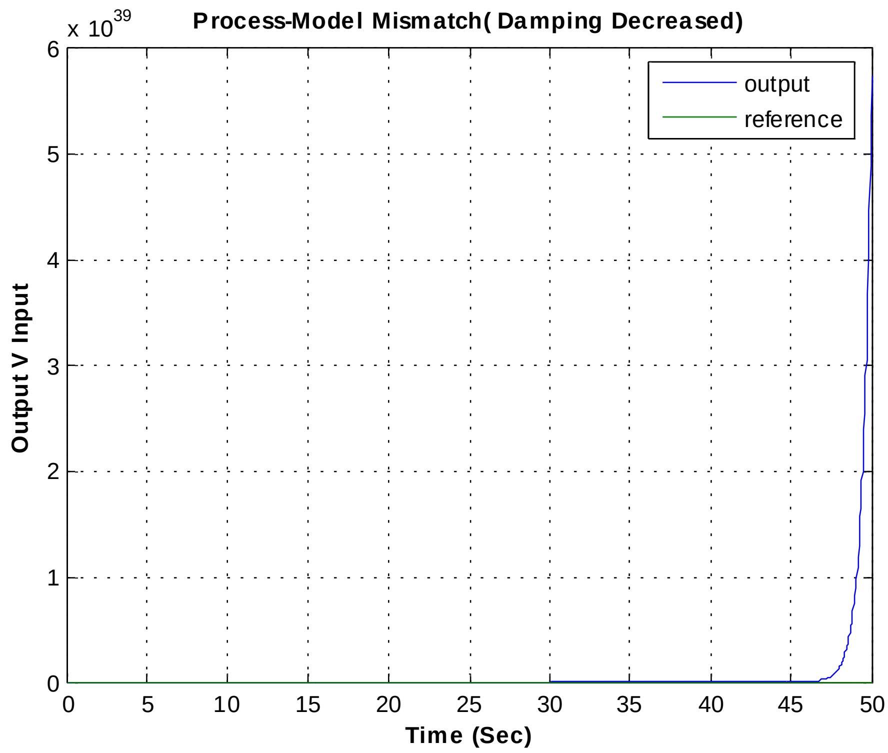
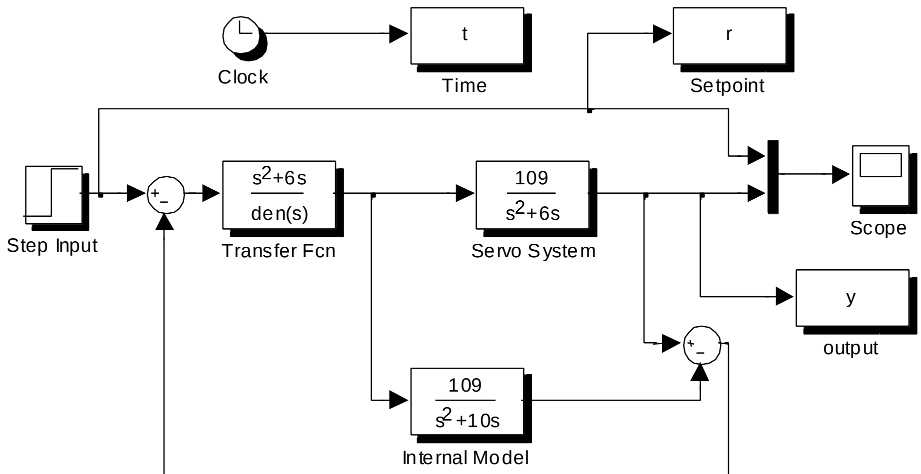



**Figure 5A.4: Reduction in Damping in the Model**




**Model M5A.4: Process-Model Mismatch (Damping Increased)**

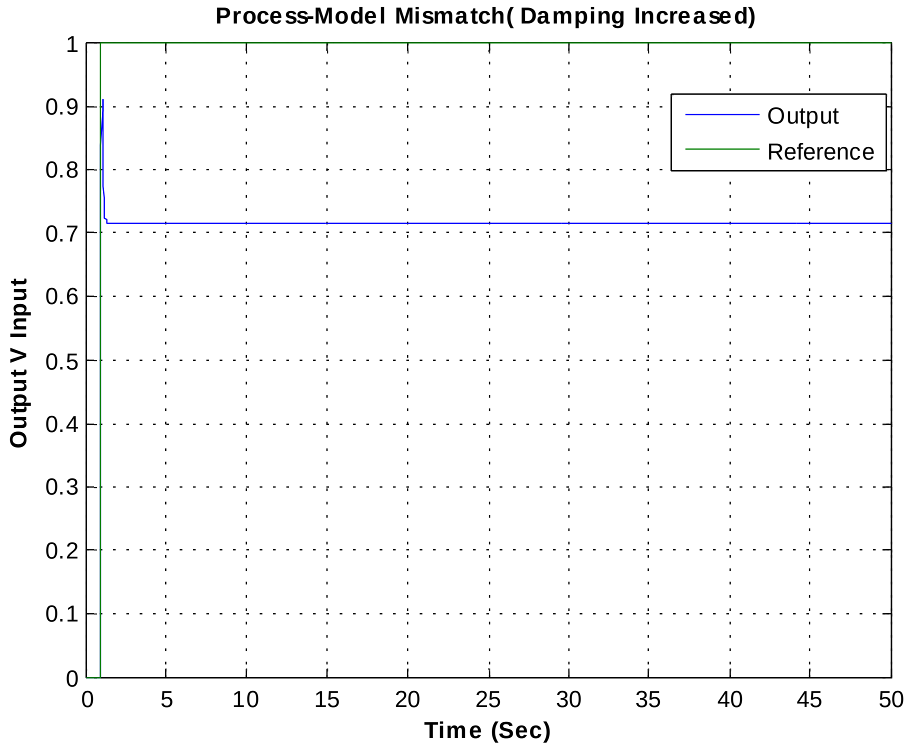
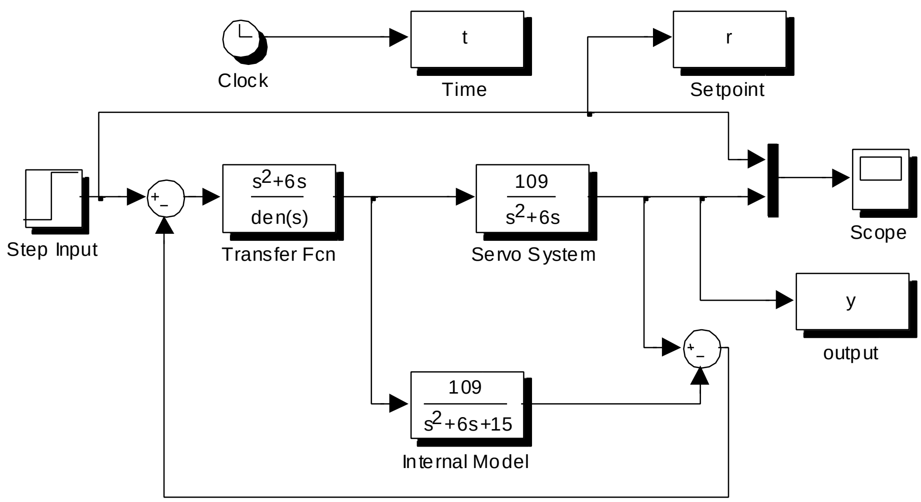



