University of Southern Queensland
Faculty of Health, Engineering & Surveying

Developing a Numerical Model for the Design of Sheet Pile Walls

A dissertation submitted by

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Abstract

This project involves the investigation, development and validation of cantilevered and anchored sheet pile wall models. The effect of sheet pile construction penetrating a sandy soil are investigated by analysing numerical outputs such as the wall deformation, ground settlement and maximum bending moment. The numerical analysis is completed using an industrially known computer software program: Fast Lagrangian Analysis of Continua (FLAC).

The quality of the FLAC models used to obtain the numerical solutions was validated for accuracy against available analytical solutions. The aims of this project were to gain sufficient knowledge on sheet pile wall design methods, better known as the limit equilibrium methods; develop an automatic Excel spreadsheet as a design tool for solving any sheet pile wall design problem; and be able to easily validate the accuracy of the obtained numerical solutions by comparing the numerical and analytical solutions.

The main focus of the investigation was to develop new cantilever and anchored sheet pile wall models for a specific geotechnical problem of a sheet pile penetrating a sandy soil in the presence of a water table. This was done by validating the numerical models and undertaking parametric studies varying specific parameters to investigate thoroughly the behaviour of the sheet pile wall system.

This research concludes that the analytical methods provide a basic understanding of the soil-wall system behaviour; however, the hypothesis on which these methods are based makes them necessarily conservative, due to the number of assumptions required to simplify the design procedure. Numerical modelling in FLAC produces more accurate results and, by undertaking advanced parametric studies, indicates the actual behaviour of the soil-wall system in the real world. The development of numerical models, undertaking of parametric studies and validation of solutions by comparing against analytical method solutions are areas deserving of further research, as this will lead to more effective sheet pile wall designs in the engineering industry.

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I certify that the ideas, designs and experimental work, results, analysis and conclusions set out in this dissertation are entirely of my own effort, except where otherwise indicated and acknowledged.

I further certify that the work is original and has not been previously submitted for assessment in any other course or institution, except where specifically stated.

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Date ______________________________

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With appreciation

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Nomenclature

The principal symbols used are presented in the following list. Locally used notation and modifications, such as by addition of a subscript or superscript, and a symbol that has different meanings in different contexts are defined where used.

K_a	Rankine's Active Pressure Coefficient
K_p	Rankine's Passive Pressure Coefficient
ϕ	Un-drained internal friction angle of the soil
$\emptyset'$	Drained internal soil friction angle
σ'	Pressure at a particular depth
$\gamma\,'$	Effective Soil unit weight
γ	Soil unit weight
L	Sheet Pile Length
γ_{sat}	Saturated unit weight of the soil
γ_w	Unit weight of water
D	Penetration depth of sheet pile
P	Total active pressure behind sheet pile wall
z	Depth below the ground surface
$\bar{z}$	Point of zero shear force below the ground surface
A	Constant (in Chapter 3 section 3.5.3)
FOS	Factor of safety
$c\,'$	Drained soil cohesion
M_{max}	Maximum bending moment exerted on sheet pile wall
F	Anchor force
$D_{theoretical}$	Theoretical penetration depth of the sheet pile wall
D_{actual}	Actual penetration depth of the sheet pile wall
A	Area (in Chapter 3 section 3.5.5)
n	Number of elements
P	Pressure applied over an area (in Chapter 3 section 3.5.5)
dA	Area differential

Chapter 1: Introduction

This project investigates the suitability of modelling various geotechnical sheet pile wall problems using an explicit finite difference program, Fast Lagrangian Analysis of Continua (FLAC). This project encompasses research into available classical theories, current techniques of analysis and the creation of computer models. This research discusses the geotechnical problems analysed and presents the results of an investigation. The geotechnical problems to be investigated are:

- Cantilever Sheet Pile Wall Penetrating a Sandy Soil
- Anchored Sheet Pile Wall Penetrating a Sandy Soil.

1.1 Background

1.1.1 Geotechnical Stability

Ground stability must be assured prior to consideration of other foundation-related items. Foundation problems involve the support of natural soil. Stability problems often occur when building over soft, low strength soil. Problems with foundation stability can be prevented by initial recognition of the problem and appropriate design.

The design of all structures demands ultimate and serviceability limit state requirements to be satisfactory. Failure under ultimate limit state occurs when 'a collapse mechanism takes place in the ground or in some parts of the structure' (Lancellotta 1995). The failure mechanism can be divided into strength and stability components.

1.1.2 Choice of Models

The cantilever sheet pile wall was modelled as it represents further study into lateral earth pressures acting on the sheet pile wall structure. The rotation effect of the sheet pile wall at the bottom of the sheet pile tip results in much more complex lateral earth pressures developing on the sheet pile wall, and hence in more complicated solutions, only available when using numerical modelling.

The parametric study that will be conducted within this dissertation is a thorough study that aims to evaluate the effect of changing certain parameters on the behaviour of the pile-wall system. The parameters that will be investigated are:

- mesh fineness
- soil strength
- water table effect
- installation of anchor systems.

The anchored sheet pile wall model represents the possibility of decreasing the effect of the lateral earth pressures developed on the sheet pile wall. This problem was investigated to analyse the application of an anchor tie rod force on the behaviour of the sheet pile wall. Knowledge of these effects will aid in future studies within the area, as it is of upmost importance for a designer to analyse the sheet pile wall deformation for serviceability purposes and the bending moment analyses for structural design purposes. Due to its nature, FLAC has the potential to decrease the solution time and increase the accuracy of the results. The outcome will be a greater understanding of effective sheet pile wall design in the engineering industry.

1.1.3 Computational Analysis

Numerous methods have been developed to solve geotechnical stability problems by hand calculations; however, modern graphical software tools have made it possible to gain a much better understanding of the inner numerical details of soil-wall system behaviour. Comparing the numerical solutions to the analytical solutions, it is clear that more accurate solutions are now available by using modern computer software. However, to obtain useful results from a computer program, it is necessary to have an experienced user.

1.2 Aims and Objectives

The intended purpose of this dissertation is to understand the limit equilibrium methods of analysis. The study will establish the relationship between the soil-pile system by means of developing a numerical model and undertaking parametric studies using FLAC. The numerical results obtained will be validated with analytical solutions to evaluate the accuracy of FLAC and obtain more information and knowledge of the system. This will lead to more effective sheet pile wall design in the engineering industry.

The identification of appropriate milestones is an important part of reaching the major objectives within a given timeframe. The sequence of the tasks is briefly described below:

- Research background information on the application of numerical analysis for geotechnical design.
- Create a spread sheet in Excel that will automatically solve for any sheet pile wall design. Aim for this spread sheet to be useable in the engineering industry.
- Gain sufficient knowledge of the software program FLAC, to enable the writing of a script code using FLAC's inner built-in coding language, FISH. This will make it possible to create an anchored sheet pile wall model in FLAC.
- Undertake parametric studies in FLAC by means of varying specific parameters to determine the effect of the net pressure, shear forces and bending moments applied on the sheet pile wall.
- Compare the results obtained from the analytical methods with the results gathered from the numerical applied analysis to verify the numerical methods.

1.3 Overview of Chapters

This chapter overview gives a brief introduction to the task, methodology and the computer program to be utilised. Following this, each problem is investigated separately, including validation and advanced parametric studies to analyse the behaviour of the sheet pile wall. The dissertation concludes with an overall summary and an outline of possible future work.

Chapter 1: Introduction

This chapter provides an outline of the study, as well as an introduction to the problem and the essential background information. The chapter also discusses the project objectives and main aim for the dissertation.

Chapter 2: Literature Review

This chapter presents a literature review of all the past studies for the design of cantilevered and anchored sheet pile wall problems. Included within the literature review are current available analytical methods for the design of sheet pile walls, as well as findings and results from past dissertational FLAC modelling of sheet pile walls. The previous work is used to determine why additional research is necessary and the scope of the research required.

Chapter 3: Developing Tools for Sheet Pile Wall Design

In this chapter, the methodology for designing sheet pile walls is introduced. Indicated in this chapter is the development of design tools such as an automated spread sheet that can automatically solve any sheet pile wall problem, solving tedious analytical equations within seconds by simply inputting known data specified by the user. The generated design tools are then used as part of the validation process of the numerical models.

Chapter 4: FLAC Overview

This chapter presents a short introduction to the FLAC software package, as well as an overview of the FLAC script that was generated to model the geotechnical problem. The methodology used for specifying the inputs required the development of a numerical model that leads to the validation of the models and specific outputs obtained from FLAC.

Chapter 5: FLAC Analysis of Cantilever Sheet Pile Wall

Presented in this chapter is the creation of a numerical cantilever sheet pile wall model for a specific sheet pile wall problem. This chapter specifies the process required for validating numerical model graphical outputs and obtaining qualitative results. Within this chapter, advanced modelling by means of undertaking a parametric study has been presented to illustrate the overall soil-pile system behaviour.

Chapter 6: FLAC Analysis of Anchored Sheet Pile Wall

Presented in this chapter is the creation of a cantilever sheet pile wall model for the specific geotechnical sheet pile wall problem. This chapter presents the validation of the numerical model, as well as advanced modelling of anchorage sheet pile wall systems, to investigate specific parameters that have a 'real life' effect in the engineering industry.

Chapter 7: Conclusions and Future Work Recommendations

This chapter presents the overall findings presented within Chapters 3–6. This chapter presents a summary of the conclusion of the dissertation. Recommendations for further work are discussed to ensure that this work is clearly defined.

1.4 Summary

The basic understanding of the studies to be undertaken was presented in this chapter to give an overview of the chapters that follow. From this chapter, it is evident that many aspects need to be considered throughout the duration of this project. Sheet pile wall problems consist of a very complex soil-wall system and it is therefore important that all aspects of the problem are covered. The following chapter presents a detailed literature review of past studies relating to the investigations that have been conducted within this dissertation.

Chapter 2: Literature Review

2.1 Introduction

There are several sheet pile walls design methods dating back to the first half of the twentieth century. These original proposals have been continuously and may currently be being reviewed (Torrabadella 2013). Analytical methods include ‘limit stage design methods’ or ‘classical methods’ (King 1995). For establishing equilibrium of the horizontal forces and moments developed along the wall and to define the failure state point along the sheet pile and the embedment depth below the dredge line for either cantilever or anchored sheet pile walls by means of undertaking geotechnical design, calculations are required regardless of the method adopted.

The estimation of the limit equilibrium method depends on the limiting earth pressure coefficients from plastic theories. The earth pressure forces on the wall are also calculated with these plastic theory values. During the limit equilibrium condition, the equilibrium equations are used to deduce the driven depth of the sheet pile wall. A factor of safety is applied by an increase in sheet pile depth to limit the movement of the wall and take into account any possible errors in the soil parameters and analysis.

The second approach, the finite element technique, first proposed by Morgenstern and Eisentein (1970), often makes use of the finite element technique to solve the stiffness equations. Satisfactory knowledge of the stress-strain behaviours of the soil and its parameters is necessary, as this indicates the behaviour of the soil-structure system.

The limit equilibrium methods are based on the prediction of maximum excavation height, for which static equilibrium will be maintained. This is known as the classical design methods. The accuracy of the earth pressure evaluation acting on either side of the wall in the condition of limit equilibrium is very important. The generated earth pressure exerted on the sheet pile wall is due to the actual distribution and magnitude of these pressures and is dependent on the complex soil-wall interaction.

Equilibrium for an anchored sheet pile wall with only a single row of anchors can be achieved without taking into consideration the passive reaction at the bottom of the back of the sheet pile wall. However, the design method used can change depending on whether this reaction force is considered. When comparing the cantilevered and anchored sheet pile walls, the main advantage found from the anchored sheet pile wall is its ability to reduce the embedment depth of the sheet pile, thus increasing the excavation depth, which has a profitable effect on the structure (Das 1990). It is important to note that due to the anchor provided, the excavation depth can be increased, but the structure behaves like a cantilever sheet pile only until the anchor is placed (Torrabadella 2013).

2.2 Background Information

Retaining walls are used to hold back soil and maintain a difference in the elevation of the ground surface. Retaining walls can be classified into two categories of structure: rigid or flexible. A wall is considered rigid if it moves as a unit and does not produce wall deformation. Most gravity walls such as masonry walls, simple concrete walls or reinforced concrete walls can be considered rigid. Flexible walls, by contrast, undergo wall deformations. The most common flexible sheet piles are steel sheet piles, due to their tolerance of large deformation occurrences. Typical examples of these two types of retaining wall are indicated in Figure 1-1.

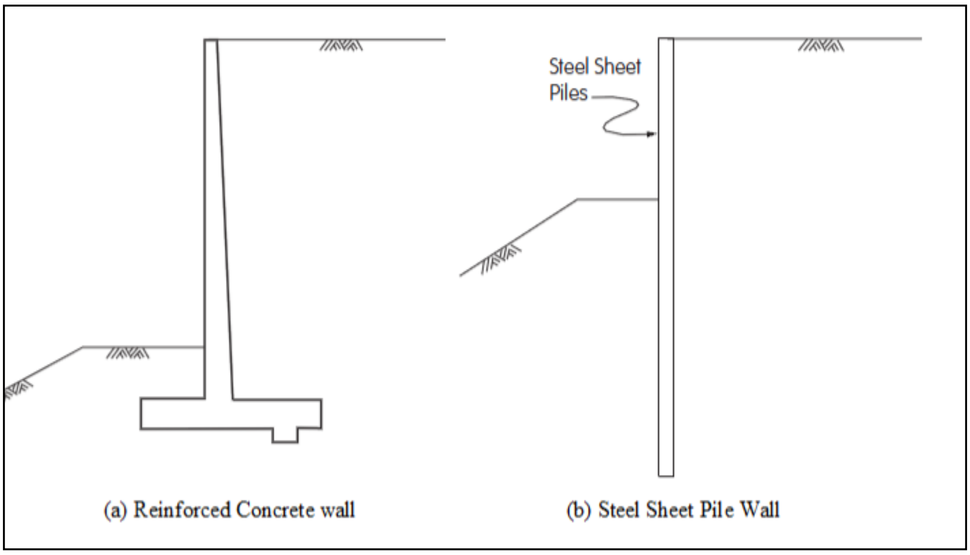

Figure 1-1: Retaining walls: (a) rigid wall, (b) flexible wall (Ramadan 2013)

Sheet pile walls consist of driven, vibrated or pushed interlocking pile segments embedded into soils to resist horizontal pressures. The sheet pile walls are constructed by driving the sheet piles into a slope or excavation. They are considered most cost-effective where retention of higher earth pressures of soft soils is required. Sheet piles have a significant advantage in that they can be driven to depths below the excavation bottom and so provide a control to heaving in soft clays or piping in saturated sand.

Sheet piles can function as temporary or permanent structures and are most often used in excavation projects. Temporary sheet piling structures are used to control or exclude earth or water and allow the continuation of permanent work. Permanent sheet piling is commonly used as a retaining structure, and at times as part of the structure of underground buildings (Paikowsky & Tan 2005).

When sheet pile walls are constructed, important design parameters are introduced that are often difficult to evaluate, making the design process complex and protracted. The generation of an automatic design tool in Excel to solve any sheet pile wall problem would help to overcome these design difficulties and time issues; not only by leading to easier evaluation, but also by making it possible to obtain results quickly for undertaking the validation process.

Numerical modelling has evolved over the years. Research has found that these numerical methods for the design of sheet pile walls are very useful and can be used to obtain information that is unavailable when using analytical methods for the design of sheet pile walls (Smith 2006; Bilgin 2010); that is, the wall deformation, ground settlement and possible surface failures. This research uses FLAC to develop its numerical model. FLAC is a popular industrially known design tool, used to solve geotechnical problems.

2.2.1 Sheet Pile Wall Materials

Sheet pile walls are made of different kinds of materials such as wood, concrete, steel or aluminium. The material selection depends on a number of factors, including strength and environmental requirements. The designer must consider the possibility of material deterioration and its effect on the structural integrity of the system. Most enduring structures are constructed of steel or concrete. Concrete is capable of providing a long service life under normal conditions, but has relatively high initial costs when compared to steel sheet piling. Concrete piling is also more difficult to install than steel piling. Long-term field observations indicate that steel sheet piling provides a long service life when properly designed (Ramadan Amer 2013).

The steel sheet pile alternative is the most popular due to its strength, ease of handling and construction. Steel sheet piles are available in various cross-section shapes. They can have problems with corrosion that can be prevented by coating. They can be used above or below water provided the required protection is applied (Bowles 1988).

Their advantages are:

- resistant to high driving stresses
- relatively lightweight
- reusable
- long service life
- easy to increase length by welding
- joints are less likely to deform
- can produce a watertight wall.

Other materials such as vinyl, polyvinyl chloride and fiberglass are also available. These pilings have very low structural capacities and function in tieback situations. When compared to other materials, only short lengths of pile are available. The designer for each sheet pile application when using one of the above-mentioned materials must carefully evaluate the properties of the specific material obtained from the manufacturer (Paikowsky & Tan 2005).

Steel is the most common material used for sheet pile walls and is thus considered as the main sheet pile wall material in this dissertation.

2.2.2 Construction of Sheet Pile Walls

The construction of sheet pile walls may involve either excavation of soils in front or backfilling of soils behind the wall; that is, fill construction or cut construction. Fill wall construction refers to a wall system in which the wall is constructed from the base of the wall up to the top: also called 'bottom-up' construction. Cut wall construction refers to a wall system in which the wall is constructed from the top of the wall down to the base, concurrent with excavation operations: known as 'top down' construction (Zhou 2006). These construction procedures generate different loading conditions in the soil and thus different wall behaviour should be expected (Das 1990).

Sheet pile walls are widely used in excavation support systems, cofferdams and cut-off walls under dams, slope stabilisation, waterfront structures and floodwalls. Sheet pile walls used to provide lateral earth support could be either cantilever or anchored depending on the wall height. Recently, land owners have been seeking to maximise the usage of their land by designing basements up to their land boundaries, with little regard for the subsoil and site condition restraints. The result is that various deep excavations are carried out in close proximity to existing buildings and infrastructures, increasing the importance in design of considering the safety of neighbouring structures (Kasim 2011).

2.2.3 Cantilever Sheet Pile Walls

Cantilever sheet pile walls are usually used with low wall height between 3 and 6 m, and sometimes less due to limitations in availability of certain section modulus and their costs (*Geotechnical design procedure for flexible wall systems* 2007). Cantilever sheet pile walls are suitable for places with tight space constraints due to the narrow base width of the cantilever wall. This type of sheet pile wall depends on the passive resistance of the foundation material in front of the wall and the moment resisting capacity of the piles for stability (Figure 1-2). Therefore, it should not be used where the foundation material may be removed during wall service life (Caltrans 2004).

Figure 1-2: Cantilever Sheet Pile (Hauraki Pilling LTD)

2.2.4 Anchored Sheet Pile Walls

Anchored sheet pile walls are required when the wall height exceeds 6 m or when the lateral wall deflection is limited for design consideration (Leila & Behzad 2011). Anchoring the sheet pile wall requires less penetration depth and also less moment to the sheet pile because it will drive additional support by the passive pressure on the front of the wall and the anchor tie rod. Anchored sheet pile walls are typically constructed in cut situations, and may be used for fill situations with special design considerations to protect the anchor from construction damage from fill placement or fill settlement (*Geotechnical design procedure for flexible wall systems* 2007).

Several types of anchors can be used with sheet pile walls, such as dead-man and grouted tiebacks. Temporary support can also be provided for the walls by making use of struts, braces and rakers (*Geotechnical design procedure for flexible wall systems* 2007). The selection of the most suitable type of anchor generally depends on the soil type, presence of groundwater and cost considerations (Elias & Juran 1991). For situations in which one or more levels of anchor are required, it is most suitable to make use of grouted tiebacks, whereas the suitability of tie dead-man anchors is typically limited to situations requiring a single level of anchor (Caltrans 2004).

Horizontal struts need to be used when the width of excavation is small and when their usage does not affect the construction of permanent elements; inclined rakers are used for wide excavation. According to Gulhati and Datta (2008), grouted tiebacks and dead-man anchors are used when there is available underground space beyond the excavated area. This space should be free from the foundations and the underground utilities of adjacent structures.