**Figure 5A.5: Increase in Damping in the Model**




**Model M5A.5: Process-Model Mismatch (Complex Poles-Stable)**

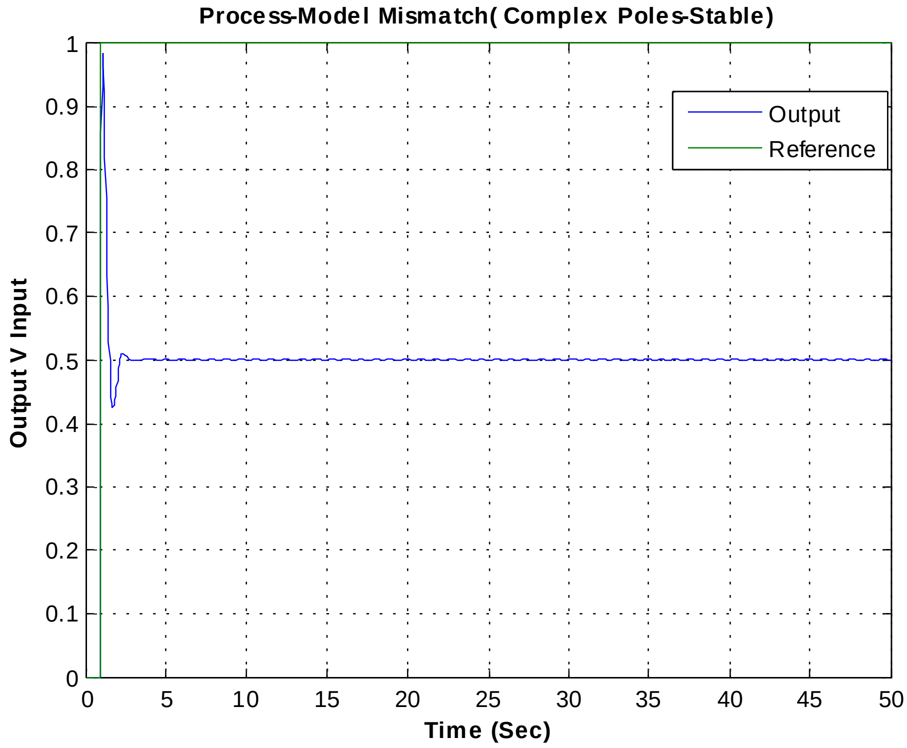
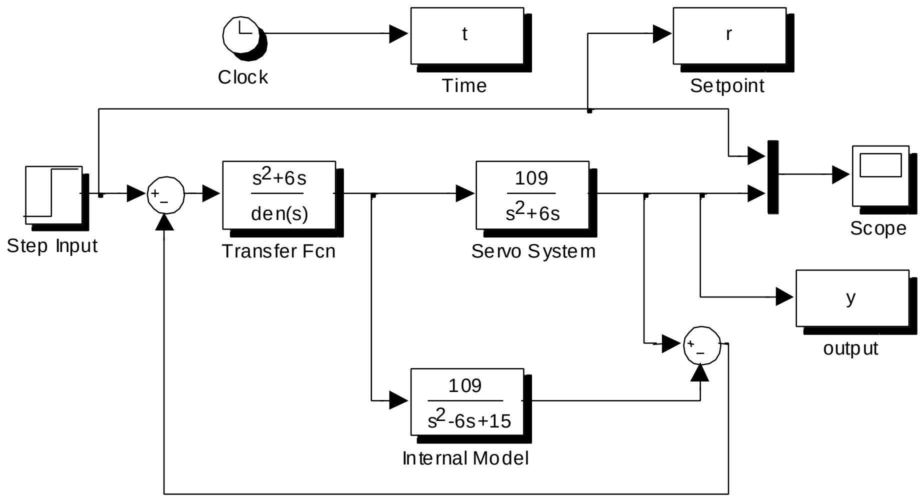



**Figure 5A.6: Model with Stable Complex Poles**




**Model M5A.6: Process-Model Mismatch (Complex Poles-Unstable)**

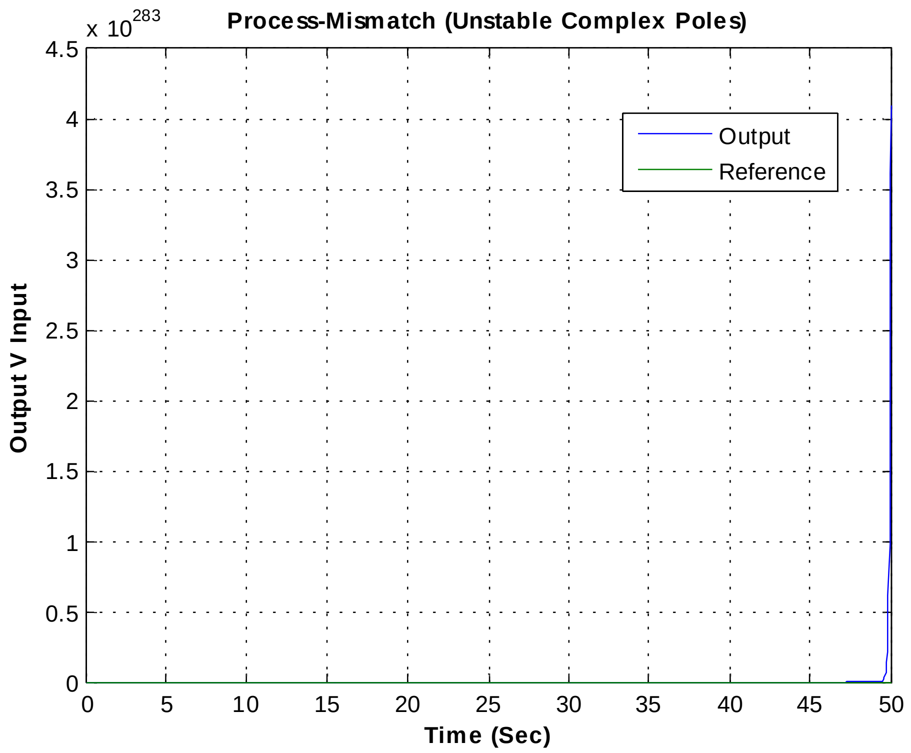
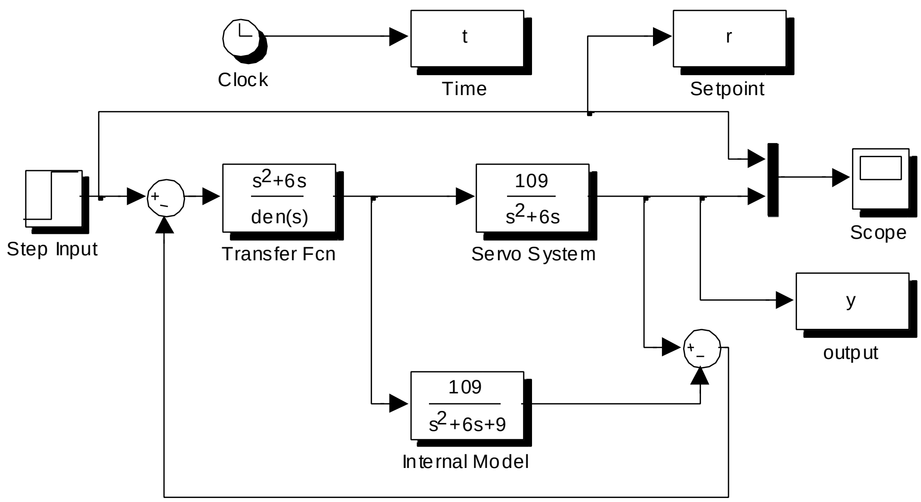



**Figure 5A.7: Model with Unstable Complex Poles**




**Model M5A.7: Process-Model Mismatch (Imaginary axis Poles)**

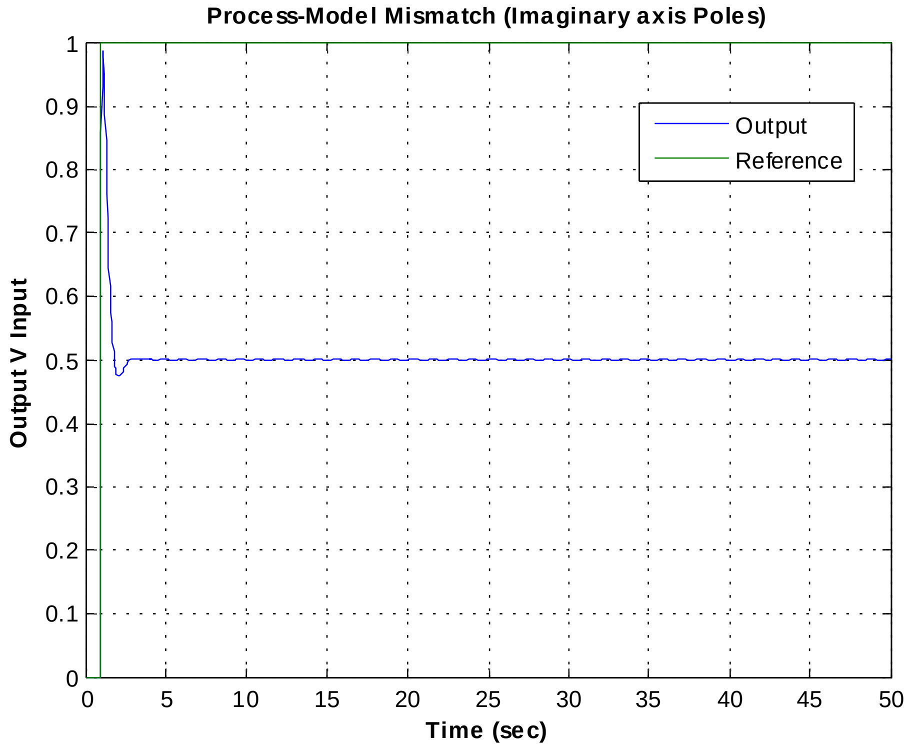



**Figure 5A.8: Model with Poles on the Imaginary axis**

# APPENDIX C

$$G_{PID}(s) = \frac{G_{IMC}(s)}{1 - G_{IMC}(s)G_{pm}(s)} \quad \textit{Equation B1}$$

where $G_{pm}(s)$ and $G_{IMC}(s)$ are as discussed earlier. i.e.