Figure 1-3: Macalloy Anchored Sheet Pile (Iceland, 2002)

2.2.5 Sheet Pile Wall Failure Mechanisms

When analysed as retaining structures, several failure modes for a sheet pile system must be considered in the design process (US Army Corps of Engineers, 1996). These failures include deep-seated failure, rotational failure due to pile penetration inadequacy, overstressing of the sheet pile and anchorage component failure. An investigation of the load capacity of piles subjected to combined loading was performed, as second-order bending effects reduce the lateral load capacity of the wall when piles are exposed to combined axial and lateral loads (Greimann 1987).

Deep-seated failure occurs when the complete soil mass, containing the retaining wall system, rotates along a single failure surface. This type of failure is classed as a soil failure only, independent of the structural capacities of the wall and any anchorage system (Paikowsky & Tan 2005). Another form of rotational failure occurs when the retaining wall rotates due to the exerted soil pressures. This type of failure can be prevented by adequate wall penetration into the soil or by implementing an anchorage system.

The other failures that may occur in retaining wall systems are sheet pile overstressing, passive anchorage failure, tie rod failure and wale system failure (Figure 2-4). In the case of pile overstressing due to both lateral and axial loads, a plastic hinge leading to failure will develop.

When the anchor moves laterally within the soil due to the force exerted on it, a passive anchorage failure will occur. The tie rod may fail if the required tensile capacity is not adequate, and the wale system may undergo a bearing failure if the loads are not evenly distributed (Evans 2010).

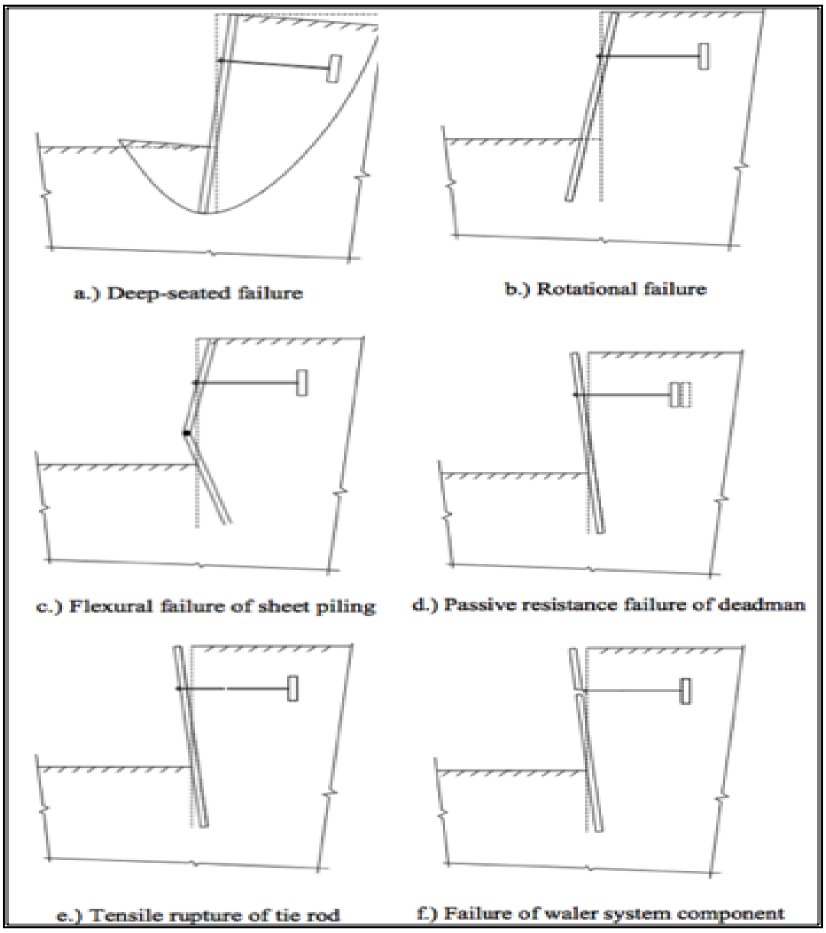

Figure 1-4: Failure modes for anchored sheet pile walls (Caltrans 2004)

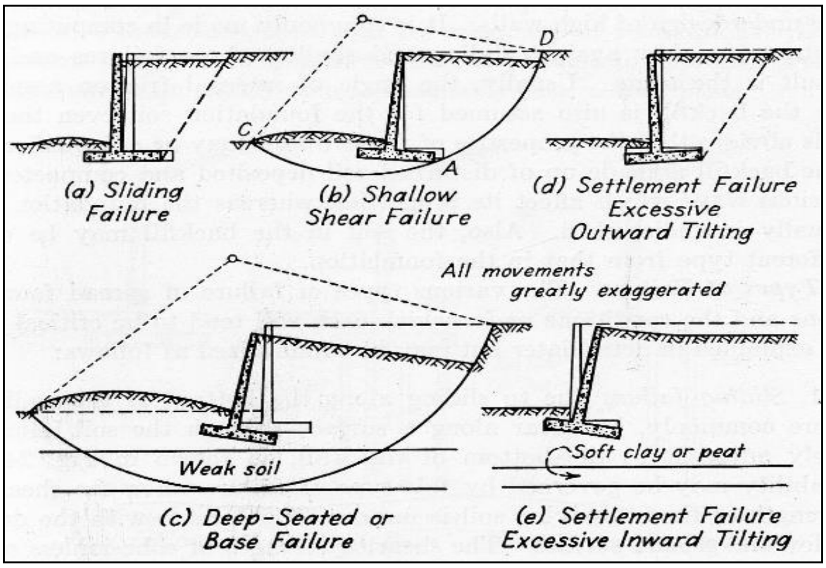

Figure 1-5: Failure modes for cantilevered sheet pile walls (Leila & Behzad 2011)

2.3 Classical Design Methods

There are several design methods that make different assumptions and hence make different simplifications of the net pressure distribution exerted along the sheet pile wall. In this section, the classical design methods of sheet pile walls are discussed. The current limit state design method most commonly used in the United Kingdom (UK) is the UK method, as described by Padfield and Mair (1984). In the United States (US), the USA method, or gradual method, as described by Bowles (1996), is the most commonly used limit state design method. Suggesting a rectilinear pressure distribution leads to the simplifying of the net pressure distribution along the sheet pile wall. An analytical limit equilibrium approach has been suggested by King (1995), involving an empirically determined parameter. The net pressure distribution has been examined using finite element analysis by Day (1999).

Due to the vast number of parameters that require consideration when evaluating sheet pile wall design, some of the theories presented below have limitations that lead to the restriction and exclusion of their usage in current sheet pile wall design. After undertaking thorough research of the classical sheet pile wall design methods, a particular sheet pile wall design method was selected for designing the sheet pile walls by hand in this research project. This methodology will be furthered discussed in

Chapter 3. In addition, in Section 2.4, discussion is presented of some dissertations on numerical sheet pile wall design (Smith 2006; Ramadan 2013; Torrabadella 2013).

2.3.1 Padfield and Mair (1984) Design of Retaining Walls in Stiff Clays

The full UK method gets its name in contrast to the simplified method, described below. In the full method, the active limit state is assumed to be reached in the back of the wall above the rotation point, and the passive limit state is assumed to be reached in front of the wall between the dredge line and the rotation point. Supposedly, an overturn in the normal pressure direction is to be produced at the rotation point, below which the full passive pressure is moved behind the wall and the active to the front. This causes a sudden jump in the earth pressure, which is needed to prescribe moment equilibrium.

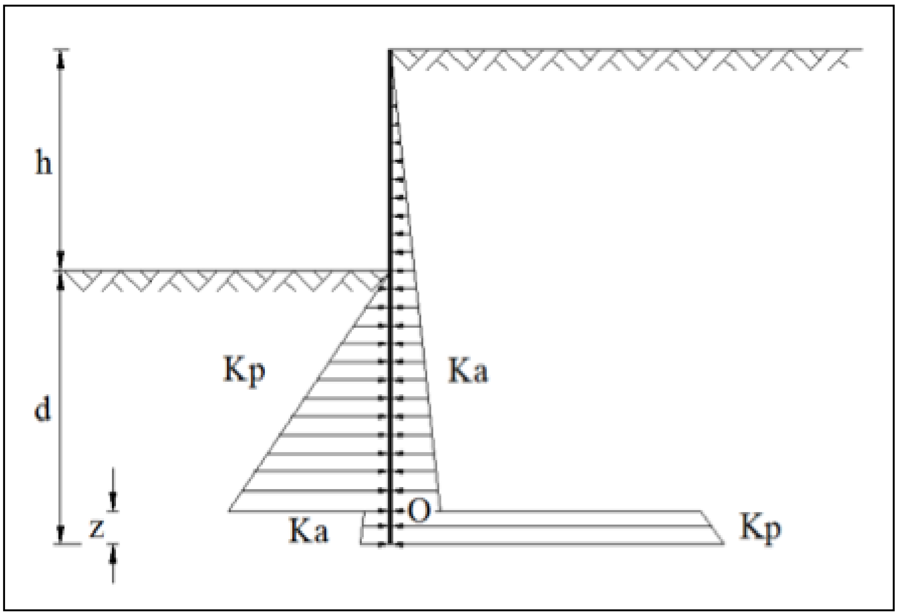

Figure 2-1: Full method (Padfield & Mair 1984)

Due to the complexity of the full method, a simplification was recommended by Padfield and Mair (1984). As shown in Figure 2-2, the earth pressure below the rotation point can be replaced by an equivalent concentrated force acting on point O, represented as the resultant force. The value for the depth d has been found to be considerably lower than compared to the value calculated by the full method. Thus, the simplified method is slightly more conservative than the full method, although it leads to appreciably similar results.

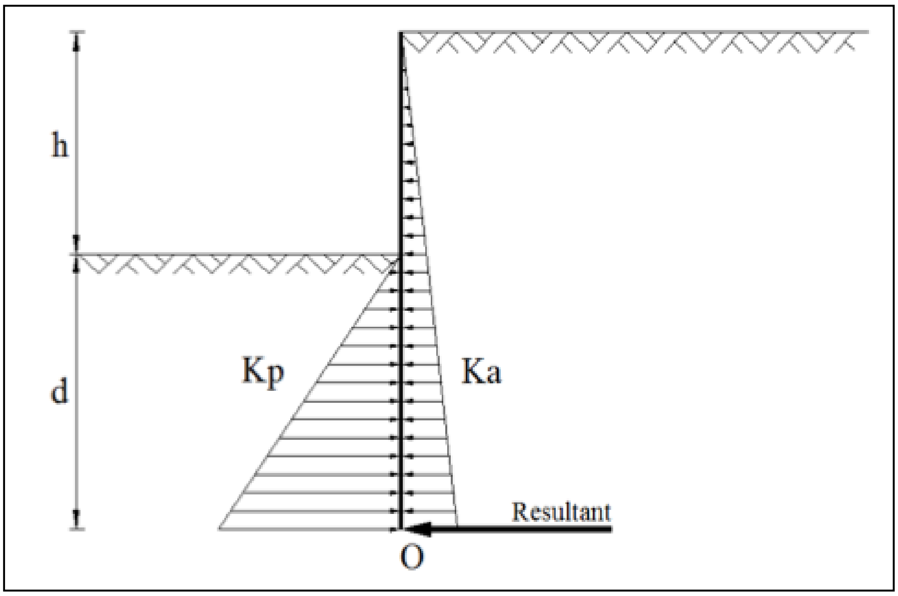

Figure 2-2: Simplified method (Padfield & Mair 1984)

2.3.2 Bowles (1988) Foundation Analysis and Design

A rectilinear net earth pressure distribution was proposed by Bowles (1988) in which the active earth pressure in the back of the wall above the dredge line and passive earth pressure in front of the wall immediately below the dredge line were fully mobilised even before failure. The design depth of penetration was calculated by finding the z in Figure 2-3, corresponding to the maximum net earth pressure in front of the wall, satisfying both equilibrium of horizontal forces and moments about the bottom of the wall.

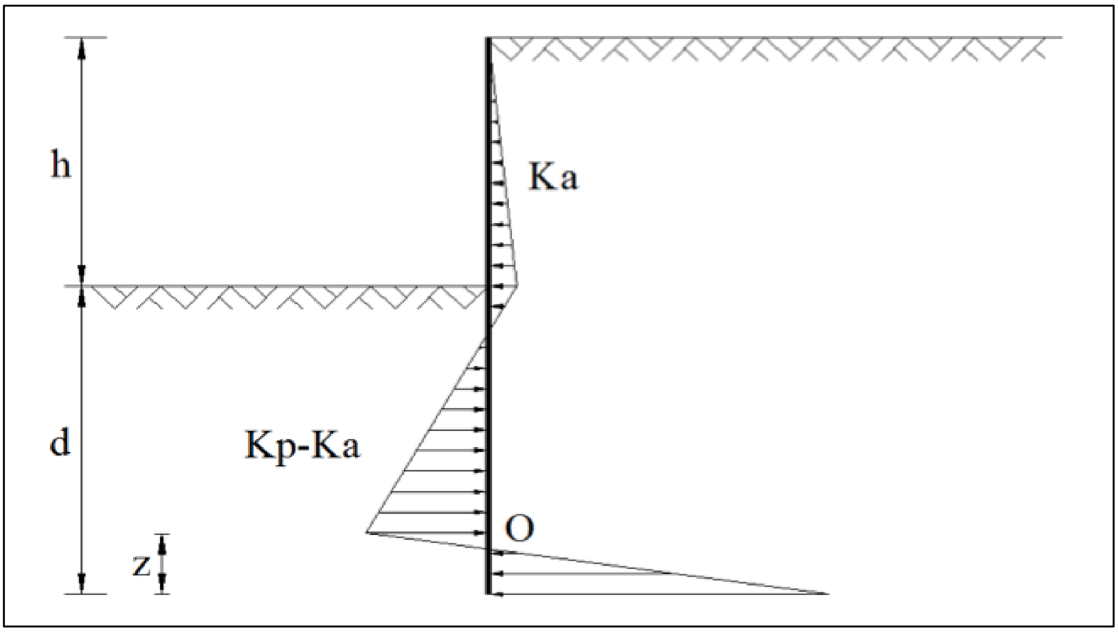

Figure 2-3: Rectilinear Earth Pressure Distribution (Bowles 1988)

A slightly different approach was later reviewed that does not involve the hypothesis of a sudden change in the earth pressure distribution. The assumption made in this method is to consider the transition zone at which the net earth pressure gradually changes its direction from the front to the back of the wall. The rotation point is where the transition occurs and is also assumed to be linear. This gradual method is also known as the general rectilinear net pressure method or the USA method, as presented by Skrabl (2006) and Day (1999).

2.3.3 Day (1999) Net Pressure Analysis of Cantilever Sheet Pile Walls

Day (1999) presented a finite element study in which the net earth pressure over the sheet pile wall was examined. In the finite element study conducted by Day (1999), five case studies were considered, consisting of wall heights of 10 m and soil friction angles ranging between 20 and 50 degrees with variable excavation depths. The results indicated that a dependent relationship exists between the point of zero net pressure and the ratio between the active and passive pressure distributions (Figure 2-4).

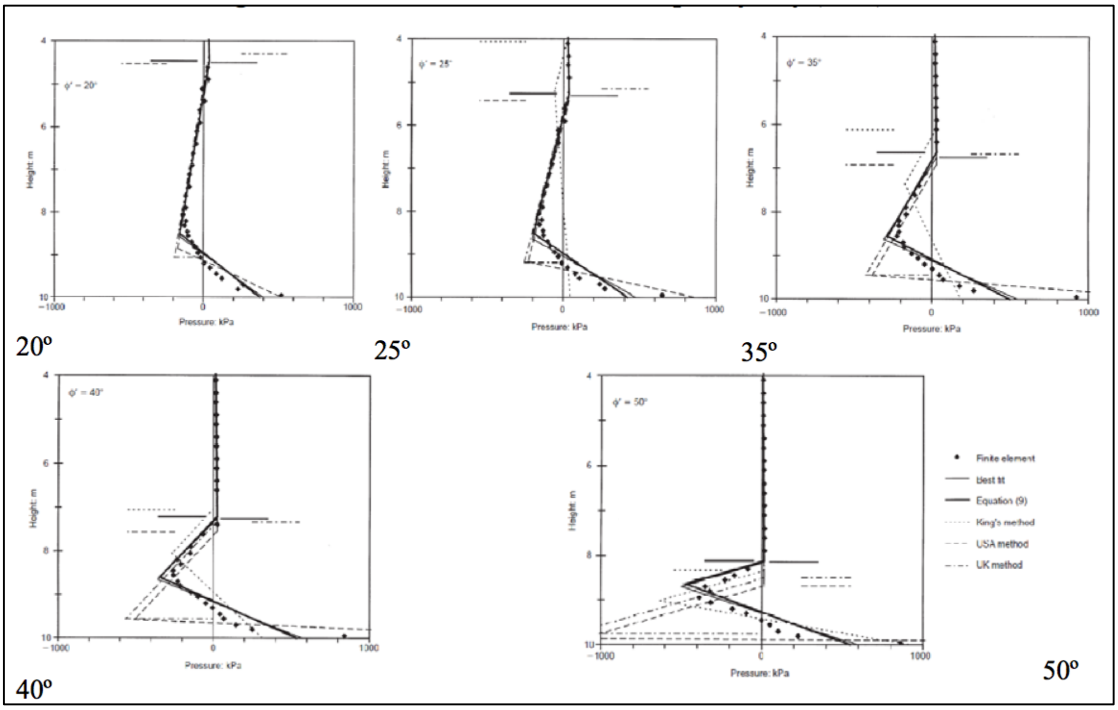

Figure 2-4: Case studies by Day (1999)

Day proposed an equation to define the point of zero pressure (Figure 2-5). This equation proposed a linear relation between the position of the point of zero pressure and the ratio of Kp to Ka. The proposal by King (1995) that $\varepsilon' = 0.35$ is generally conservative.

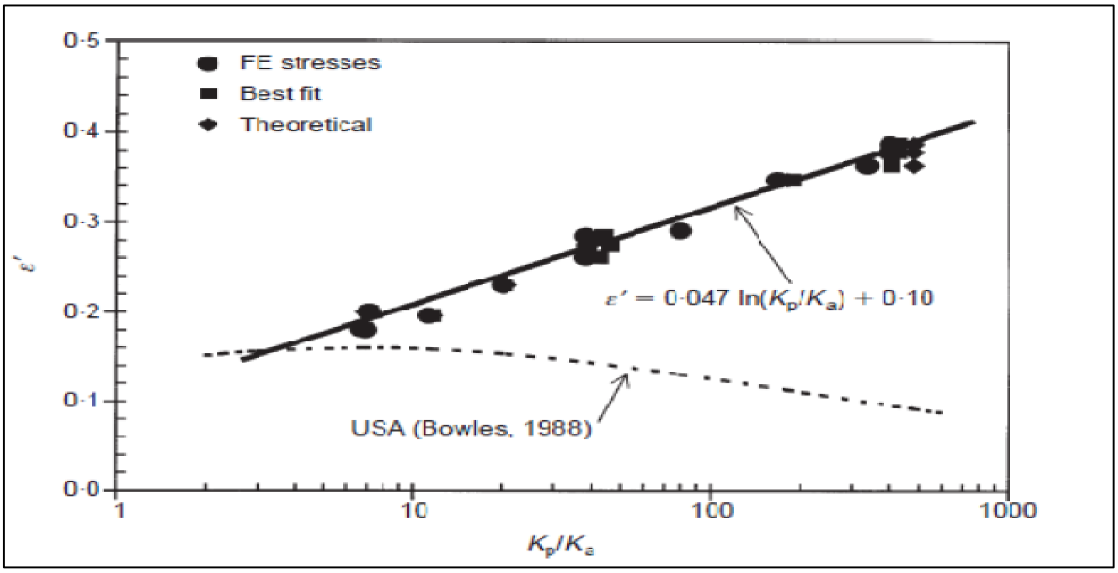

Figure 2-5: Point of zero net earth pressure, presented by Day (1999)

The rectilinear net pressure distribution and pressure coefficients predicted by Caquote and Kerisel are more accurate than the existing design methods commonly used in the UK and US. According to Day (1999), the predictions for both the critical retained height and the bending moment distribution using the empirical equations agree excellently when compared to the finite element numerical results for cantilever sheet pile walls. The finite element results are in fact in better agreement with Caquote and Kerisel's results than the existing design analytical methods.

2.3.4 Das (1990) Principles of Foundation Engineering

Cantilever sheet pile walls are usually recommended for retaining walls of moderate height (6 m or less, measured above the dredge line). According to Das (1990), such piles act as wide cantilever beams. The basic principles proposed by Das (1990) are explained in the figure on the following page, which indicates the nature of lateral yielding of a cantilever wall penetrating a sand layer below the dredge line.

The wall rotates about a point O (Figure 2-6 [a]). The hydrostatic pressures on either side of the sheet pile wall are assumed to cancel each other out; thus, only considering the effective lateral soil pressure below the dredge line to act on the sheet pile was assumed. In zone A, the lateral pressure is just the active pressure from the land side; however, in zone B, there will be active pressure from the land side as well as passive pressure from the water side due to the yielding occurrence of the wall. The condition in zone C is reversed, which is below the point O. The actual net pressure distribution on the wall is shown in Figure 2-6 (b), and a simplified version is illustrated in Figure 2-6 (c).

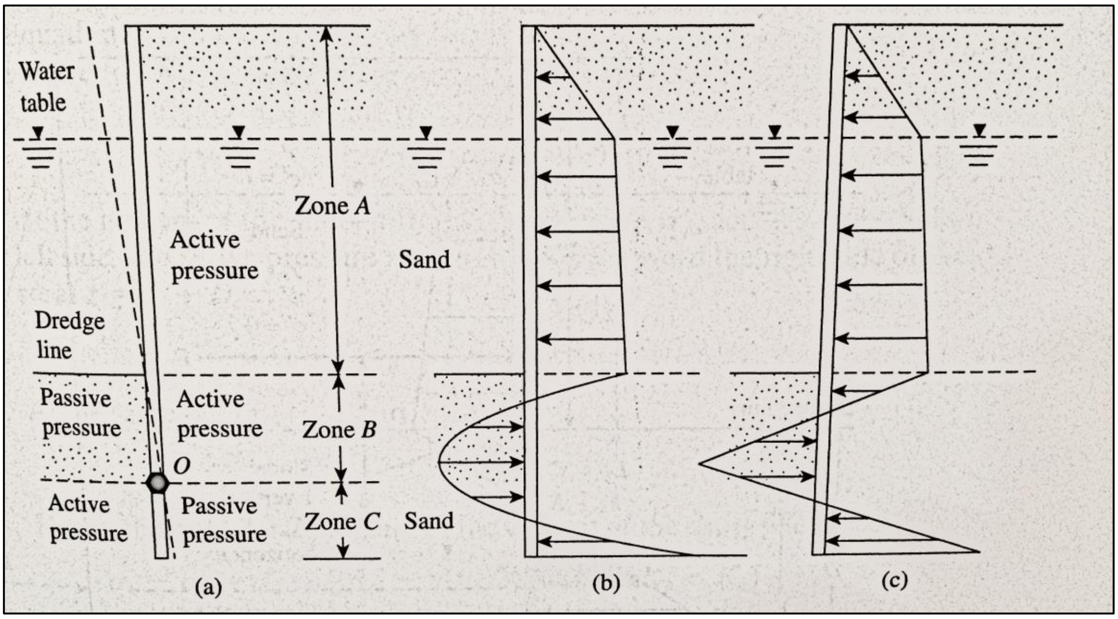

Figure 2-6: Cantilever sheet pile penetrating sand (Das 1990)

When the height of the backfill material behind a cantilever sheet pile wall exceeds 6 m, anchor sheet pile wall becomes more economical. According to Das (1990), this type of construction is referred to as an anchored sheet pile wall or an anchored bulkhead. Das specifies that the presence of anchors decreases the penetration depth of the sheet pile and reduces the cross-sectional area and weight of the sheet piles. However, Das (1990) suggests that the anchors be designed with care.

The two basic methods of designing anchored sheet pile walls are (a) the free earth support method and (b) the fixed earth support method. According to Das (1990), the free earth support method involves a minimum penetration depth to be obtained and the absence of a pivot point for the static system (Figure 2-7).

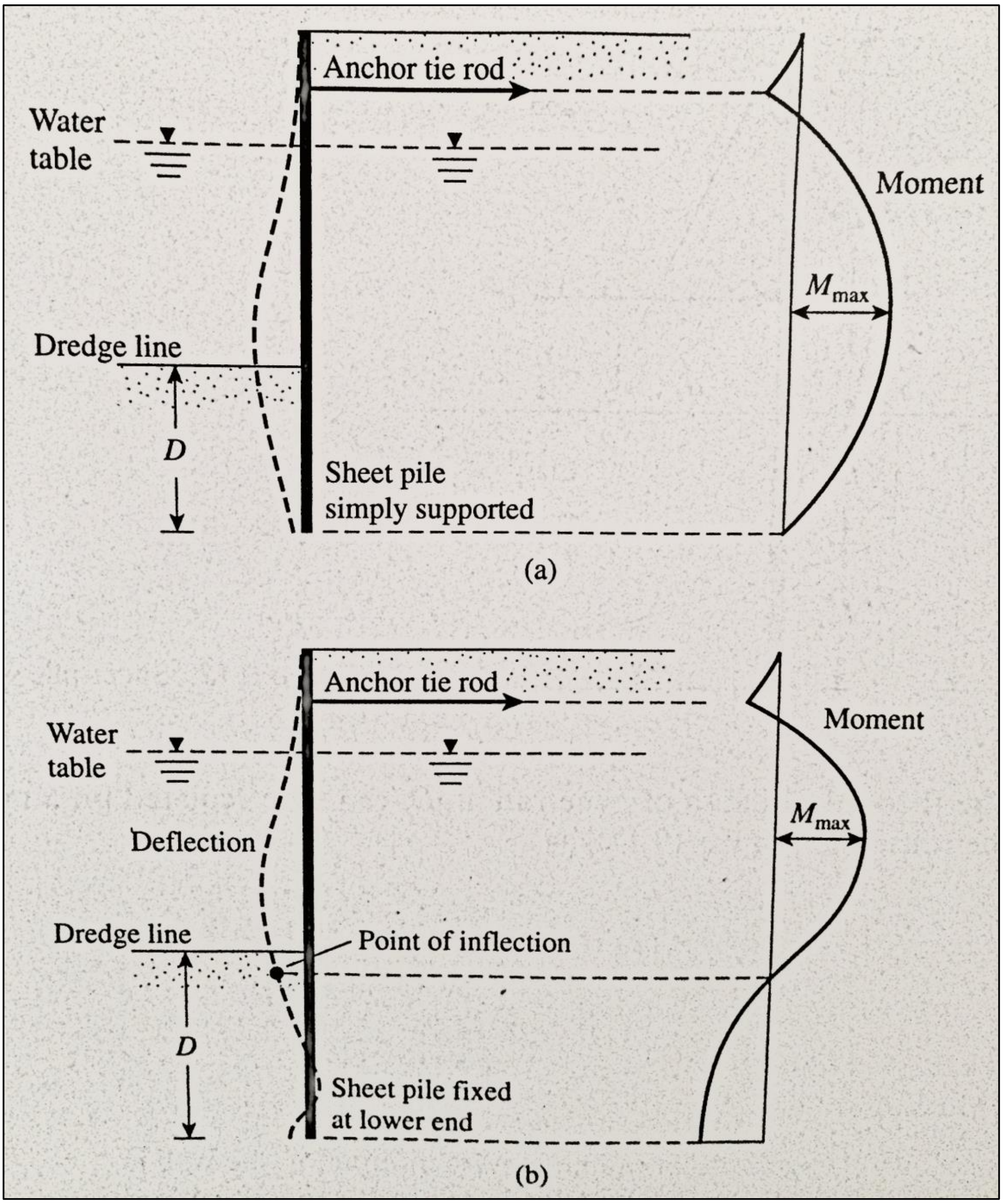

Figure 2-7: Nature of variation of deflection and moment for anchored sheet piles: (a) free earth support method; (b) fixed earth support method (Das 1990)

Fixed Earth Support Method for Anchored Piles

In the fixed earth support method, the sheet pile is embedded deeply in comparison with the height above the dredge level in such a way as to ensure that the passive pressure in front of the wall is no longer fully mobilised. An overturn in the normal earth pressure is achieved by means of the increasing embedment depth. The earth pressure distribution results is similar to that achieved for the cantilever sheet pile wall (Figure 2-8). The wall behaves as if partially built-in and being subjected to bending moments (United States Steel 1975).

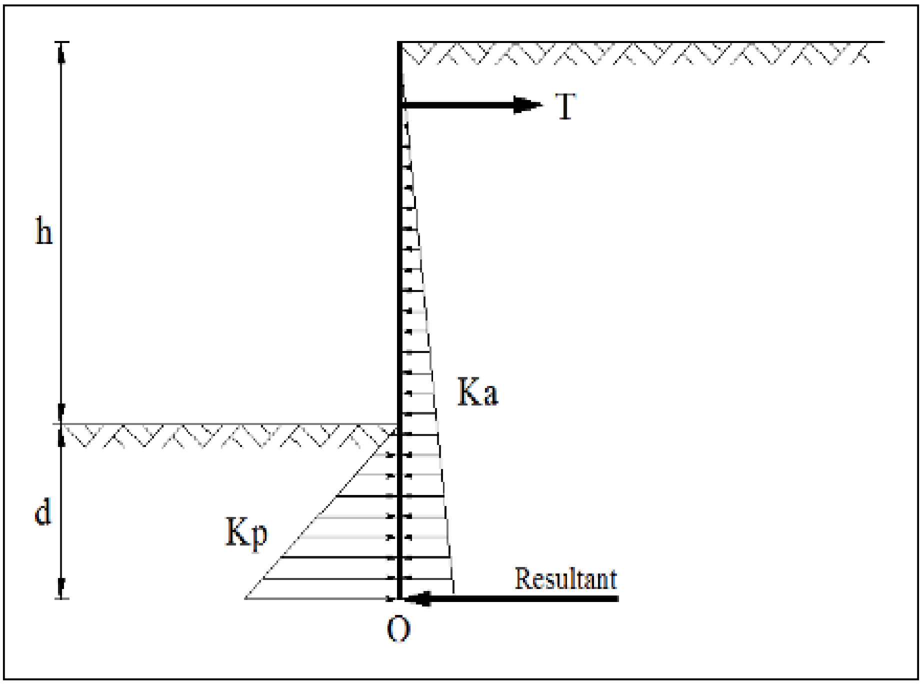

Figure 2-8: Fixed Earth Support Method (Torrabadella 2013)

Free Earth Support Method for Anchored Pile

The movement on the embedded zone of the wall has been assumed sufficient to mobilise both the active and passive pressures behind and in front of the wall, respectively. Thus, the method is based on the assumption to satisfy stability of the sheet pile against lateral displacement by means of driving the sheet pile only deep enough to withstand such pressures (Shanmugam 2004; Das 1990). The entire depth of embedment mobilises the shear strength of the soil (Figure 2-9).

Proceeding then by means of summing the moments with respect to the point of applied anchor force and equating the expression to zero, the minimum embedment depth is calculated to provide equilibrium.

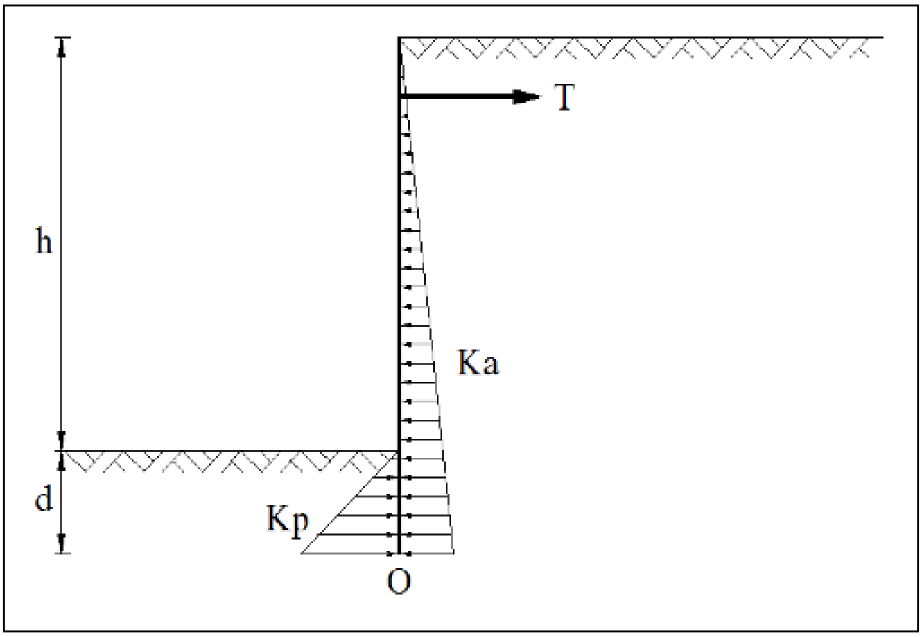

Figure 2-9: Free Earth Support Method (Torrabadella 2013)

The theory and assumptions made by Das (1990) for the development of the lateral earth pressures exerted on the sheet pile wall are based on Rankine theory. There are two commonly accepted methods for calculating simple earth pressure (Keystone Retaining Wall Systems 2003): Coulomb and Rankine theory. The Coulomb theory was developed in 1776, while the Rankine theory was developed in 1857. These theories, which remain the basis for present-day earth pressures calculation, are based on the fundamental assumptions that the retained soil is:

- cohesionless
- homogenous
- isotropic
- semi-finite
- well drained.

The active earth pressure calculation requires that the wall structure rotates or yields sufficiently to engage the entire shear strength of the soils involved to create the active earth pressure state. The amount of movement highly depends on the soil that is involved.

Both theories use identical parameters; however, Coulomb wedge theory calculates less earth pressure than Rankine theory (Figure 2-10). This indicates that the results obtained from the Rankine theory will be more conservative. Das (1990) made use of these conservative methods for the design of sheet pile walls.

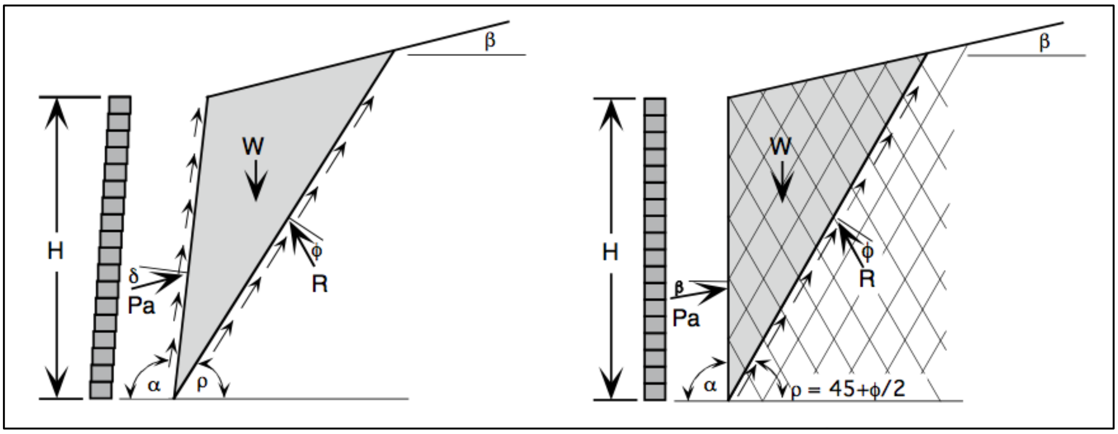

Figure 2-10: (a) Coulomb wedge analysis, (b) Rankine 'state of stress' analysis (Keystone Retaining Wall Systems 2003)

2.3.5 Blum's (1931) Equivalent Beam Method Theory for Anchored Piles

Blum's equivalent beam method theory is used to find the embedment depth, by analysing the sheet pile as a beam structure. The beam is divided into two sections: an upper beam and a lower beam. In the upper beam, the net pressure acts against the back of the wall; in the lower part of the beam, the net pressure action is placed in front of the wall.

The moments are taken around the point in line with the anchor force for the upper part of the beam to find the force R_b; in the lower beam, moments are taken at the bottom to find the embedment depth (Figure 2-11). The embedment depth must be increased to ensure that the reaction R_c can be engaged (Azizi 2000; Bowles 1996; Tsinker 1997).

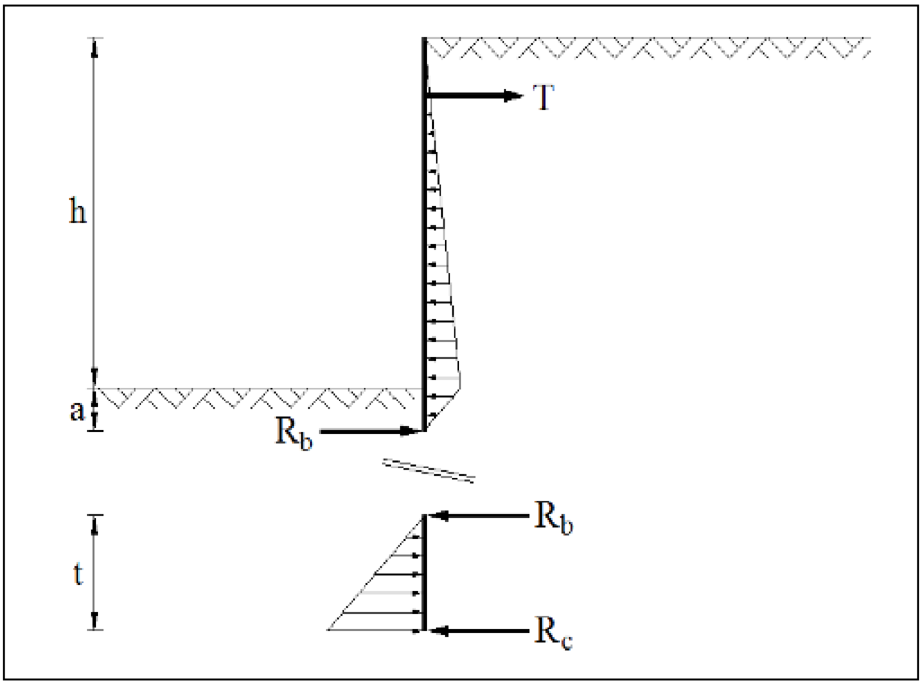

Figure 2-11: Blum's equivalent beam for anchored sheet pile wall design (Torrabadella 2013)

2.3.6 Conclusion of Classical Method Design

The comparison of the method proposed by Das (1990) with other currently used methods has shown that the results obtained compare well with the numerical finite element results provided by Day (1999) and Smith (2006). Using the analytical method proposed by Das (1990) can thus be considered successful for validating numerical solutions for cantilever sheet pile wall models against the analytical solutions. This method is used in the relevant chapters that follow.

2.4 Numerical Analysis and Dissertations

2.4.1 Smith (2006), Development of Numerical Models for Geotechnical Design

Smith (2006) investigated a cantilever sheet pile wall penetrating sand in the absence of a water table using the finite difference method software, FLAC. The numerical results obtained from the numerical model developed in FLAC were then compared to the analytical solutions and the advantages and disadvantages were discussed. The depth of embedment was then varied to identify the effect exerted on the sheet pile wall by analysing the bending moment, wall deflection and ground settlement. Smith's (2006) investigation demonstrated that FLAC produced similar results to the limit equilibrium methods. The outputs obtained were also found to be more accurate when compared to the limit equilibrium method solutions. Smith (2006) suggested the possible future work of undertaking numerical parametric studies using the cantilevered sheet pile wall model to develop an anchored sheet pile wall model. Performing parametric studies was also deemed valuable for the advanced analysis of the behaviour of the sheet pile walls.

2.4.2 Bilgin (2010), Numerical Studies of Anchored Sheet Pile Wall Behaviour Constructed in Cut and Fill Conditions

Construction of sheet pile walls involves either excavation in front or backfilling of soil behind the wall. Different loading conditions in the soil are generated due to the construction procedures, generating different wall behaviours. The conventional methods used in the design of anchored sheet pile walls, which are based on the limit equilibrium approach, do not consider the method of construction. However, continuum mechanics numerical methods, such as the finite element method, make it possible to incorporate the construction method into the analysis and design of sheet pile walls. This allows for the analysis of the soil-wall system, to obtain more viable and accurate solutions. Bilgin (2010) investigated the effect of wall construction by varying soil conditions and wall heights using finite element modelling. The construction method's influence on the wall behaviour in terms of wall deformation, wall bending moments and anchor forces were investigated, with Bilgin (2010) concluding that construction using backfilling produces significantly higher bending moments and wall deformations. These findings indicate that there are limitations to be considered when using the limit equilibrium methods, and that more information can be obtained by undertaking numerical analysis (Bilgin 2010).