$$G_{pm}(s) = G^{+}_{pm}(s)G^{-}_{pm}(s) \quad \textit{Equation B2}$$

and

$$G_{IMC}(s) = G_{f}(s)G_{c}(s) \quad \textit{Equation B3}$$

where

$$G_{c}(s) = G^{+}_{pm}(s)^{-1} \quad \textit{Equation B4}$$

For the system to be controlled, as has been an assumption that the model is an exact representation of the process, it is the intention of this section to show the mathematics behind the mathematical representation of the IMC-PID form which proceeds as follows:

For the process it has already been shown as an assumption that:

$$G_{pm}(s) = G_{p}(s) = \frac{109}{s(s+6)} \quad \textit{Equation B5}$$

$$G_{f}(s) = \frac{1}{(1+\tau_{f}s)^{2}} \quad \textit{(the filter) Equation B6}$$

Using equation B4:

$$G_{c}(s) = \frac{s(s+6)}{109} \quad \textit{Equation B7}$$

As a consequence of Equation B3:

$$G_{IMC}(s) = \frac{s(s+6)}{109(1+\tau_f s)^2} = \frac{s^2+6s}{109\tau_f^2 s^2 + 218\tau_f s + 109} \quad \ldots .B8$$

Hence, using equation B1:

$$G_{IMC}(s)G_{pm}(s) = \frac{s(s+6)}{109(1+\tau_f s)^2} \times \frac{109}{s(s+6)} = \frac{1}{(1+\tau_f s)^2} \quad \ldots .B9$$

Then

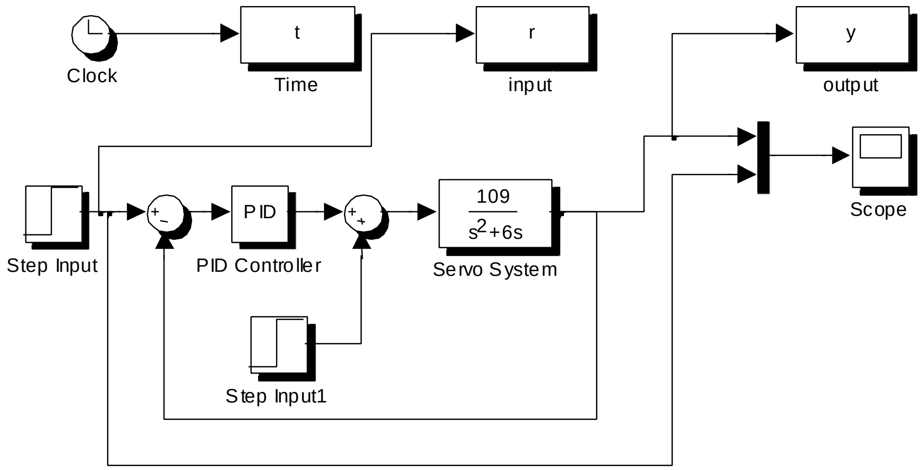
$$1 - G_{IMC}(s)G_{pm}(s) = \frac{(1+\tau_f s)^2 - 1}{(1+\tau_f s)^2} \quad \text{Equation B10}$$

Thus using Equation B1:

$$G_{PID}(s) = \frac{s(s+6)}{109(1+\tau_f s)^2} \times \frac{(1+\tau_f s)^2}{(1+\tau_f s)^2 - 1} \quad \text{Equation B11}$$

$$G_{PID}(s) = \frac{s(s+6)}{109(\tau_f^2 s^2 + 2\tau_f s)} = \frac{s+6}{109\tau_f^2 s + 218\tau_f} \quad \ldots ..B12$$

# APPENDIX D




**Model M6C.1: IMC-PID (Step Disturbance at input of Plant)**

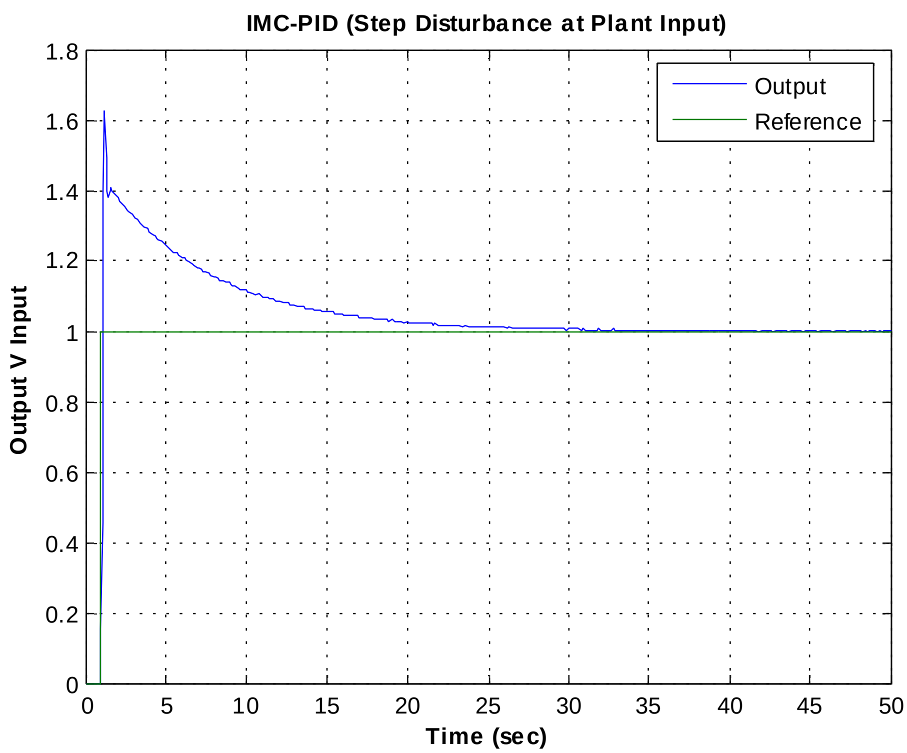
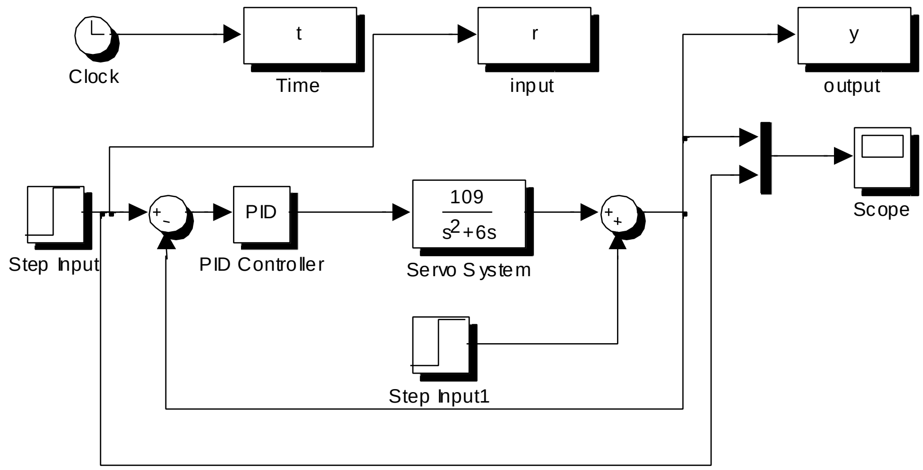



**Figure 6C.1: IMC-PID (Step Disturbance at Input of Plant)**




**Model M6C.2: IMC-PID (Step Disturbance at output of Plant)**

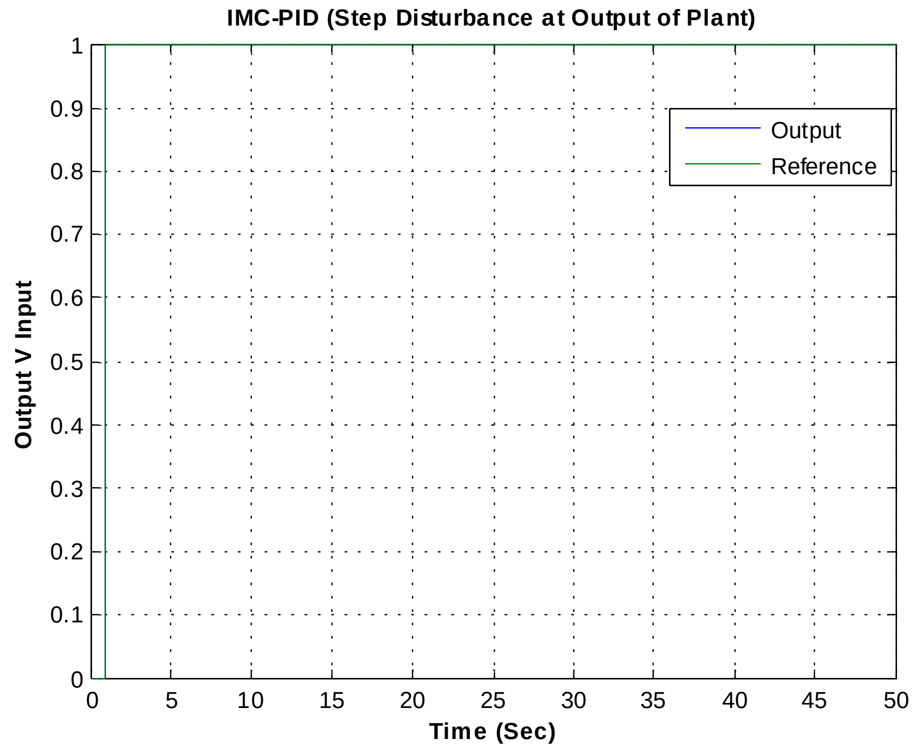
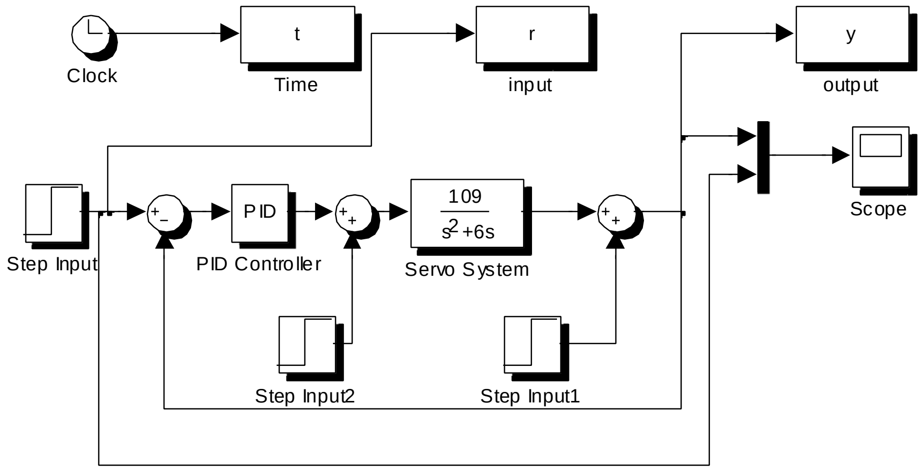