2.4.3 Bilgin (2012), Lateral Earth Pressure Coefficient for Anchored Piles

According to Bilgin (2012), the design of anchored sheet pile walls established by the conventional methods is based on the lateral force and moment equilibrium of active and passive earth pressure and anchor forces. Bilgin (2012) carried out a parametric study using both conventional and numerical methods to investigate the behaviour of a single-level anchored sheet pile wall. The effect on the wall lateral earth pressures, wall moments and anchor forces was investigated. The results obtained indicated that the free earth support method over-estimates the bending moments, whereas the anchor forces were underestimated. Interestingly, new lateral earth pressure coefficients that took the stress concentration around the anchor level into account were used in the design, which led to more realistic earth pressure distributions acting on the wall, as well as more accurate anchor sheet pile wall designs.

2.4.4 Ramadan (2013), Effect of Wall Penetration Depth on the Behaviour of Sheet Pile Walls

The purpose of this dissertation was to analyse the wall penetration depth on sheet pile wall behaviour. According to Ramadan (2013), important serviceability considerations are not considered when using the limit equilibrium methods. This is because information about the wall deformation cannot be obtained by these analytical methods. Ramadan (2013) investigated wall behaviour by varying the soil conditions for both the cantilever and anchored sheet pile walls. Finite element analysis was then used to perform numerical modelling to analyse the behaviour of the walls and the structural response. It was found that wall deformations reduce with increasing wall penetration depth for both wall types and the bending moments significantly reduced with increasing wall penetration depth.

2.4.5 Torrabadella (2013), Numerical Analysis of Cantilever and Anchored Sheet Pile Walls at Failure and Comparison with Classical Methods

Torrabadella (2013) analysed the influence of the initial stress state condition on the horizontal displacement of sheet pile walls. It was found that for K_0 values between 0.7 and 0.9, minimum movement was registered at the top of the pile; however, the initial stress state also depended on the soil friction angle. Depending on the initial stress state, the wall movement was found potentially to change up to 40%. The influence of the construction procedure also had a critical effect on the wall movement. For anchored piles, it was found that when the anchors were pre-stressed, movement was absorbed, limiting wall strains. In contrast to cantilever sheet pile walls, the maximum horizontal displacement was found at a particular depth and not at the ground surface. A direct effect between the anchor force and horizontal wall displacement was found. Torrabadella (2013) also found that the limit equilibrium methods corresponded well with the numerical methods for both cantilever and anchored sheet pile walls.

2.4.6 Zhai (2009), Comparison Study for the Seismic Evaluation of Anchored Sheet Pile Walls

In Zhai’s (2009) study, the seismic stability and deformation of the channel bank and the anchored sheet pile wall subjected to a design earthquake load were investigated by analysing the results obtained from three different engineering approaches: the limit equilibrium methods, the p-y method and the time history soil structure (SSI) analysis method. It was found that the values obtained using FLAC (as the SSI method) for the maximum bending moment and anchor rod force were about 55% and 73% of the values obtained from the earth pressure method. For seismic stability, the system was found to be unstable when using the earth pressure method, but stable when using the SSI method (Zhai 2009).

2.5 Numerical Modelling Methods

Most engineering problems involve complex physical phenomena (Chaskalovic 2008). To gain a good understanding of these phenomena, engineers normally make simplified assumptions that allow the formulation of mathematical models (Pastor & Tamagnini 2004; Wood 2003).

Numerical analysis has evolved over the past few decades (Chaskalovic 2008), followed by prompt advances and improvements in modern computer technology (Rao 2005; Zienkiewicz, Taylor & Zhu 2013; Desai and Christian 1977). This will lead to the ability to undertake procedures, algorithms and other numerical techniques capable of solving ever more complex engineering problems. However, it is important for an engineer to know that with these numerical methods certain limitations, uncertainties and approximations need to be considered (Wood 2003). This leads to more computationally based studies being carried out in the geotechnical engineering industry. It is important that the results obtained from the numerical methods are validated against conventional or analytical methods (Pande & Pietruszczak 2004).

2.5.1 Industrially Commonly Known Numerical Analysis

The most common numerical techniques used currently in the geotechnical engineering industry are the finite difference method (FLAC) and the finite element method (PLAXIS). Finite difference methods were almost exclusively used in obtaining numerical solutions for geotechnical problems prior to the establishment of the finite element methods. The finite element method is considered one of the most important developments in civil engineering of the twentieth century (Papadrakakis 2001).

2.5.2 Background of FLAC Software

FLAC is a two-dimensional (2D) explicit finite difference software program, developed by Dr Peter Cundall in 1986 (*FLAC 2D online manual* 2009). This software makes it possible to visualise the behaviour of the structure in the soil, rock or any other material that may undergo plastic flow. A grid of the materials can be formed that represents elements or zones that can be adjusted by the user. This explicit, Lagrangian calculation scheme and the mix-discretisation zoning technique used in FLAC ensure the highly accurate modelling of flow and plastic collapse. Large 2D calculations can be made without the need for massive memory requirements due to no matrixes being formed.

FLAC was originally developed for geotechnical and mining engineers. This software offers a wide range of capabilities, including for solving complex problems in mechanics. The FLAC software has special built-in functions that make it unique. The application range of FLAC is extensive because it is equipped with 11 built-in constitutive models, five optional facilities and several kinds of structure elements as well as a built-in coding language, FISH (Shen 2012).

Other element structures present in FLAC include beam, anchor, pile and shell structures. These elements are used to create more realistic models of geotechnical engineering problems in the software. It will be useful to design an anchored sheet pile wall model in FLAC. The build-in coding language (FISH) can also be used to define new functions and variables to meet user demands.

2.5.3 FLAC Software Advantages and Disadvantages

The FLAC software, used here to develop a numerical model for the design of sheet pile walls, has several advantages over other methods (*FLAC 2D online manual* 2009):

- The mix-discretisation zoning method is more accurate than the reduced integration method generally used to simulate the plastic flow of materials.
- The explicit methods used decrease the time needed to solve non-linear equations.
- The full dynamic equation of motion is used, making the software more suitable to simulate problems involving vibration, failure and large deformations.
- The element numbering is done in row and column formatting.

There are also some disadvantages when using FLAC that need to be considered (*FLAC 2D online manual* 2009):

- More time is needed to reach convergence for a linear problem than when using the finite element methods.

- FLAC depends on the ratio of maximum and minimum natural periods of the system for the convergence velocity.

Thorough research has shown that FLAC is an excellent software choice for modelling any geotechnical engineering model. Therefore, FLAC is used to undertake the numerical modelling in this dissertation.

Chapter 3: Developing a Design Tool for Sheet Piles Walls

3.1 Introduction

Nowadays, the engineering profession is discovering and using the computational powers of computer spread sheets in practice. They are used in bid preparation, budgeting, control, engineering design computation and many other areas. However, the computational power of the computer spread sheet is only the beginning of what can be accomplished. The success of geotechnical works relies on the proper planning, analysis and design of sheet pile walls. The analytical methods normally consist of many equations and may take a long time to solve by hand. This chapter gives an overview of how the tedious equations obtained by the analytical methods for the design of sheet pile walls are used to develop design tools in an Excel spread sheet that can automatically solve any sheet pile wall design problem in a matter of seconds.

Presented within this chapter is an explanation of the analytical procedure necessary for the design of sheet pile walls, as well as the advantages and disadvantages of these analytical methods. The development of the sheet pile wall design tool is explained, and different geotechnical problem examples and output solutions are given.

3.2 The Analytical Methods

A sheet pile wall is an alternative to using a gravity retaining wall to support retained material. It consists of vertical structural elements implanted at adequate depth into the soil beneath the specific granular material to be retained (Day 1999). Several sheet pile walls design methods exist, dating back to the first half of the twentieth century. These original proposals have been continuously and may currently be being reviewed. To define the embedment depth below the dredge line for cantilever and anchored sheet pile walls, geotechnical design calculations using analytical methods are used for establishing equilibrium of the horizontal forces and moments developed along the wall (Figure 3-1).

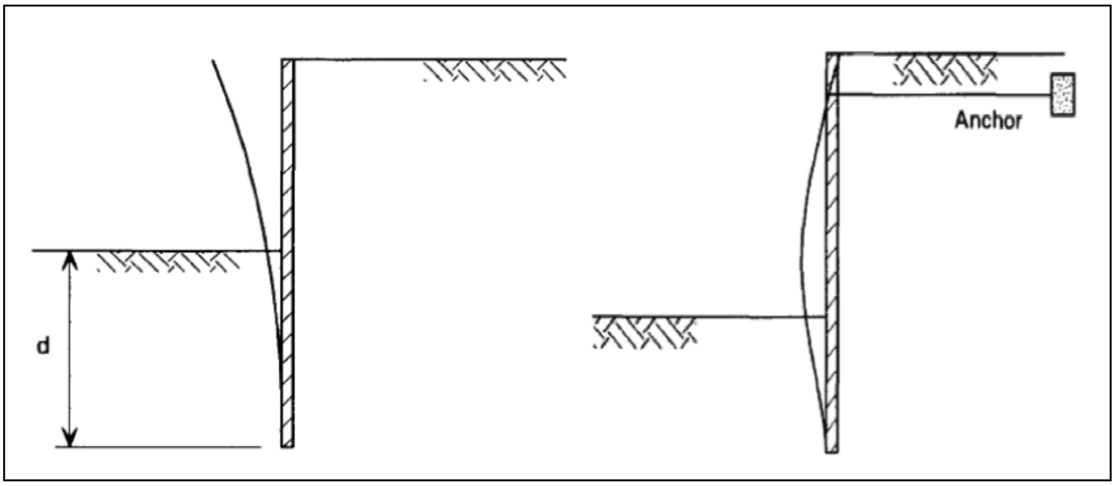

Figure 3-1: Displacement of Sheet Pile Wall: (a) Cantilever (b) Anchored (Yandzio 1998)

3.3 Design Procedure for Cantilever Sheet Pile Wall

Cantilever sheet pile walls are usually recommended for walls of moderate height (6 m or less, measured above the dredge line). In such walls, the sheet piles act as a wide cantilever beam above the dredge line. The net lateral pressure distribution on a cantilever sheet pile wall can be explained by the basic principles of Das (1990), with the aid of Figure 3-2 (a).

It has been assumed that the straight planes represent the ground and failure surfaces and that the resultant force acting on the backfill slope is acting in a parallel direction. Both active and passive pressure zones will develop on either side of the sheet pile wall, as indicated in Figure 3-2 (b).

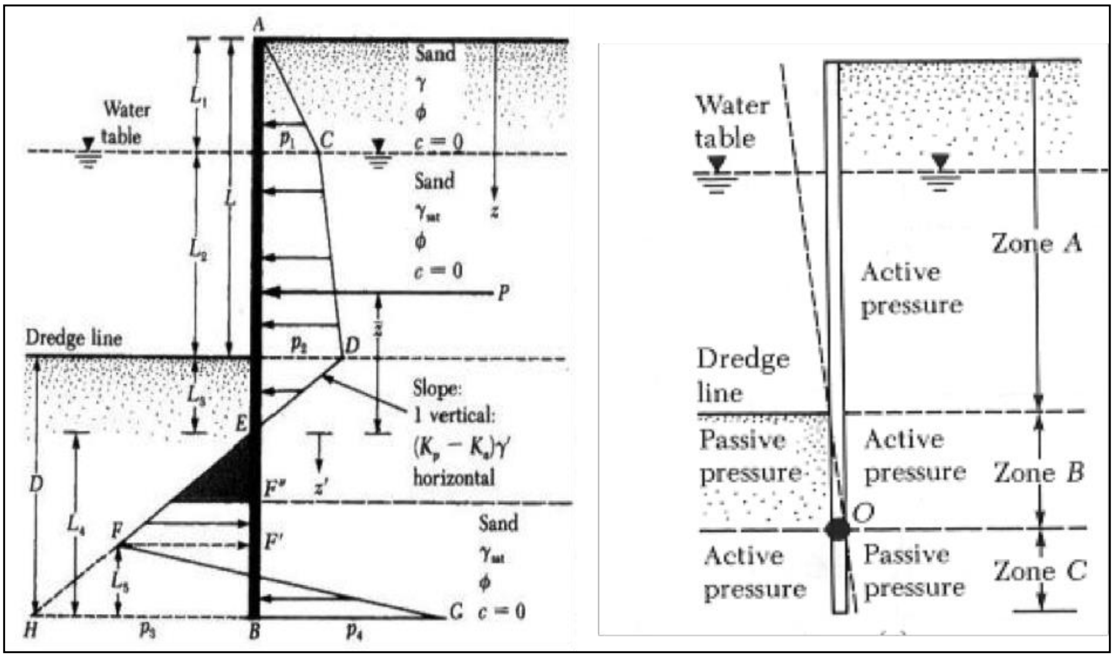

Figure 3-2: (a) Cantilever Pile Penetrating a Sandy Soil, (b) Active and Passive Pressure Distribution (Das 1990)

Due to this development of both active and passive pressures, it is necessary to determine the Rankine's active and passive pressure coefficients:

$$K_a = \tan^2(45 - \phi/2) \tag{3-1}$$

$$K_p = \tan^2(45 + \phi/2) \tag{3-2}$$

Where

ϕ - Angle of friction of sand

It is important to note that after conducting a geotechnical survey, the designer will know certain input parameters. This is important information, as it gives knowledge about the type of soil, the friction angle of the soil, the length above the dredge line and the soil cohesion.

Knowing this input data, the active pressure on the right side of the sheet pile wall can be determined:

$$\sigma'_1 = \gamma L_1 K_a \tag{3-3}$$

$$\sigma'_2 = (\gamma L_1 + \gamma' L_1)\, K_a \tag{3-4}$$

Where

γ - Unit weight of the soil above the water table

$\gamma\,'$ - Effective unit weight of the soil = $\gamma_{sat} - \gamma_w$

At the level of the dredge line, the hydrostatic pressure on both sides of the wall is equal in magnitude and hence cancels out. As indicated in Figure 3-2 (a), the net pressure will be equal to zero at the point E. Hence, using the ratio given as 1 vertical to $\gamma'(K_p - K_a)$ in the horizontal, the unknown length L_3 can be determined:

$$L_3 = \frac{\sigma'_2}{\gamma'(K_p - K_a)} \tag{3-5}$$

The total pressure above the dredge line can now be determined by applying the area of known pressure exerted on the sheet pile wall and summing all the forces in the horizontal:

$$\mathrm{P} = 0.5\,\sigma_1^{'}\mathrm{L}_1 + \sigma_1^{'}\mathrm{L}_2 + 0.5(\sigma_2^{'} - \sigma_1^{'})\mathrm{L}_2 + 0.5\sigma_2^{'}\mathrm{L}_3 \tag{3-6}$$

Summing the moments of all the pressure forces exerted on the wall about point E and dividing by the total pressure force P will provide the distance $\bar{z}$ from E to the force P.

$$\bar{z} = \left[\begin{matrix} 0.5\sigma_1^{'}\mathrm{L}_1 * \left(\frac{L_1}{3} + L_2 + L_3\right) + \\ \sigma_1^{'}\mathrm{L}_2 * \left(L_3 + \frac{L_2}{2}\right) + \\ 0.5(\sigma_2^{'} - \sigma_1^{'})\mathrm{L}_2 * \left(L_3 + \frac{L_2}{3}\right) + \\ 0.5\sigma_2^{'}\mathrm{L}_3 * \frac{L_3}{3} \end{matrix}\right] \Big/ P \tag{3-7}$$

Thus, the only unknown is the length of L_4, which is determined by deriving four equations containing the unknown length L_4 by:

- the formation of an equation for p_3 using the given ratio of 1 vertical to $\gamma'(\mathrm{K_p} - \mathrm{K_a})$ in the horizontal (3-8)
- determining the net pressure p_4 at the bottom of the sheet pile by subtracting the total active pressure from the total passive pressure (3-9)
- summing the moments about the point B at the bottom of the sheet pile (3-10)
- deriving an equation for the length L_5, which forms a part of the unknown length L_4 (3-11).

$$\sigma_3^{'} = \gamma'\mathrm{L}_4(\mathrm{K_p} - \mathrm{K_a}) \tag{3-8}$$

$$\sigma_4^{'} = \sigma_5^{'} + \gamma'\mathrm{L}_4(\mathrm{K_p} - \mathrm{K_a}) \tag{3-9}$$

$$P(L_4 + \bar{z}) - (0.5L_4\sigma_3^{'})\left(\frac{L_4}{3}\right) + 0.5L_5(\sigma_3^{'} + \sigma_4^{'})\left(\frac{L_5}{3}\right) \tag{3-10}$$

$$L_5 = \sigma_3^{'}\mathrm{L}_4 - 2\mathrm{P} \Big/ \sigma_3^{'} + \sigma_4^{'} \tag{3-11}$$

These four equations are then rearranged to determine L_4, solving an equation to the fourth power:

$$L_4^4 + A_1L_4^3 - A_2L_4^2 - A_3L_4^1 - A_4 = 0 \tag{3-12}$$

Where A_1, A_2, A_3 and A_4 are given by Das (1990):

$$A_1 = \frac{\sigma'_5}{\gamma'(K_p - K_a)} \tag{3-13}$$

$$A_2 = \frac{8P}{\gamma'(K_p - K_a)} \tag{3-14}$$

$$A_3 = \frac{6P[2\bar{z}\gamma'(K_p - K_a) + p_5]}{\gamma'^2 (K_p - K_a)^2} \tag{3-15}$$

$$A_4 = \frac{P[6\bar{z}p_5 + 4P]}{\gamma'^2 (K_p - K_a)^2} \tag{3-16}$$

Where p_5 is the passive pressure applied above point E.

The decline in active pressure immediately above point E due to the large passive pressure being exerted on the left side of the sheet pile wall is given by:

$$p_5 = \left(\gamma L_1 + \gamma' L_2\right)K_p + \gamma' L_3 (K_p - K_a) \tag{3-17}$$

Knowing the length L_4, the sheet pile penetrating depth is simply:

$$D_{theoretical} = L_3 + L_4 \tag{3-18}$$

It is important for designers to note that a certain factor of safety (FOS) has to be satisfied to avoid any possibility of soil-system failure. It is at the discretion of the designer to apply a FOS to the calculated sheet pile penetrating depth or to decrease the overestimated Rankine's passive pressure coefficient. According to Das (1990), it is recommended to apply a FOS of between 1.5 and 2.

As already mentioned, it is important to determine the maximum bending moment distributed on the sheet pile wall for design purposes. Thus, the sheet pile is analysed as a normal beam to find the point of zero shear force:

$$Z' = \sqrt{\frac{2P}{(K_p - K_a)\gamma'}} \tag{3-19}$$

Knowing the maximum bending moment will occur at this point, the moments about the point of zero shear force are summed:

$$\mathrm{Mmax} = \mathrm{P}(\bar{z} + Z') - [0.5\gamma' Z'^2(\mathrm{K_p} - \mathrm{K_a})](\frac{Z'}{3}) \quad (3\text{-}20)$$

Table 3-1: Analytical Results for Cantilever Pile

Parameters	Results	
Length (m)	L4	5.95
Theoretical Penetration Depth (m)	Dt	6.51
Factor of Safety	FOS	1.40
Actual Penetration Depth (m)	Da	9.12
Total Wall Length (m)	Ltot	18.12
Maximum Bending Moment (kN.m)	Mmax	741

Obtaining these solutions using the tedious analytical equations to be solved by hand takes a long time and is prone to human error. Thus, being able to solve many different sheet pile wall problems in a matter of minutes would be useful for the engineering industry.

3.4 Cantilever Sheet Pile Problem Description

The following example is solved analytically using the procedure detailed in Das (1990). The example is then solved in an Excel spread sheet developed by this study so that the relevance of developing design tools for cantilevered sheet pile walls can be understood.

After conducting a geotechnical survey, certain input parameters will be known. These input parameters give important information such as the length (L) above the dredge line, the cohesion (c) of the soil, the friction angle ϕ and the unit weight γ of the soil.

For the example in Figure 3-3, the cantilever sheet pile wall is penetrating a sandy soil and therefore has zero cohesion. The friction angle ϕ and unit weight γ of the sandy soil were obtained from Das (1990). The solutions obtained for this example using the analytical limit equilibrium methods are tabulated in Section 3.5.

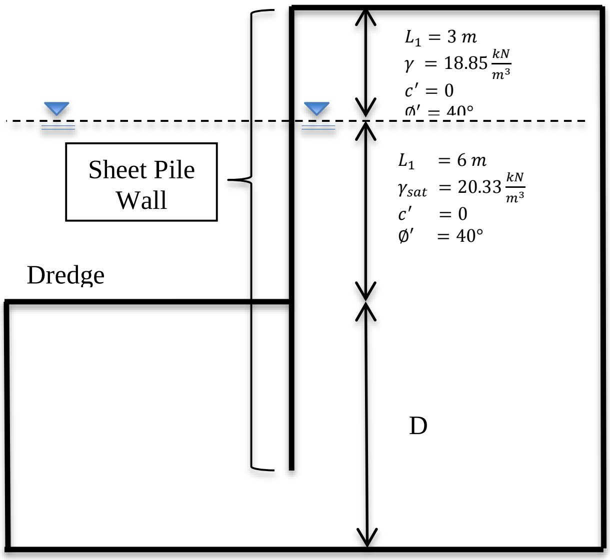

Figure 3-3: Cantilever Sheet Pile Problem Definition (Das 2007)

3.5 Development of Excel Spread Sheet for the Cantilever Pile Problem

The aim in developing the Excel spread sheet was that it could automatically solve complex derived analytical equations by means of a user inputting known data (Table 3-2) into the spreadsheet.

Table 3-2: User Input Parameters

Parameters above the dredge line			Parameters below the dredge line		
Depth (m)	L1	3	Depth (m)	L2	6
Unit weight of soil ($\frac{kN}{m^3}$)	γ_{dry}	18.85	Unit weight of soil ($\frac{kN}{m^3}$)	γ_{sat}	20.33
Cohesion of soil ($\frac{kN}{m^2}$)	c1	0	Cohesion of soil ($\frac{kN}{m^2}$)	c2	0
Angle of Internal Friction (Degrees)	Ø	40	Angle of Internal Friction (Degrees)	Ø	40
Effective Unit weight of soil ($\frac{kN}{m^3}$)	γ'	0	Effective Unit weight of soil ($\frac{kN}{m^3}$)	γ'	10.52

3.5.1 Known Geotechnical Input Data

The input data give an outline of the design problem, such that the total depth above the dredge line is the sum of length 1 and length 2, giving 9 m. Normally, cantilever sheet pile walls are used for heights of less than 6 m (Das 1990), indicating that this sheet pile wall is very long. The unit weight of the dry soil at a depth of 3 m from the ground surface is 18.8 ${kN}/_{m^3}$ and the unit weight of the saturated soil below the water table is 20.33${kN}/_{m^3}$. The effective unit weight of the soil is found by subtracting the unit weight of water (9.81${kN}/_{m^3}$) from the saturated unit weight of the soil, which gives 10.52${kN}/_{m^3}$. The soil type is classified as a sandy type soil. Therefore, the cohesion of the soil above and below the water table is equal to zero. The internal friction angle of the sandy type soil is 40 degrees.

3.5.2 Designer Selection of Factor of Safety

As mentioned, when using the analytical methods, it is recommended to apply a FOS either at the beginning of the problem or at a later stage by increasing the theoretical penetrating depth of the sheet pile. This decision influences the final solutions obtained for the total length of the sheet pile and the point on the structure at which zero shear force occurs; hence, the maximum bending moment will also be affected.

The designer using the Excel spread sheet is given the option of inputting the FOS value required and selecting one of two options, as follows:

(1) Kp (Rankine's passive pressure coefficient) is used when the FOS should be applied to the calculated theoretical penetration depth of the sheet pile. Then the non-factorised passive pressure coefficient will be used (Kp).

$$Kp = 3.25 \qquad \text{from} \quad (3\text{-}2)$$

(2) Kp design (Rankine's passive pressure coefficient after an applied FOS). If applying the FOS directly to the Rankine's passive pressure coefficient, the pressure coefficient will be reduced (Kp design).

Kp design = Kp/FOS (3-21)

= 2.5

Since the FOS was selected as 1.4 (Table 3-3), it was assumed that the output results and solutions for using both cases of Rankine's passive pressure coefficient would be equal. However, after analysing the solutions, this was found not to be the case.

Table 3-3: Designer Selection for Kp

FOS	Kp	Kp design
1.4	4.6	3.29

3.5.3 Automatic Analytical Analysis

A separate section of automatic analysis was next derived in Excel to solve all the analytical equations. This is identified in the Excel spread sheet under automatic analysis:

Table 3-4: Automatic Analysis of Analytical Equations

Parameters	Outputs
K_a	0.22
$K_a - K_p$	4.38
$\sigma_1^{;}$	12.30
$\sigma_2^{;}$	26.02
L_3	0.56
P_1	18.45
P_2	114.96
P_3	7.35
P	140.75
$\bar{z}$	3.62
$\sigma_5^{;}$	576.40
A_1	12.51
A_2	24.43
A_3	361.77
A_4	866.73
L_4^4	23.79
Y	0.00
$\sigma_3^{;}$	274.15
$\sigma_4^{;}$	850.55
L_5	1.20

If a FOS is applied to Rankine's passive pressure coefficient before the automatic analysis commences, to reduce the passive pressure coefficient, the theoretical depth obtained when using a factored passive coefficient is found to be 6.06% smaller than an un-factored passive pressure coefficient (Table 3-5).

Table 3-5: Effect of Kp Selection on Sheet Pile Wall Length

Parameters	Kp design	Kp	Percentage difference (%)
Total theoretical length	16.51	15.51	6.06
Total actual length	16.51	18.12	8.88
Point of zero shear force	2.95	2.47	16.27

The application for reducing the passive pressure coefficient compared to applying a FOS to the theoretical penetration depth will lead to an 8.88% decrease for the actual factored penetrating sheet pile wall length.

The point of zero shear force on the sheet pile wall will decrease by 16.27% when applying a FOS during the automatic analysis. This affects theoretical penetration depth.

An indirect relationship was found between the actual wall penetration depth and the point of zero shear force on the pile. If the actual wall penetration depth increases, the point of zero shear force decreases.

3.5.4 Important Output Values

The important output values such as length L_4, theoretical depth D_t, actual depth D_a and maximum bending moment M_{max} were next obtained (Table 3-6).

Table 3-6: Important Theoretical Output Solutions

Parameters	Kp design	Kp	Percentage difference (%)
Theoretical Penetration Depth	7.51	6.51	13.31
Actual Penetration Depth	7.51	9.12	17.65
Maximum Bending Moment	787	741	5.84

It can be seen that when a FOS is applied to the passive pressure coefficient compared to applying the FOS to the theoretical pile penetration depth, the theoretical penetration depth of the sheet pile increases by 13.31% with the reduction of Rankine's passive pressure coefficient. This causes the pile penetration depth to remain constant for both theoretical and actual wall penetration depths.

Applying a FOS to the theoretical penetration depth will lead to an increase of 17.65% to the actual penetration depth. The increase of the actual penetration depth reduces the maximum bending moment by 5.84% for the un-factored passive pressure coefficient compared to the factored passive pressure coefficient. This formulates an indirect relation between the theoretical and actual penetration depths, as well as between the actual penetration depth and the maximum bending moment obtained.

3.5.5 Graphical Visual Representation

As the analytical methods and calculations do not indicate the outputs graphically, a simple table has been developed to give graphical visual outputs for the deformation, shear force and bending moments distributed along the sheet pile wall.

The pressure diagram in Figure 3-4 was established by knowing the pressure at certain points on the sheet pile wall as calculated using the analytical equations.

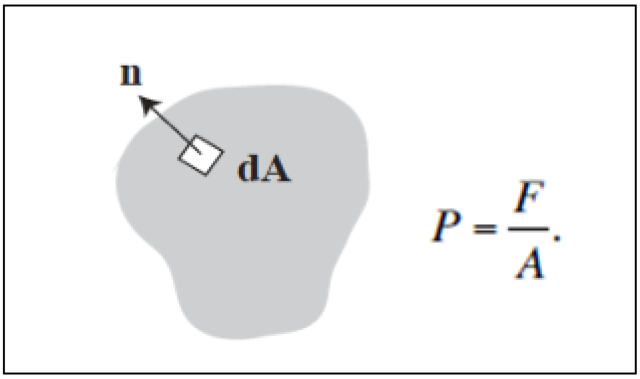

Figure 3-4: Hydrostatic equilibrium of fluid motion (Szolga 2010)

Where P is the pressure (kN/m^2)

F is the force (kN)

A the Area in (m^2)

By interpolating between the known depths, it was possible to find the pressures corresponding to the increasing depths. The shear force at known depths were calculated for a 1m-wide strip using equation (3-22) and similarly interpolating between two values and multiplying by a half to find the specific shear force at a particular depth. The maximum bending moment was calculated using equation (3-23) for specific depths and interpolating between values for increasing sheet pile depth.

$$\text{Net ShearForce} = \text{pressure} * (\text{length} * \text{width}) \quad (3\text{-}22)$$

$$\text{Net Bending Moment} = \text{force} * \text{distance} \quad (3\text{-}23)$$

From the hydrostatic equilibrium of fluid motion, the force applied on an object is a vector, while the pressure is a scalar. For a force produced by pressure, it is necessary to consider a surface with a certain area and direction.

In statics, moments are effects (of a force) that cause rotation. When commuting equilibrium, it is necessary to calculate the moment for every force that has been generated on the object. The moment has a magnitude equal to the product of the force magnitude F and the perpendicular distance from the point to the line of action of the force (Figure 3-5).

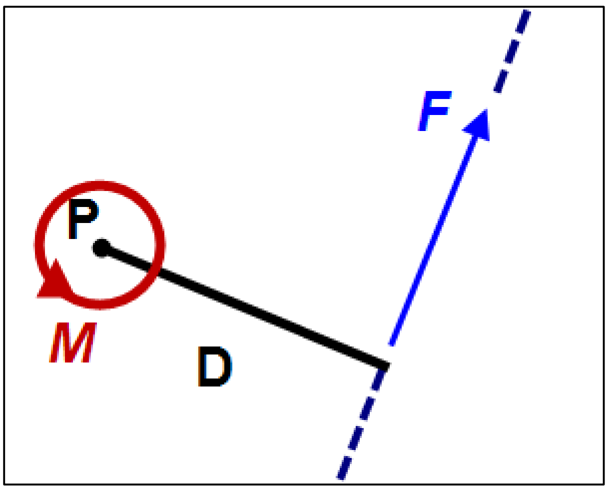

Figure 3-5: System of forces and moments (Szolga 2010)

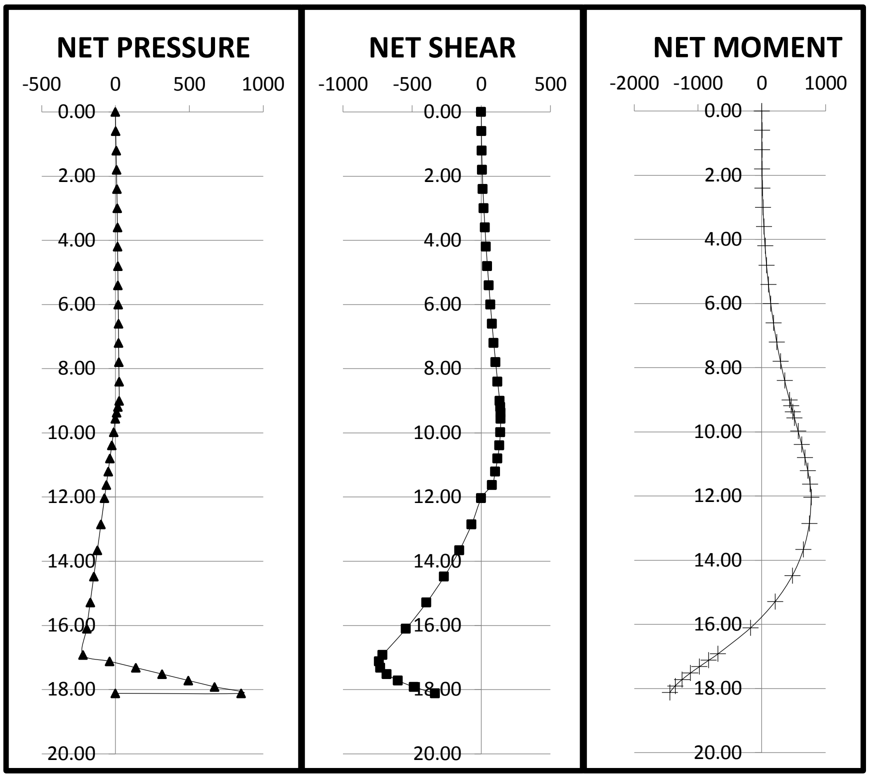

Figure 3-6: Visual Diagrammatic Output Figures for a cantilever sheet pile

3.6 Design Procedure for Anchor Walls

Anchored walls, also known as tieback walls, with a single row of anchors, are able to achieve equilibrium without the necessity of considering the passive reaction at the bottom of the back of the wall. Depending on the method of design, it may be required to take the passive reaction force into account. The main advantage of an anchored sheet pile when compared to the classical cantilever sheet pile is the ability of the anchor force to reduce the embedment depth of the penetrating pile, thus increasing the excavation depth, which in turn makes the structure more profitable. However, some disadvantages have been found that need to be considered, such as that until the anchor is placed, the structure behaves as a simple cantilever sheet pile wall.

All the equations as described for cantilevered analytical design are similarly used for the anchored sheet pile wall design, up until the point of zero shear force and maximum bending moment need to be calculated. This is because this sheet pile type has the extra unknown anchor tie rod force, as well as the requirement to sum all the moments about the point at which the anchor force is placed, instead of around the sheet pile tip.

To obtain this force, it is necessary to sum all the forces in the horizontal direction and equate that to zero. This is achieved by subtracting the pressure force exerted on the sheet pile due to triangle EFB from the total force exerted on the sheet pile above the point E, indicated by the force P, to establish the equation (3-24):

$$\mathrm{F} = \mathrm{P} - 0.5(\gamma' (K_p - K_a))\, L_4^2 \qquad (3\text{-}24)$$

Then, instead of summing the moments about the pile tip to find the length L_4, it is now required to sum all the moments about the point O as shown in Figure 3-7, which is at the point of the anchor tie rod force, to equate the equation (3-25), rearranging to solve for the unknown length L_4:

$$L_4^3 + 1.5L_4^2(l_2 + L_2 + L_3) - [\frac{3P(L_1+L_2+L_3)}{\gamma' K_p - K_a}] \qquad (3\text{-}25)$$

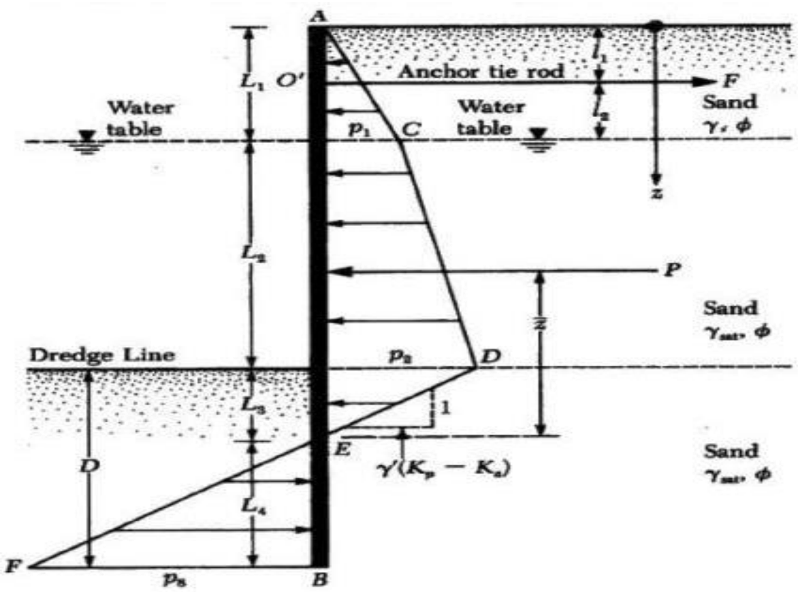

Figure 3-7: Anchored sheet pile penetrating a sandy soil (Das 1990)

The theoretical penetration depth can now be added = $L_3 + L_4$

According to Das (1990), for anchored sheet pile wall models, it is recommended to increase the theoretical depth by about 30–40%, to take the actual construction process into consideration. Thus, the actual penetrating depth of the sheet pile = 1.3 or $1.4D_{theoretical.}$

If a FOS is applied to Kp at the beginning of the design procedure, then the increase in theoretical depth is not required. According to Das (1990), the maximum theoretical moment to which the sheet pile will be subjected occurs at a depth between z = L1 and z = L1 + L2. The depth of zero shear force and hence maximum moment may be calculated by making a cut on the structure, analysing the structure as a beam and summing the moments around that point:

Point of zero shear = $0.5 \times \sigma'_1 L_1 - F + \sigma'_1 (z - L_1) + 0.5 \times K_a \gamma' (z - L_1)^2$ (3-26)

Thus, once the point of zero shear is determined, the maximum bending moment can easily be found.

$$M_{max} = -(0.5 \times \sigma'_1 L_1) \times \left[x + \left(L_1/3\right)\right] + F(x+1) - \sigma'_1 x \times \left(\frac{x}{2}\right) - 0.5 K_a \gamma' (x^2) \frac{x}{3} \quad (3\text{-}27)$$

The solutions obtained for this example when using the analytical limit equilibrium methods are tabulated in Table 3-7.

Table 3-7: Analytical Results for Anchored Pile

Parameters		Results
Length (m)	L4	6.63
Theoretical Penetration Depth (m)	Dt	2.22
Factor of Safety	FOS	1.40
Actual Penetration Depth (m)	Da	3.11
Total Wall Length (m)	Ltot	12.11
Maximum Bending Moment (kN.m)	Mmax	180

3.7 Anchored Sheet Pile Problem Description

The following example is solved analytically using the analytical design approach. The example is then solved in an Excel spread sheet developed in the paper so that the relevance of developing design tools for sheet pile walls can be understood.

After conducting a geotechnical survey, certain input parameters will be known. These input parameters give important information such as the length (L) above the dredge line, the cohesion of the soil, the friction angle ϕ and the unit weight γ of the soil. For the example in Figure 3-8, the cantilever sheet pile wall is penetrating a sandy soil and therefore has zero cohesion. The friction angle ϕ and unit weight γ of the sandy soil were obtained from Das (1990).

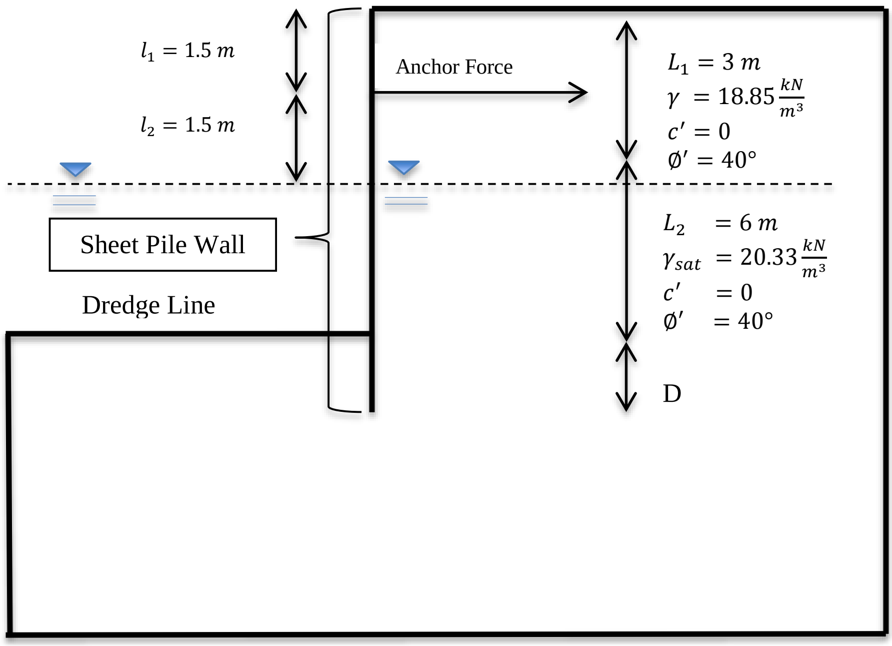

Figure 3-8: Anchored sheet pile problem definition (Das 2007)

3.8 Development of Excel Spread Sheet for Anchor Wall Problem

The design of the Excel spread sheet was aimed at developing a tool to solve automatically the complex derived analytical equations, by requiring a user to enter known input data into clearly labelled cells.