**Figure 6C.2: IMC-PID (Step Disturbance at Output of Plant)**




**Model M6C.3: IMC-PID (Step Disturbance at both Input and Output of Plant)**

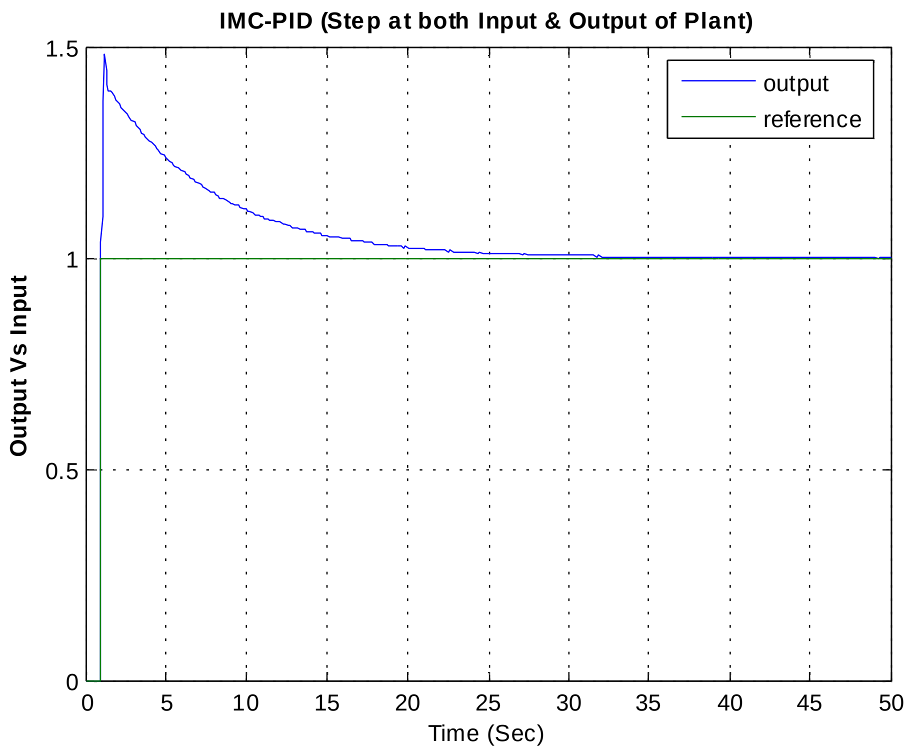
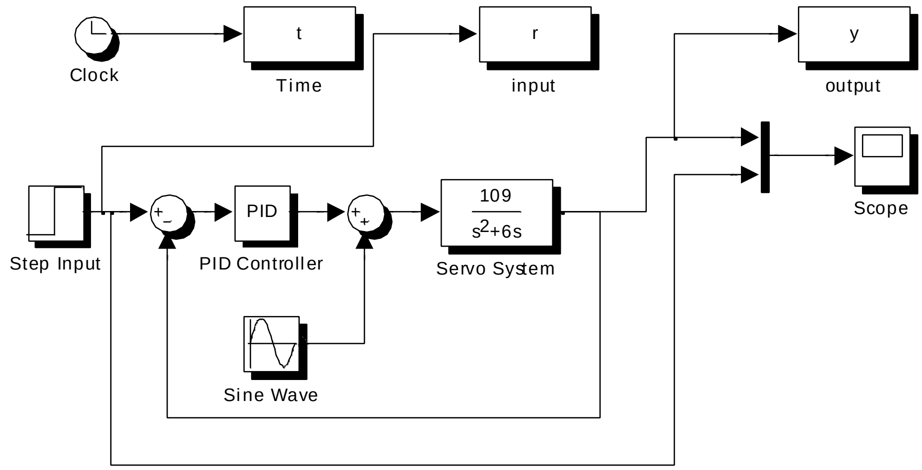



**Figure 6C.3: IMC-PID (Step at both Input &Output of Plant)**




**Model M6C.4: IMC-PID (Sinusoidal Disturbance at Input of the Plant)**

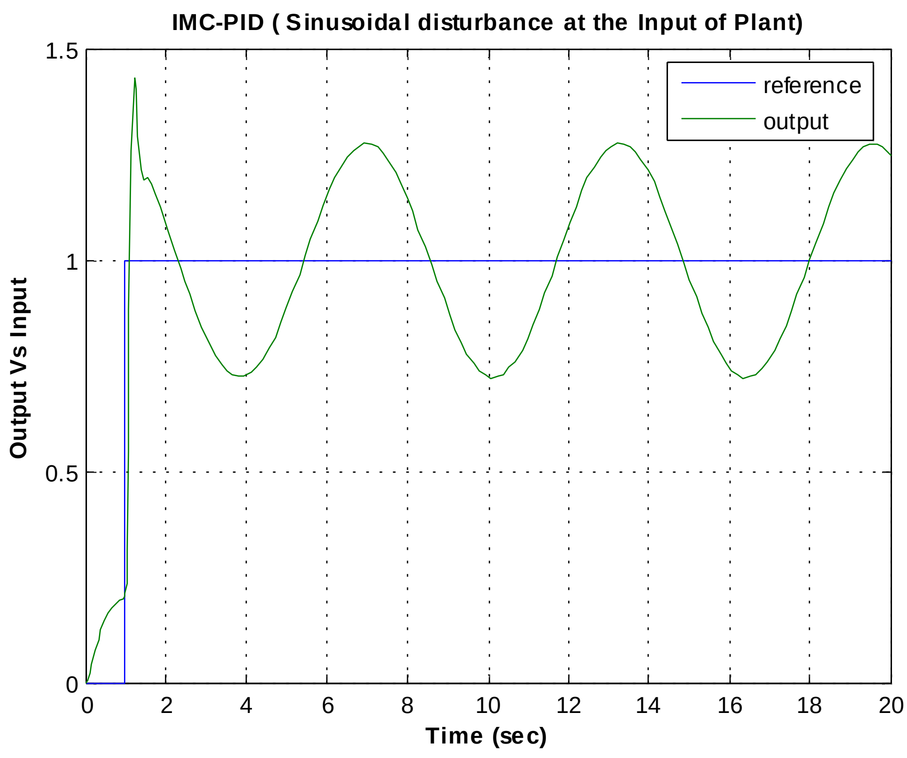
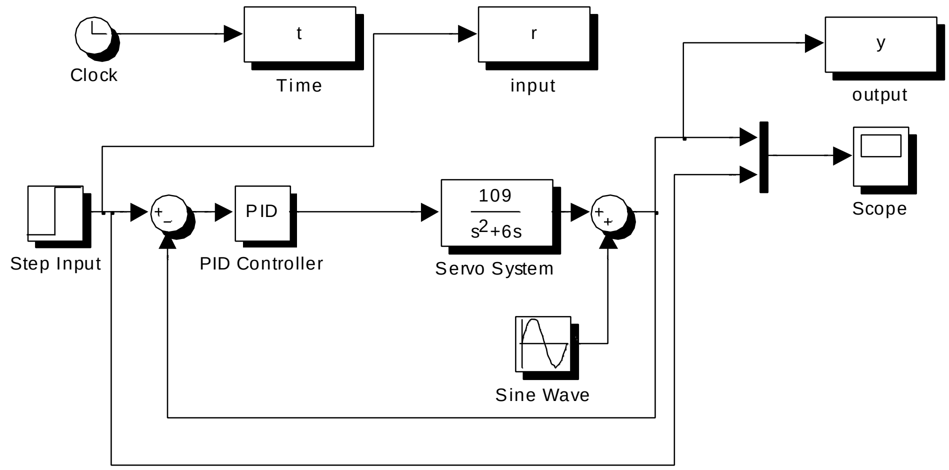