3.8.1 Known Input Data

As the input data required were clearly explained for cantilevered sheet pile wall Excel spread sheet development, this shall not be repeated here, as the only difference is that the occurrence of a depth above (l_1) and below (l_2) the anchor tie rod force needs to be specified by the user.

Table 3-8 gives the known data to be entered by the user.

Table 3-8: User input data

Parameters above the dredge line			Parameters below the dredge line		
Depth (m)	L1	3	Depth (m)	L2	6
Unit weight of soil ($\frac{kN}{m^3}$)	γ	18.85	Unit weight of soil	γ	20.33
Cohesion of soil ($\frac{kN}{m^2}$)	c1	0	Cohesion of soil	c2	0
Angle of Internal Friction (degrees)	$\emptyset$	40	Angle of Internal Friction	$\emptyset$	40
Effective Unit weight of soil ($\frac{kN}{m^3}$)	γ'	0	Effective Unit weight of soil	γ'	10.52
Length above and below anchor force (m)	$l_{1/2}$	1.5			

3.8.2 Automatic Analytical Analysis

Tables 3-9 and 3-10 provide the data for automatic analytical analysis.

Table 3-9: Automatic analysis of Analytical Equations

Parameters	Outputs
K_a	0.217442832
K_p	4.598909932
$K_p - K_a$	4.3814671
$\sigma_1^{'}$	12.29639215
$\sigma_2^{'}$	26.02138371
L_3	0.564540485
P_1	18.44458823
P_2	114.9533276
P_3	7.345062291
P	140.7429781
z	3.619987666
L_4^4	6.625354562
Y	-6.98236E-05
L_4	1.65633864
$\sigma_3^{'}$	76.34567309
$\sigma_4^{'}$	652.7186084
$\sigma_8^{'}$	-86.14711324

Table 3-10: Effect of Kp Selection on Sheet Pile Wall Length

Parameters	Kp design	Kp	Percentage difference (%)
Total theoretical length	11.74	11.22	4.43
Total actual length	11.74	12.11	3.06
Point of zero shear force	6.89	6.66	3.34

After making a comparison between the two different scenarios of Rankine’s applied passive pressure coefficient, similar results were obtained to those from the cantilever sheet pile wall analysis.

Applying a FOS to the Rankine's passive pressure coefficient led to an increase of theoretical wall penetration depth, giving a 4.43% difference between the different applications of applied FOS.

The actual depth of the pile saw a 3.06% increase of wall depth after the FOS was applied for both scenarios of varying passive pressure coefficients. This led to a 3.34% reduced point of zero shear force and indicated an indirect relation between the actual wall penetrating depth and the occurrence of the point of zero shear.

3.8.3 Important Output Values

The theoretical wall penetration depth decreased by 19% when a FOS value was applied when determining the actual wall penetration depth, leading to the actual penetrating depth of the sheet pile increasing by 11.9% (Table 3-11).

Table 3-11: Important Output Solutions

Parameters	Kp design	Kp	Percentage difference %)
Theoretical Penetration Depth	2.74	2.22	19.0
Actual Penetration Depth	2.74	3.11	11.9
Anchor Tie Rod Force	83	78	6.0
Maximum Bending Moment	209	180	13.9

The increased actual wall penetrating depth led to an indirect relation with the anchor tie rod force, resulting in a decrease of 6% for a FOS applied to the theoretical penetration wall depth.

The maximum bending moment decreased with the original increase of the theoretical penetration depth; whereas the maximum bending moment increased by 18% when the FOS was applied to the passive pressure, thus decreasing the passive pressure effect throughout all the calculations. This is a major effect, as it suggests that the passive pressure has a dramatic effect on the maximum bending moment exerted on the sheet pile. This leads to more conservative solutions, which is undesirable when considering costings. Engineers should always undertake a cost analysis, optimising the cost benefit analysis while providing quality designs that are safe and will not lead to failure.

3.8.4 Anchored Sheet Pile Wall Graphical Visual Representation

The visual diagrams of the pressure forces, shear forces and bending moment forces exerted along the sheet pile wall depth were similarly established as previously explained for the cantilever visual representation analysis. The only difference between the cantilever and anchored visual diagrams was the presence of the anchor tie rod force, as the applied shear force exerted on the sheet pile due to the anchor tie rod force needs to be considered. If the pressure force is acting towards the sheet pile, it was taken as a positive force. Thus, the anchor shear force due to the anchor tie rod force at 1.5 m had to be subtracted, as it was acting away from the sheet pile wall, as indicated in Figure 3-9, at the point of 1.5 m depth.

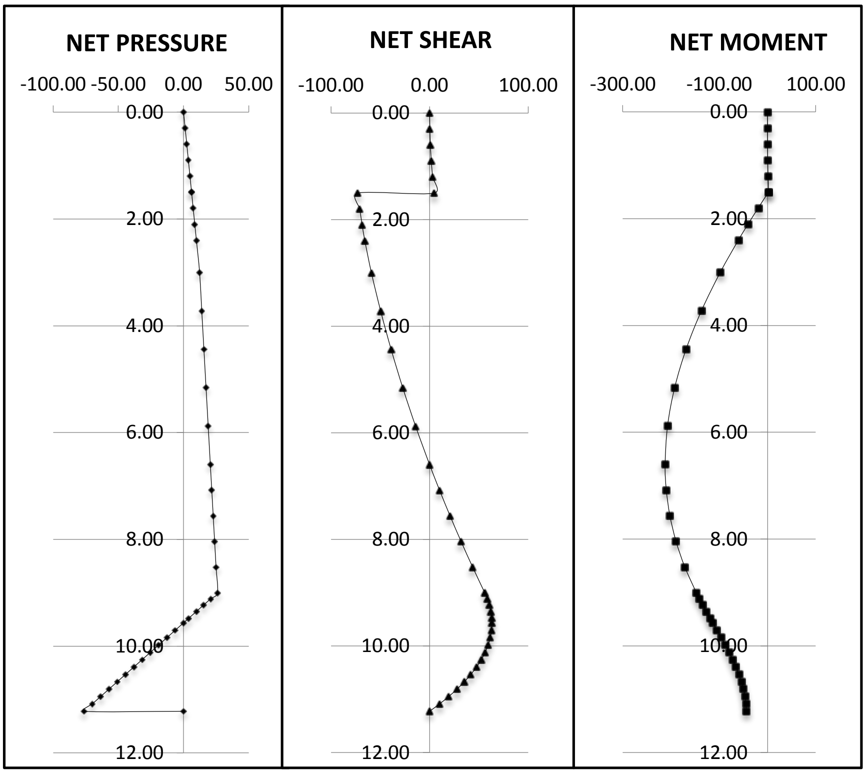

Figure 3-9: Visual Diagrammatic Output Figures for an anchored sheet pile

The visual outputs created in Excel for both the cantilevered and anchored sheet pile wall bending moments diagrams are similar to those for bending moment occurrence with increasing wall depth (Das 1990) Figure 6.7 (b) and Figure 6.15 (b). The visual diagrams are thus found to be viable and capable of being used to give a basic understanding of how the sheet pile wall will behave under pressure force distribution, shear force distributions and bending moment distribution along an increasing sheet pile depth.

3.9 Comparison between Cantilever and Anchored Pile Outputs

3.9.1 Effect of Different Factors of Safety Applications

Table 3-12: Comparison between Cantilever and Anchored Sheet Pile Wall

Cantilever Sheet Pile Wall Analysis			Anchored Sheet Pile Wall Analysis			
Parameters	Kp design	Kp	Kp design	Kp	Percentage difference (%) Kp design	Percentage difference (%) Kp
Total theoretical length	16.51	15.51	11.74	11.22	28.9	27.7
Total actual length	16.51	18.12	11.74	12.11	28.9	33.2
Point of zero shear force	2.95	2.47	6.89	6.66	57.2	62.9

When comparing the cantilever and anchored sheet pile wall analysis solutions of the effect of the Rankine's passive pressure coefficient, it can be seen that there is a good correspondence between the different types of sheet pile walls. The theoretical depth of sheet pile wall decreases for both scenarios of applying the FOS value of 1.4, when comparing the cantilever and anchored sheet pile wall.

The total actual wall length remains constant when compared against the actual penetration depth. This is expected, as the FOS is already considered when calculating the theoretical penetration depth. This was not the case when analysing the FOS application being applied to the theoretical penetrating wall, which leads to the distinct increase of 33.2%.

The point of zero shear force increases along the pile depth when an anchor force is applied to the sheet pile for both scenarios of applied FOS Kp design and Kp, being 57.2% and 62.9%, respectively. The presence of the anchor tie rod force has a major effect on the point at which zero shear force will occur, which will in turn have a major effect on the maximum bending moment exerted on the sheet pile wall. The occurrence of the point of zero shear force being below the dredge line is also an interesting finding, as Das (2007) assumes that the bending moment should occur at a point below the water table but above the dredge line. This was found not to be true; rather, the maximum bending moment for anchored sheet pile walls occurs at a point just below the dredge line.

3.9.2 Effect of Different Factor of Safety Applications

The presence of the anchor tie rod force for anchored sheet pile walls decreases the actual penetration depth below the dredge line dramatically. Applying the FOS to the passive pressure coefficient results in a 63.52% decrease of actual penetration depth below the dredge line, or a 65.89% decrease when the FOS is applied to the theoretical penetrating depth below the dredge line (Table 3-13).

Table 3-13: Comparison between Cantilever and Anchored Sheet Pile Wall

Cantilever Sheet Pile Wall Analysis			Anchored Sheet Pile Wall Analysis			
Parameters	Kp design	Kp	Kp design	Kp	Percentage difference (%) Kp design	Percentage difference (%) Kp
Actual Penetration Depth	7.51	9.12	2.74	3.11	63.52	65.89
Anchor Tie Rod Force	-	-	83.00	78.00	0.00	0.00
Maximum Bending Moment	787	741	209	180	73.43	75.72

Comparing the bending moment results for the cantilever sheet pile analysis with the anchored sheet pile analysis results, it was found that the maximum bending moment exerted on the sheet pile decreased substantially, with 73.43% for FOS applied to the passive pressure and 75.72% for the FOS applied to the theoretical wall penetration depth below the dredge line.

3.10 Conclusion

Designing sheet pile walls using the analytical methods is a very tedious and time-consuming procedure. In the engineering industry, time is money, and a design tool that could solve these equations automatically with the same accuracy and ability to create visual solutions, all in a fraction of the time with only the necessity of inputting known data, would be extremely valuable. The solutions obtained using the Excel spread sheet are similar to those derived using the analytical methods. Thus, the Excel spread sheet has been proven accurate and successful.

The analytical methods have been found to be conservative due to the necessity of making several simplifications and assumptions. Important design information such as ground settlement and possible surface failures cannot be obtained from the analytical methods. It is thus proposed to use FLAC for future preferences to attain such critical information to provide greater accuracy of results.

Chapter 4: FLAC Overview

4.1 Introduction

This chapter presents a short introduction to the FLAC software and explains the necessary principles required for the use of this software. Several geotechnical numerical modelling analyses were completed using FLAC. This chapter provides an explanation of the FLAC program, highlighting its advantages and disadvantages for undertaking the numerical analysis. This is important information for any designer using numerical methods, as every software has its own limitations that need to be recognised to enable a better understanding of particular outcomes.

FLAC is a 2D explicit finite difference program for engineering mechanics computation (*FLAC 2D online manual* 2009). Explicit finite difference indicates the solution of the problem being modelled by using a time-stepping procedure. FLAC contains a very powerful built-in programming language called FISH (FLACish) that enables the user to write single script files of code for increasing the usefulness and usability of this software. The program simulates the behaviour of structures built of soil, rock or other materials that may undergo plastic flow. These structures are described by the behaviour of the elements according to a suggested linear or non-linear stress/strain relationship (Das 1990).

Several different versions of FLAC are currently available, the most current of which is version 6. For this dissertation, version 4 has been adopted due to its availability. According to the establishment by Lyle (2009), there are only marginal differences between the two versions, with the major difference being the speed improvements obtained from FLAC version 6. This difference between the two versions would not compromise the accuracy of the results. Thus, for the purpose of this dissertation, FLAC 2D was solely used to undertake the research. FLAC 2D indicates only two directions, i and j, when undertaking the computations. Three-dimensional FLAC versions are available; however, given the lack of time to learn a complex computer language, it was decided to use the 2D version of FLAC.

4.2 Major FLAC Features

According to the FLAC manual, the FLAC software has a number of major features (*FLAC 2D online manual* 2009):

- Large strain simulations of continua, with optional interface that is able to distinctively simulate planes along which slip and/or separation can occur.
- Obtaining stable solutions from the provided explicit solution scheme when compared to unstable physical processes.
- Availability to model groundwater flow, with full coupling to mechanical calculation (including negative pore pressure, unsaturated flow and phreatic surface calculation).
- Selection of multiple structural elements (including non-linear material behaviour).
- Full library of material models (e.g., elastic, Mohr-Coulomb plasticity, ubiquitous joint, double-yield, strain-softening, modified Cam-Clay and Hoek-Brown).
- Statistical distribution of any property for generating plots of virtually any problem variable with extensive facilitation.
- Extra user-defined features such as the built-in language FISH (e.g., new constitutive models, new variables or new commands).

To obtain a thorough understanding of FLAC programming, reasonable time and effort is required. Experience is required to achieve accurate and effective results. Due to a lack of experience using the program, it was necessary to carefully analyse all results to ensure valid outputs. The results obtained from the FLAC software program will therefore be validated as discussed in Chapter 5 to ensure quality solutions.

4.3 FLAC Model Analysis

The built-in programme language FISH gives the possibility to use the command-driven software mode. Compared to the menu-driven mode in FLAC, the command-driven mode was found to reduce the software's performance of unnecessary repetitive tasks, allowing for faster result assembly. The text file storing facility (known as a script file) can be modified quickly and easily.

In the developed FLAC script based on the procedure outlined in Example 4.13 and Installation of a Triple Anchored Excavation Wall in Sand (p 17-1) (*FLAC 2D online manual* 2009), the basic steps undertaken to analyse cantilevered and anchored sheet pile wall problems are as follows:

1. Create a mesh and define the various input variables for the mesh. Assume a length for both horizontal and vertical direction. Define the number of blocks per metre run. Specify the model Mohr of soil. (This model is the conventional model used to represent shear failure in soils and rocks.) Input the soil properties.
2. Remove a complete column for the positioning of the sheet pile wall and shift the right hand side block to the left with one single block difference to establish a double coordinate system for creating an interface between the wall and the soil.
3. Fix the boundary conditions to allow for possible horizontal collapse and vertical displacement.
4. Specify the magnitude of gravity.
5. Insert the sheet pile wall and its specific properties.
6. Provide an interface between the pile and the soil on which sliding or separation can occur. Attach the two sub grids created in step 2.
7. Set history to set small to avoid large settlement from occurring at the top of the sheet pile. Solve the model elastically to reach equilibrium and save the current state.
8. Reset all displacement back to zero before excavating the soil.
9. Excavate the soil on one side of the sheet pile, solve this process and save the excavated state—hence the cantilever sheet pile wall model.
10. Install a cable structural element. Attach the cable element to the pile structure. Specify the cable position and properties.
11. Solve the anchor sheet pile wall model and save the anchor model.
12. Save the graphical and numerical data produced during the solution phases of FLAC into a specified folder.

4.3.1 FLAC Input Variables

Table 4-1: Grid Generation

X_Element size
Y_Element size
Grid size i,j

Table 4-2: Soil Properties

Bulk modulus (GPa)	3e9
Shear modulus (GPa)	1e9
Cohesion (kg/m^2)	0
Internal Friction Angle (degrees)	40
Dilation	0
Tension	0
Dry sand density (kg/m^3)	1922
Saturated sand density (kg/m^3)	2072
Effective sand density (kg/m^3)	1072

Table 4-3: Pile Element Properties (ArcelorMittal 2013; Das 2007)

Elastic Modulus of steel (GPa)	200
Area (m^2)	0.02
Second moment of inertia (m^4)	4.5e-6
Friction between Sand and Pile (degrees)	0
Penetration (D) depth of pile - cantilever (m)	9.12
Penetration (D) depth of pile – anchor (m)	3.11

Table 4-4: Rod Anchor Properties (Ischebeck; FLAC 2D online manual 2009)

Elastic modulus of steel (GPa)	200
Area (m^2)	0.0015
Yield strength (MPa)	1e10
Grout shear stiffness (MPa)	1e8
Intrinsic shear strength (MPa)	1e8
Friction between Tie Rod and Pile (degrees)	0
Tie rod length (m)	14
Tie rod element spacing (m)	1.2

Table 4-5: Solving Analysis

Solve elastic	(obtain initial soil stresses)
Strain	(set large or small)
Solve	(before excavation)
Reset Displacement	(y_disp = x_disp = 0)
Solve	(after excavation)
Solve	(after anchor installation)

For accurate modelling and evaluation of geotechnical sheet pile wall problems, it is essential that all the input variables are correctly evaluated and entered within the script, as this is part of the validation process that takes a very long time and may lead to obtaining incorrect and invalid solutions if not done correctly.

4.3.2 FLAC Output Variables

Table 4-6: Output Variables

Grid.jpeg	
X_disp.jpeg	(also textual form)
Y_disp.jpeg	(also textual form)
Ssr.jpeg	(also textual form)
Pile Moment.jpeg	(also textual form)
Plasticity.jpeg	(also textual form)
Pile X_disp.jpeg	(also textual form)
Cable axial_force.jpeg	(also textual form)

These outputs, along with the importance of the information, will be discussed in detail in Chapters 5 and 6.

4.3.3 Data and Result Extraction

To ensure the accuracy and acceptability of the results obtained in FLAC, it was necessary to extract the output solutions and export the data into Excel, where plots were generated to determine inaccuracies visually. These inaccuracies were tracked by searching for missing data in the Excel spreadsheet. Overall, the methodology was successful in obtaining high quality results from the FLAC analysis model.

4.4 Chapter Summary

This chapter presented a brief introduction to 2D-analysis using FLAC, its features and reason for selection. The two advanced models—the cantilever and anchored sheet pile wall models—introduced in this chapter will be further discussed in Chapters 5 and 6, respectively. The following chapters will discuss the creation of a typical FLAC script required inputs by a user, the typical outputs achieved from FLAC modelling and the export of these outputs into Excel to allow for comparison of the results.

Chapter 5: Numerical Analysis of Cantilever Sheet Pile Walls

5.1 Introduction

The objective of this chapter is to investigate the analysis of a cantilever sheet pile wall penetrating a sandy type soil in the presence of a water table using the FLAC software. The FLAC results will then be compared to the results from the Excel spread sheet consisting of the analytical equations. The parameters that will be investigated within the parametric study include:

- fineness of mesh
- effect of soil strength
- effect of water table.

The variables such as the maximum bending moment, wall deflection and ground settlement will be investigated.

5.2 Background Information

Cantilever sheet pile walls are flexible structures. Due to the wall being flexible, when the sheet pile wall moves away from the soil, it forms an active pressure zone; however, when the wall moves into the soil, it forms a passive pressure zone (Figure 5.1). This leads to the formation of pressure distributions on either side of the sheet pile wall.

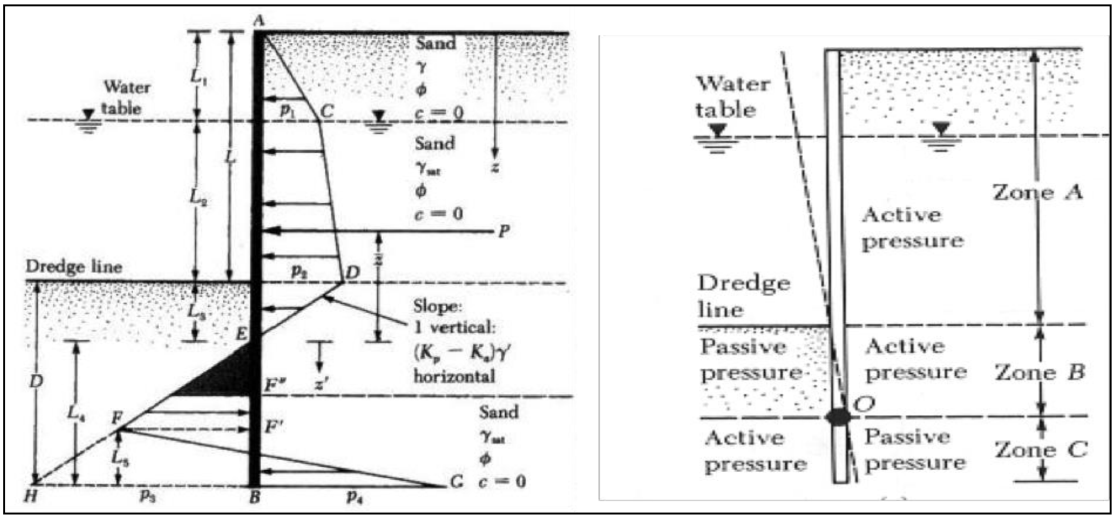

Figure 5-1: (a) Cantilever Pile Penetrating a Sandy Soil, (b) Active and Passive Pressure Distribution (Das 1990)

To obtain the total net pressure distribution exerted on the sheet pile wall, it is required to add all the pressure distributions together.

For a design engineer, it is of upmost importance to determine the maximum bending moment exerted on the sheet pile wall for structural design purposes and the net pressure distribution exerted along the pile depth for stability purposes (Coduto 2001) (Figure 5-2).

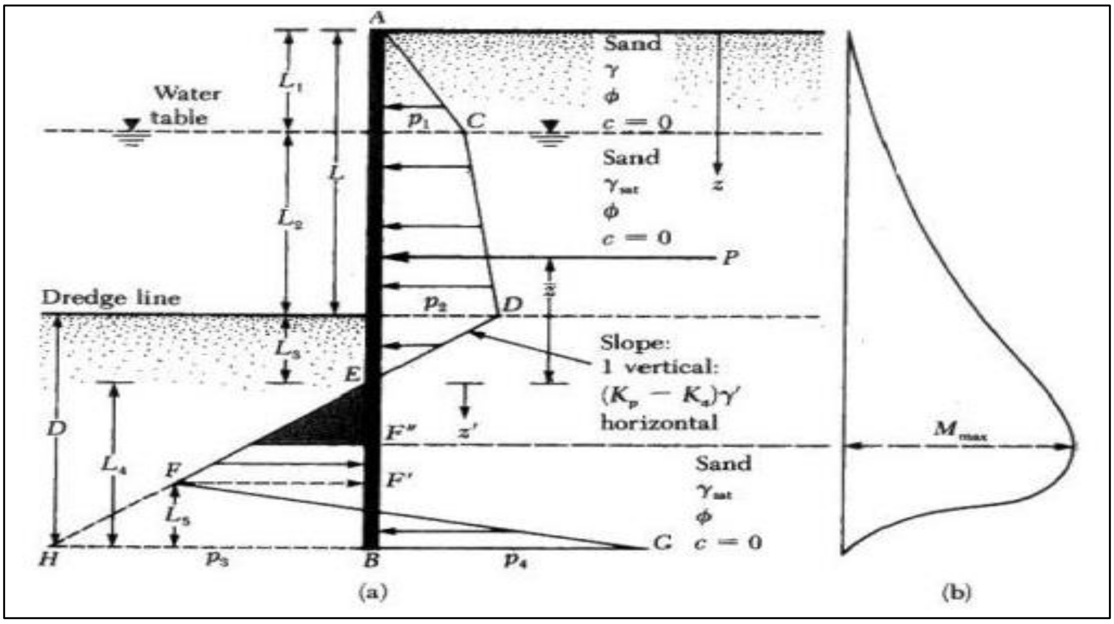

Figure 5-2: Cantilever Sheet Pile Penetrating Sand: (a) Net Pressure Variation Diagram; (b) Moment Variation (Das 2007)

Based on the classical earth pressure theory, several methods commonly utilise the limit state methodologies (e.g., the UK and USA methods) for analysing cantilever sheet pile walls using the active and passive lateral pressures that act on the wall. The design methods are based on the fact that force and moment equilibrium are required for determining the minimum required wall penetration depth and the maximum bending moment. A FOS should be applied to the passive pressures to take any uncertainties in the soil condition, method of stability analysis and loading conditions into account, and to restraint the soil movements to an acceptable level (Potts & Fourie 1984).

This dissertation's investigations use FLAC software for undertaking a numerical examination of cantilevered sheet pile walls as an alternative design analysis to the limit equilibrium methods.

5.3 Problem Description

Figure 5-3 shows the problem to be investigated. The main parameters to be investigated are the wall horizontal displacement, maximum bending moment and ground settlement.

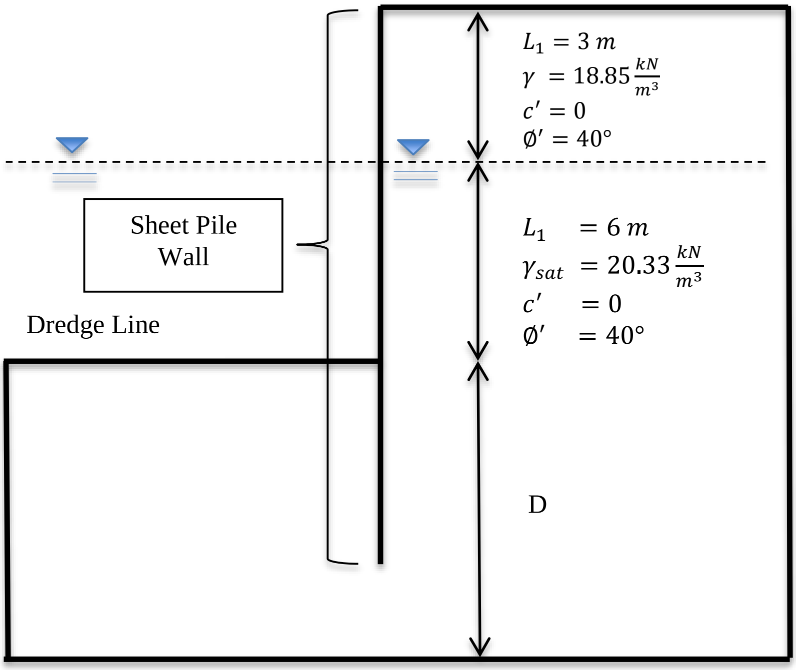

Figure 5-3: Problem to be Investigated

5.4 Analysis using FLAC

FLAC is a finite difference program and does not approach this problem in the same way as the analytical methods. When using the limit equilibrium methods for solving this problem, the penetration depth of the sheet pile is found to be below the dredge line. Conversely, FLAC requires the penetration depth of the sheet pile to be entered before it can be determined whether the system is stable. Thus, the overall soil/pile system behaviour can be examined by varying several input parameters.

5.4.1 Creation of Model

The first step is to build the geometry of the problem, assuming an initial length in the x and y directions. The soil layer boundaries and material properties are then defined. Construction elements like walls and anchors are placed next. The soil/wall system is then created using interface properties, which are then defined. Finally, the mesh is generated. Over the years, several modes have been developed for representing the soil behaviour. These include the linear elastic model, perfectly plastic model, hyperbolic model and the Mohr-Coulomb model. Selecting the model to be used for modelling the soil is extremely important and is dependent on several factors, as specified by Ramadan (2013). Initially, a very course mesh was assumed to analyse the specific problem (Figure 5-4). This was adjusted upon gaining a greater familiarity with the FLAC software.

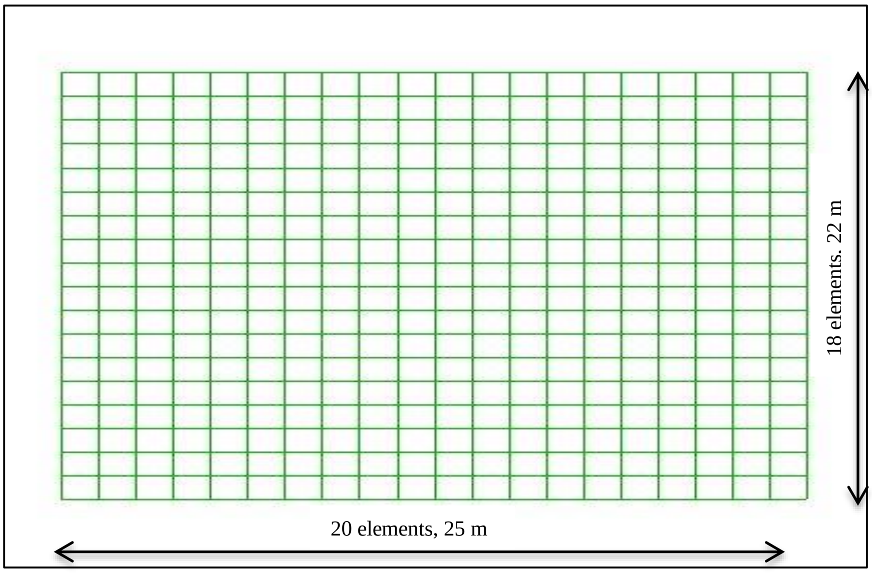

Figure 5-4: Assumed Course Mesh Grid

5.4.2 Soil Mass Properties

The soil acts as non-linear and irreversible when subjected to very high loads. According to *FLAC 2D online manual* (2009), a number of constitutive models are available, as FLAC is able to distinguish between several material models for soil, interfaces, plates, anchors and geogrids. The numerical analysis that was performed analysed the soil under drained soil conditions and suggested that pore water pressure could be prevented from developing. Normally, this drainage type is used for dry soils for accommodating full drainage due to high permeability. In FLAC, there are two options for modelling the soil mass:

- The initial stresses within the soil mass can be set. This initial stress condition depends on the weight of the material and the history of the formation. The stress state at that initial moment is characterised by a vertical effective stress (σ_v'), whereas the horizontal earth pressure is determined by the lateral earth pressure (or at rest) coefficient (K_0).
- All of the above recommendations may be ignored if and only if the soil mass is solved to reach equilibrium using the 'solve elastic' at the first instance, thus automatically creating the initial stresses due to the weight of the soil mass.

5.4.3 Construction of Sheet Pile

The sheet pile was constructed by inserting a structural beam element into the mesh. The surface of this sheet pile was modelled as perfectly smooth so as not to form any friction between the soil and the sheet pile. This was done to ensure accuracy when comparing the numerical results with the analytical solutions. (A frictionless effect between the sheet pile wall and soils was originally assumed by Das [2007].) The sheet pile wall properties were specified according to ArcelorMittal (2013) a leading steel manufacturing company. These properties were specified in Chapter 4.

To create this interface between the pile and the grid, it was required to remove a complete column (Figure 5-5) where the pile would be constructed using the ‘model null’ command to generate a double coordinate system to which to attach the pile.

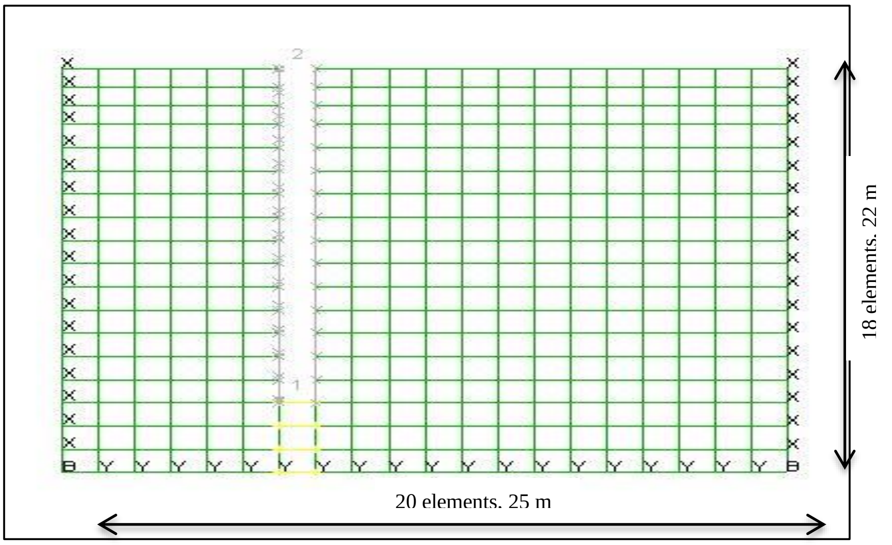

Figure 5-5: Column Removed for Sheet Pile Construction

The sub grid on the right then had to be incrementally moved to the left according to the grid ratio as specified by the user. The sheet pile was then installed, as shown in Figure 5-6.

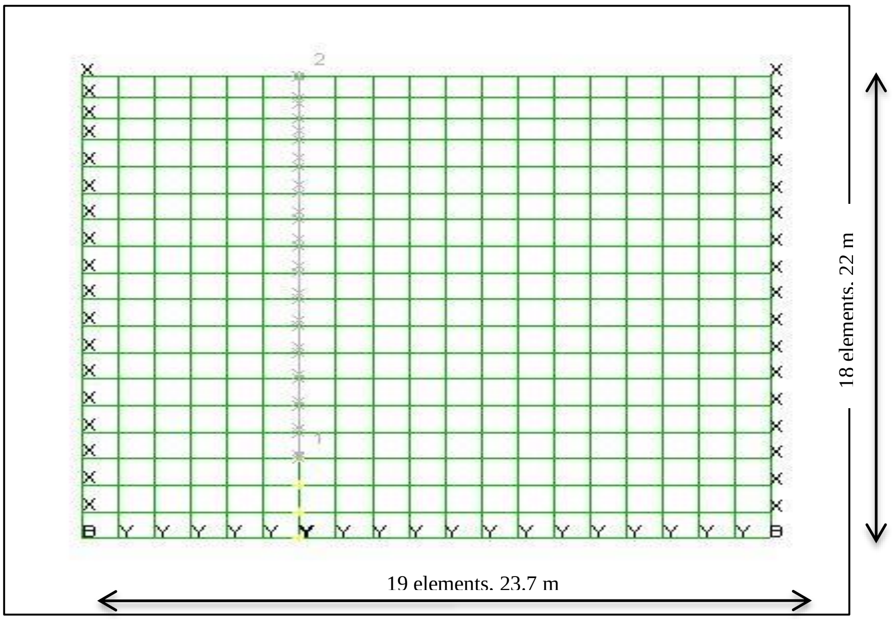

Figure 5-6: Constructed Sheet Pile Wall Model

5.4.4 Interface Properties

According to the FLAC manual, an interface between the soil/wall systems is represented as normal, and there is shear stiffness between the two planes (Figure 5-7). For either side of the interface, FLAC uses contact logic similar to that used by the finite element method.

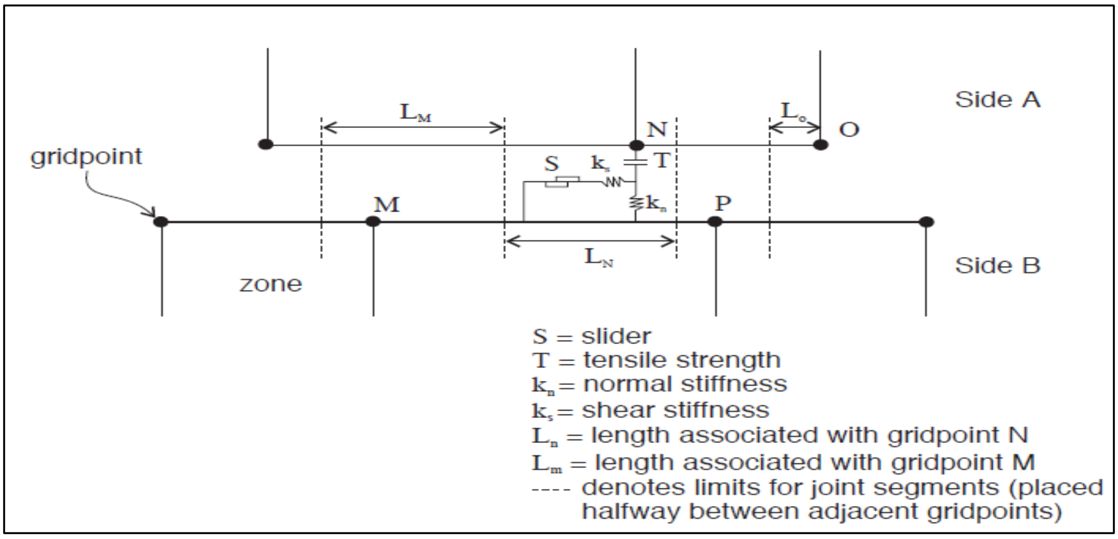

Figure 5-7: An Interface Represented by sides a, and b, connected by shear $(\boldsymbol{k_s})$ and normal $(\boldsymbol{k_n})$ stiffness springs (*FLAC 2D online manual* 2009)

The code keeps a list of the grid points (i, j) that lie on each side of any particular surface. Each point is then taken and checked for contact with its closest neighbouring contact point on the opposite side of the interface. Referring to Figure 5-6 on the segment M-P, the grid point marked N is checked for contact between the specified segment. The length L_N is defined for the contact of N between the segment M-P. The length is equal to half the distance to the nearest grid point to the left of N, plus half the distance to the nearest grid point to the right, irrespective of the side on which the neighbouring grid point is located. This will ensure that the entire interface is divided into contiguous segments, each controlled by a grid point. The interface was thus successfully installed, creating a valid interface between sub grids and between the element and the grid.

5.4.5 Soil Excavation

The required depth above the dredge line to the left of the sheet pile was then excavated using the same 'model null' command.

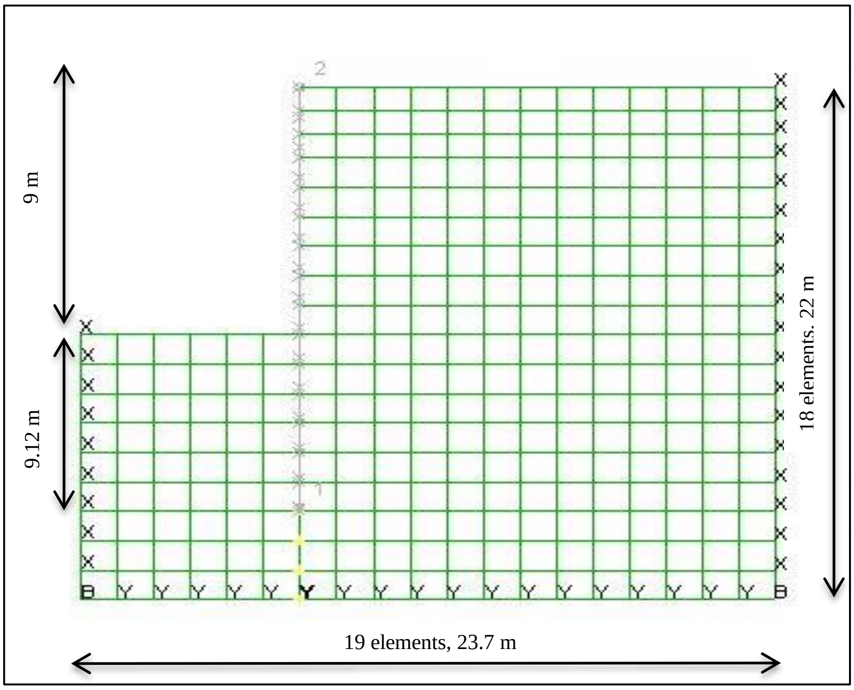

Figure 5-8: FLAC Model Containing Course Mesh

After excavating the soil, the model was solved again using the 'solve' command to investigate the effects and behaviour of the sheet pile wall by interpreting specific FLAC outputs such as maximum bending moment, maximum wall horizontal deformation and failure surface.

5.4.6 FLAC Results

Establishing whether the created model is correct forms part of the model validation process. This process is time consuming for unexperienced FLAC users and might be frustrating; however, being able finally to interpret correctly the outputs obtained from FLAC is extremely rewarding.

Failure Surface

Plotting the failure surface on a shear strain rate plot gives a good indication of whether the grid size of the model is acceptable.

Figure 5-9 (a) is the initially assumed model grid. The failure surface in Figure 5-9 (b) is not fully contained within the initially assumed grid. Thus, it was required to increase the grid in the x-direction (that is, widen it in the horizontal direction).