**Figure 6C.4: IMC-PID (Sine Disturbance at Input of the Plant)**




**Model M6C.5: IMC-PID (Sinusoidal Disturbance at Output of the Plant)**

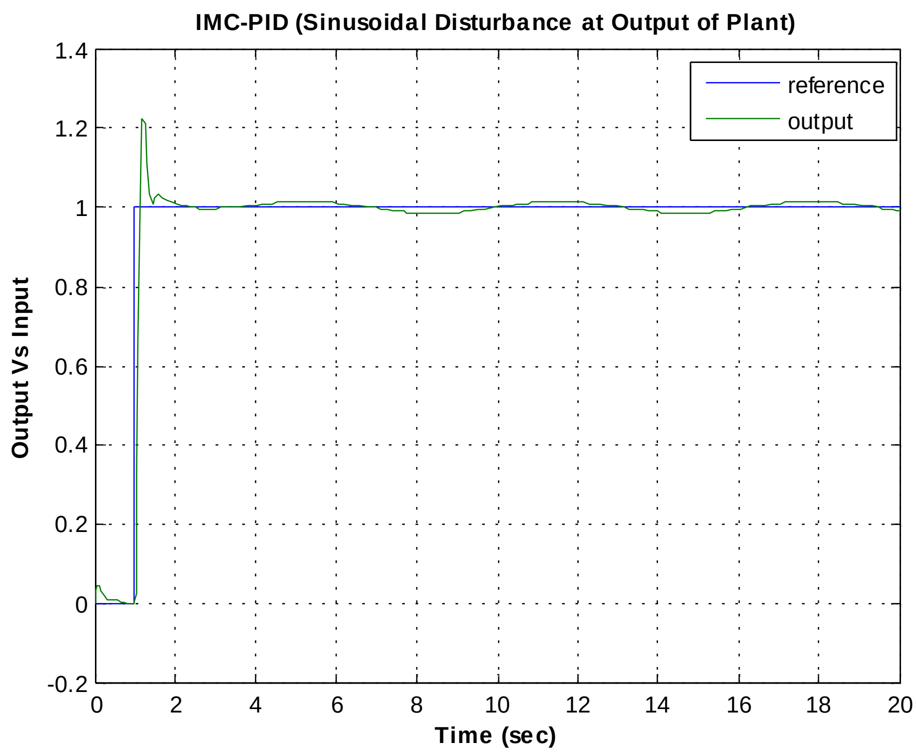
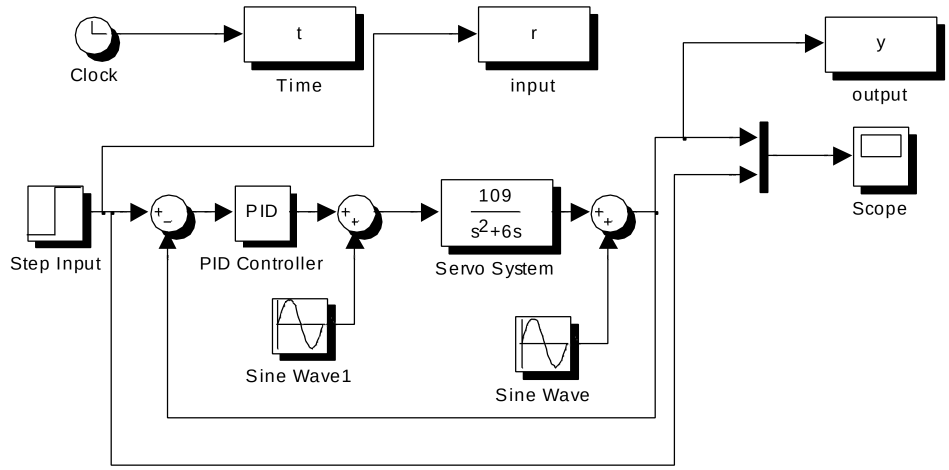



**Figure 6C.5: IMC-PID (Sinusoidal Disturbance at Output of the Plant)**




**Model M6C.6: IMC-PID (Sinusoidal Disturbance at Output & Input of the Plant)**

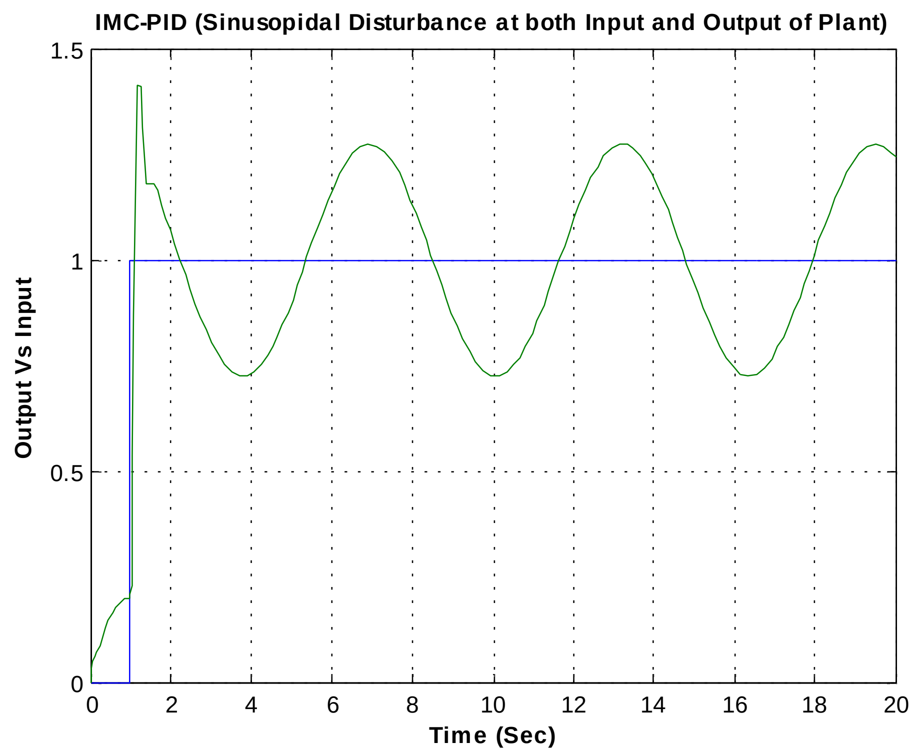



**Figure 6C.6: IMC-PID (Sinusoidal Disturbance at both Input & Output of plant)**

# APPENDIX E

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Table I. IMC-Based PID Controller Parameters[a]

| | model | $y/y_s = \tilde{g}_+ f$ | controller | $k_c k$ | $\tau_I$ | $\tau_D$ | $\tau_F$ | comments |
|---|---|---|---|---|---|---|---|---|
| A | $\frac{k}{\tau s + 1}$ | $\frac{1}{\epsilon s + 1}$ | $\frac{1}{k}\frac{\tau s + 1}{\epsilon s}$ | $\frac{\tau}{\epsilon}$ | $\tau$ | – | – | – |
| B | $\frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)}$ | $\frac{1}{\epsilon s + 1}$ | $\frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k\epsilon s}$ | $\frac{\tau_1 + \tau_2}{\epsilon}$ | $\tau_1 + \tau_2$ | $\frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$ | – | – |
| C | $\frac{k}{\tau^2 s^2 + 2\zeta\tau s + 1}$ | $\frac{1}{\epsilon s + 1}$ | $\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k\epsilon s}$ | $\frac{2\zeta\tau}{\epsilon}$ | $2\zeta\tau$ | $\frac{\tau}{2\zeta}$ | – | – |
| D | $k\frac{-\beta s + 1}{\tau s + 1}$ | $\frac{-\beta s + 1}{\epsilon s + 1}$ | $\frac{\tau s + 1}{k(\beta + \epsilon)s}$ | $\frac{\tau}{\beta + \epsilon}$ | $\tau$ | – | – | (2, 3, 5) |
| E | $k\frac{-\beta s + 1}{\tau s + 1}$ | $\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$ | $\frac{\tau s + 1}{ks(\beta\epsilon s + 2\beta + \epsilon)}$ | $\frac{\tau}{2\beta + \epsilon}$ | $\tau$ | – | $\frac{\beta\epsilon}{2\beta + \epsilon}$ | (1, 4) |
| F | $k\frac{-\beta s + 1}{\tau^2 s^2 + 2\zeta\tau + 1}$ | $\frac{-\beta s + 1}{\epsilon s + 1}$ | $\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k(\beta + \epsilon)s}$ | $\frac{2\zeta\tau}{\beta + \epsilon}$ | $2\zeta\tau$ | $\frac{\tau}{2\zeta}$ | – | (2, 3, 5) |
| G | $k\frac{-\beta s + 1}{\tau^2 s^2 + 2\zeta\tau s + 1}$ | $\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$ | $\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{k(\beta\epsilon s + 2\beta + \epsilon)s}$ | $\frac{2\zeta\tau}{2\beta + \epsilon}$ | $2\zeta\tau$ | $\frac{\tau}{2\zeta}$ | $\frac{\beta\epsilon}{2\beta + \epsilon}$ | (1, 4) |
| H | $\frac{k}{s}$ | $\frac{1}{\epsilon s + 1}$ | $\frac{1}{k\epsilon}$ | $\frac{1}{\epsilon}$ | – | – | – | – |
| I | $\frac{k}{s}$ | $\frac{2\epsilon + 1}{(\epsilon s + 1)^2}$ | $\frac{2\epsilon s + 1}{k\epsilon^2 s}$ | $\frac{2}{\epsilon}$ | $2\epsilon$ | – | – | (6) |
| J | $\frac{k}{s(\tau s + 1)}$ | $\frac{1}{\epsilon s + 1}$ | $\frac{\tau s + 1}{k\epsilon}$ | $\frac{1}{\epsilon}$ | – | $\tau$ | – | – |
| K | $\frac{k}{s(\tau s + 1)}$ | $\frac{2\epsilon s + 1}{(\epsilon s + 1)^2}$ | $\frac{(\tau s + 1)(2\epsilon s + 1)}{k\epsilon^2 s}$ | $\frac{2\epsilon + \tau}{\epsilon^2}$ | $2\epsilon + \tau$ | $\frac{2\epsilon\tau}{2\epsilon + \tau}$ | – | (6) |
| L | $k\frac{-\beta s + 1}{s}$ | $\frac{-\beta s + 1}{\epsilon s + 1}$ | $\frac{1}{k(\beta + \epsilon)}$ | $\frac{1}{\beta + \epsilon}$ | – | – | – | (2, 3, 5) |
| M | $k\frac{-\beta s + 1}{s}$ | $\frac{-\beta s + 1}{(\beta s + 1)(\epsilon s + 1)}$ | $\frac{1}{k(\beta\epsilon s + 2\beta + \epsilon)}$ | $\frac{1}{2\beta + \epsilon}$ | – | – | $\frac{\beta\epsilon}{2\beta + \epsilon}$ | (1, 4) |
| N | $k\frac{-\beta s + 1}{s}$ | $\frac{(-\beta s + 1)((\beta + 2\epsilon)s + 1)}{(\epsilon s + 1)^2}$ | $\frac{(\beta + 2\epsilon)s + 1}{ks(\beta + \epsilon)^2}$ | $\frac{\beta + 2\epsilon}{(\beta + \epsilon)^2}$ | $\beta + 2\epsilon$ | – | – | (5, 6) |