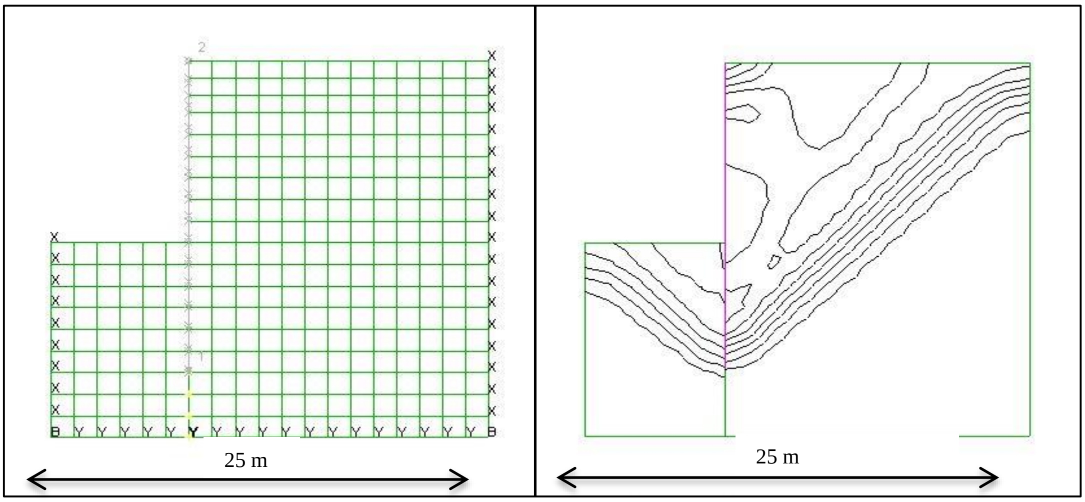

Figure 5-9: (a) Grid, (b) Failure Surface for initially assumed Model

According to the FLAC manual, the accuracy of the results depends on the fineness of the mesh. Thus, to obtain more accurate results, the next model was developed to use a fine mesh, as indicated in Figure 5-10 (a). The grid was also widened in the horizontal direction to facilitate the correct interpretation of the results obtained from FLAC.

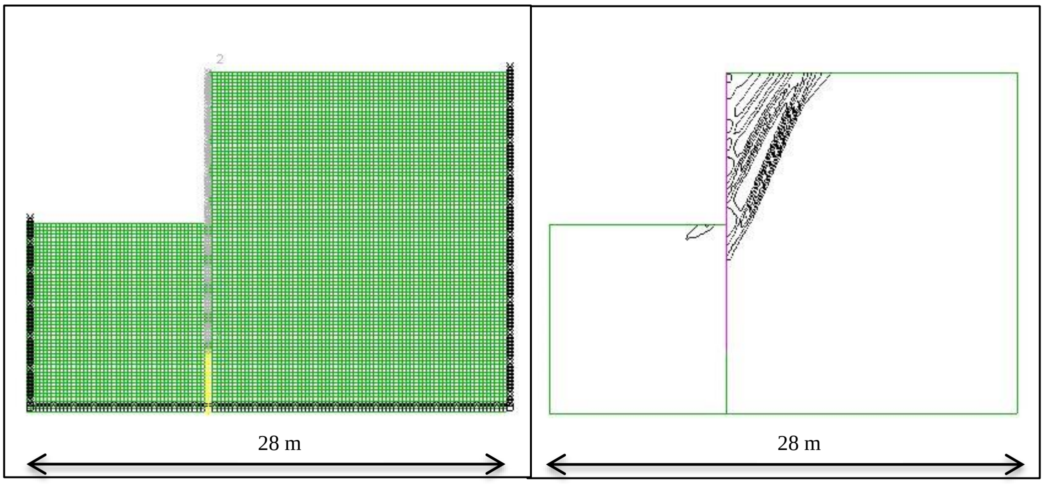

Figure 5-10: (a) Grid, (b) Failure Surface for Fine Mesh Model with Horizontal Increase

Figure 5-10: (a) Grid, (b) Failure Surface for Fine Mesh Model with Horizontal Increase

This figure indicates the failure surface in Figure 5-10 (b), which is fully contained within the grid. This indicates that the grid size is acceptable. However, when analysing the actual failure surface in Figures 5-9 and 5-10, it was thought that only a singular slip surface would occur. For both figures, this was clearly not the case; yet according to Griffiths, Fenton and Martin (2000), single slip failure surfaces do not form for cantilever sheet pile walls. Instead, the deformation on either side of the sheet pile is distributed evenly across the active and passive failure zones. Thus, the failure surface plots as shown in Figures 5-9 (b) and 5-10 (b), respectively, indicate parallel lines forming over a large area. This behaviour is thus found to be acceptable.

Maximum Bending Moment and Horizontal Wall Displacement

According to past research, numerical methods such as FLAC are very accurate when compared to analytical solutions (Smith 2006; Zhai 2009; Bilgin 2010). Accuracy of results is important for design engineers. If the forces exerted on the structure are underestimated, structure failure may result, which could lead to lives being lost and the design engineer being held accountable. By analysing the cantilever sheet pile wall behaviour in FLAC, it was possible to obtain the maximum bending moment exerted on the sheet pile wall and the maximum horizontal wall displacement. As already mentioned, these outputs are extremely important for stability and structural design purposes (Figure 5-11).

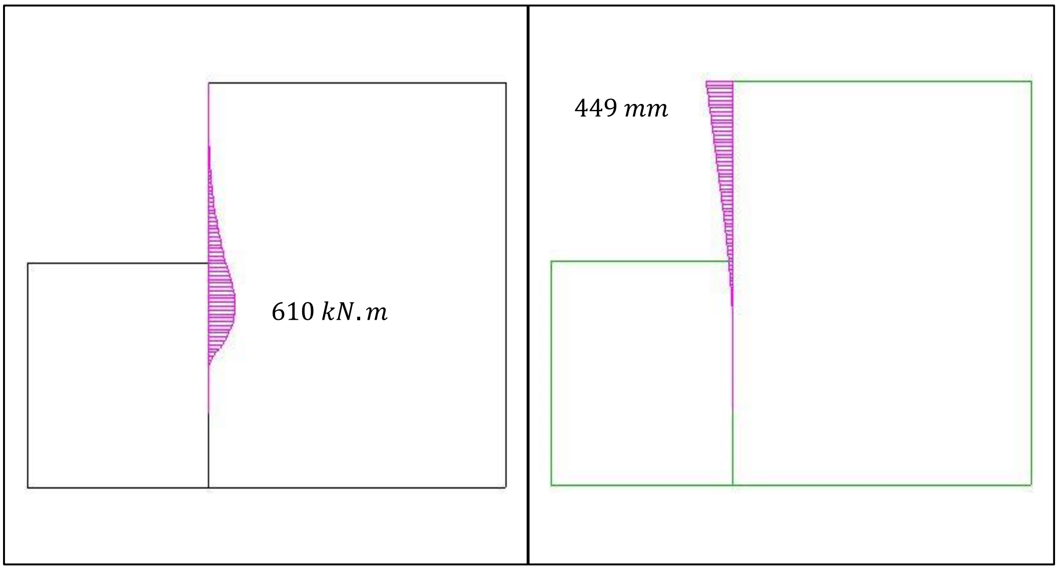

Figure 5-11: (a) Maximum Bending Moment, (b) Maximum x-Displacement

Containing these visual outputs certainly gives a good indication of the soil-wall system; however, determining the quality of the result outputs is what is important. This validation is done by comparing the FLAC output of maximum bending moment with the maximum bending moment obtained from the limit equilibrium methods as established in Chapter 3. Unfortunately, the maximum wall horizontal deflection cannot be validated, as this deflection cannot be obtained from the limit equilibrium methods. This is a limitation of using the analytical methods.

Soil Mass Failure

The failure of the soil mass can be examined by plotting the plasticity graph in FLAC (Figure 5-12). This plot indicates which elements are at yield or have undergone plastic deformation. The green elements indicate at yield elements undergoing elastic deformation, while the red elements indicate elements undergoing plastic deformation.

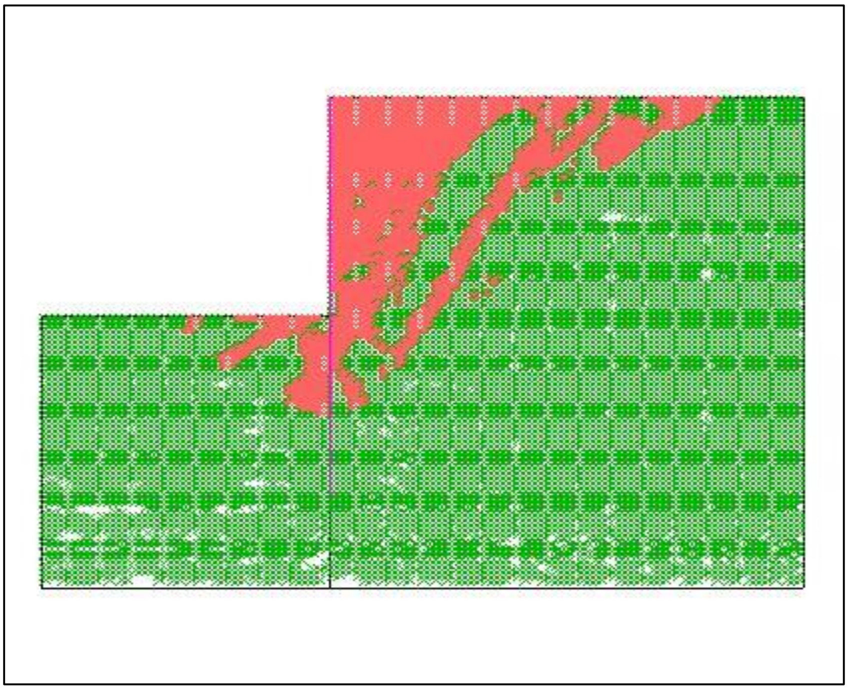

Figure 5-12: Plasticity Indicators for the Fine Mesh Model

The overall purpose of analysing this plot is to determine whether the system fails. It is thus crucial to examine the plot of plasticity indicators for each element prone to failure. The soil mass is found to fail when the plasticity elements on one side of the sheet pile connect the ground surface to the ground surface on the other side of the sheet pile via extending around the sheet pile tip. Failure also occurs when the plastic elements extend far beneath the sheet pile tip into the outer boundary of the plot.

By examining Figure 5-12, it is clear that this sort of behaviour is not occurring. Thus, it is reasonable to conclude that the cantilever sheet pile wall penetrating 18.12 m of sandy soil is safe, as failure is not occurring.

Validation of FLAC Model

The validation of this model was an important step within this chapter as it ensured the quality of the results being obtained. The maximum bending moment obtained by FLAC was plotted against the maximum bending moment obtained from the analytical solutions (Figure 5-13).

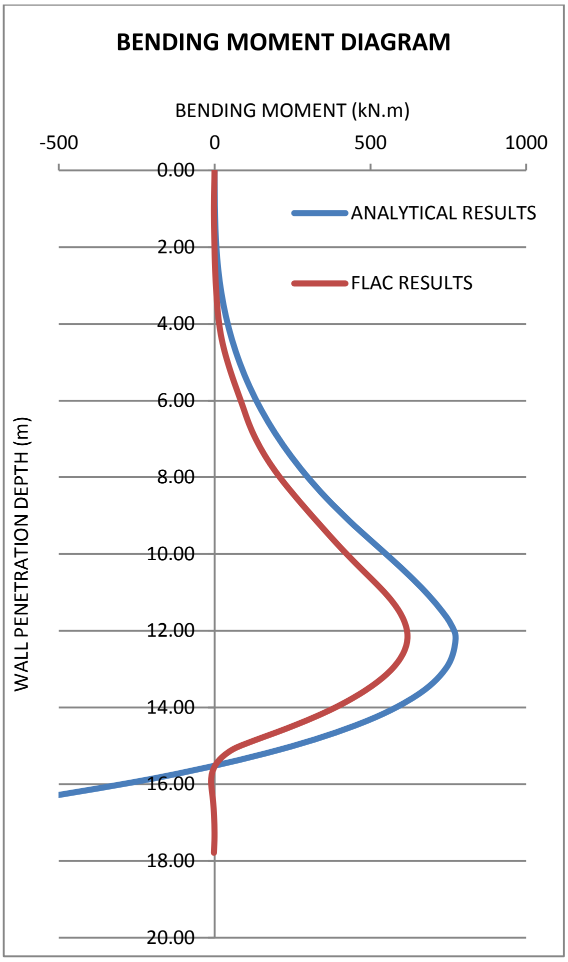

Figure 5-13: Visual Comparison of Maximum Bending Moment

As shown in Table 5-1, the maximum bending moment obtained from FLAC sees only a 17.67% reduction from the maximum bending moment obtained using the analytical methods. This gives a 82.32% comparison between the obtained solutions.

Table 5-1: Comparison of Maximum Bending Moment

Wall total Depth (m)	Analytical Solution	FLAC Solution Solution	Percentage Reduction (%)
17.8	741	610	17.67

This close comparison indicates that the results compare very well, strongly suggesting the validity of the results obtained from FLAC. The maximum bending moment obtained from FLAC is also smaller than the value obtained when using the analytical methods, indicating that the FLAC results are more accurate than are those obtained using the analytical solutions. Thus, the assumptions made by the analytical methods lead to the results being more conservative. To ensure complete validation of the model, it was necessary to validate the wall horizontal deformation and the ground surface settlement values obtained in FLAC against previously conducted work (Table 5-2).

Table 5-2: Validation of FLAC Model

Wall Penetration Depth D (m)	Maximum Wall Deflection (m)	Maximum Surface Settlement (m)	Percentage Correspondence (%)
6.88	0.984	1.65	59.63
7.57	0.780	1.23	63.41
8.08	0.667	1.00	66.70
8.60	0.545	0.812	67.11
9.12	0.459	0.662	69.33

Similar to previous research it was found that as the wall penetration depth increases both the maximum wall deflection and the maximum surface settlement decreases thus indicating a non-linear relation occurring. The percentage correspondence between the maximum wall deflection and maximum surface settlement thus linearly increases due to the non-linear relationship between the values as this is expected, since a closer correspondence would occur when values become smaller leading to a smaller difference and thus increasing the accuracy of the results.

This finding proves that this cantilever model is both valid and produces quality results.

Convergence Study

The parametric study focusses primarily on studying the effect of changing the mesh fineness to investigate the effect on the model accuracy. Varying the friction angle of the sandy soil to analyse how the soil strength affects the behaviour of the sheet pile wall. Inspecting the water table effect with varying sheet pile wall depth analysing the behaviour of the sheet pile by analysing the outputs such as the wall deformation, bending moments and ground settlement computed in FLAC.

Effect of Mesh Fineness

According to the FLAC manual, the accuracy of the results depends on the fineness of the mesh. Analysing a model containing a fine mesh will result in accurate solutions. As can be seen from Table 5-3, this is definitely the case as the mesh was increased from course to fine the maximum bending moment obtained for models containing medium and fine meshes respectively was smaller than compared to the maximum bending moment obtained by the analytical solutions.

Table 5-3: Convergence Study of Varying Mesh Fineness

Mesh fineness	Maximum bending moment (kN.m)	Maximum wall deflection (m)	Maximum surface settlement (m)	Blocks per m run
Course	837	0.678	0.830	80
Analytical Solution	741	-	-	-
Medium	680	0.503	0.688	100
Fine	610	0.459	0.662	120

Analysing the maximum bending moment obtained from the model containing a course mesh, a much larger value is obtained compared to analytical solutions. This indicating that the results obtained from FLAC are more conservative than the analytical solutions. This is quite strange and it might lead to the thought that the results are invalid. For unexperienced FLAC users this might be the case. This indicates that using such course meshes to model numerical geotechnical problems should be avoided as this may lead to inaccurate solutions. It is best to undertake modelling using a medium mesh and increasing the fineness from thereon. Therefore, the findings emphasise the importance for the necessity of skilled FLAC users when undertaking numerical analysis modelling.

Investigating the maximum wall deformation and maximum ground settlement only the results obtained from the medium and fine mesh will be taken into consideration as we have established the data from modelling with a course mesh may be misleading.

The maximum wall deformation increases with the increase of the mesh fineness, indicating a direct relation occurring. As mentioned in previous chapters obtaining accurate maximum wall deformation data is important for serviceability purposes as this information is used by engineers to determine if the structure satisfies serviceability requirements.

The maximum ground settlement increases with the increase of mesh fineness once again leading to more accurate results. The accuracy of the maximum ground settlement result is important as the ground settlement effects nearby structures. Normally, in urban environments, excavation occurs nearby other structures. If the maximum ground settlement effect has not been investigated, catastrophic building collapses may result.

Effect of Soil Friction Angle

An additional analysis was performed using relatively looser and denser sandy soils to investigate the effect of soil strength on the sheet pile wall behaviour with increasing wall penetration depth below the dredge line.

The analysis results, in terms of maximum wall displacement and maximum bending moments, for a loose sandy soil ($\emptyset = 35°$), medium dense sandy soil ($\emptyset = 40°$) and a very dense sandy soil ($\emptyset = 45°$) are given in Table 5-4.

Table 5-4: Parametric Study of Soil Friction Angle

		FLAC Analysis		Analytical Analysis
Soil Friction Angle (degrees)	Penetration Depth D (m)	Maximum Bending Moment (kN.m)	Maximum Wall Deflection (m)	Maximum Bending Moment (kN.m)
35	10.68	935	0.825	1034
40	9.12	610	0.459	741
45	7.85	394	0.250	535

The reduction of the maximum bending moment obtained by FLAC is relative to the conventional design method values. The maximum bending moment results obtained from FLAC compared to the solutions obtained by the analytical solutions are smaller, thus ensuring high quality results were obtained.

The cantilever sheet pile has less wall deformation for denser soils with decreasing penetration depth below the dredge line. This is very interesting as earlier determined for decreasing penetration depth the wall deformation increases, but increasing the soil strength leads to the opposite behaviour occurring. As the penetration depth below the dredge line decreases, the wall deflection also decreases with increasing soil strength of the soil.

Effect of the Ground Water Table

Often, sheet pile structures are built-in connection with waterfront facilities. The effect of hydrostatic pressure should be added to the earth pressure if the soil is not able to drain the water from behind the sheet pile wall. If the water level varies on either side of the sheet pile wall an unbalanced hydrostatic pressure is formed leading to increasing lateral pressure that may cause the wall to be forced in an outward direction. In Figure 5-2, the water table on both sides of the sheet pile wall is presented at the same level as suggested by Das (2007). This leads to the soil being fully submerged, causing the hydrostatic pressures at any depth from both sides of the wall to cancel out, therefore only considering the effective lateral soil pressures exerted on the wall below the water table.

As shown in Table 5-5, the maximum bending moment for the cantilever sheet pile wall model containing no water table is 23.29% larger than the maximum bending moment obtained from the cantilever sheet pile wall model with a water table present. This was expected due to the assumption made by Das.

Table 5-5: Parametric Study of Ground Water Table

	No Water Table Model	Water Table Model	
Wall Penetration Depth D (m)	Maximum Bending Moment (kN.m)	Maximum Bending Moment (kN.m)	Percentage Decrease (%)
6.64	717.00	550.00	23.29
7.00	627.00	497.00	20.73
7.55	479.00	394.00	17.74

It was assumed that the hydrostatic pressures on either side of the wall cancelled each other out, however the effective unit weight of a soil should be used in the presence of a water table. Thus obtaining a much smaller effective unit weight of the soil when compared to the normal unit weight of the soil, which was used for the model in the absence of the water table. Decreasing bending moment occurs for the increase of penetration depth below the dredge line, forming a direct relationship. It is completely acceptable for the model in the absence of a water table to expect a larger maximum bending moment exerted on the pile, due to the large soil force exerted on the pile.

5.4.7 Chapter Summary

The wall behaviour was investigated through the wall displacements, bending moments and ground settlement. The finite difference method, FLAC was used to analyse this sheet pile wall behaviour and to investigate the effect of a changing specific parameters and undertaking a parametric study to investigate the behaviour of the pile.

It was obtained that using finer mesh grids led to more accurate results as well as being able to obtain outputs that can easily be analysed thus leading to more qualitative results. It is of upmost importance to have skilled FLAC user, model geotechnical engineering problems to be able to understand the outputs obtained from FLAC as well as validating the model. As validating the model is the most critical part of numerical modelling, the results were validated by comparing the solutions to the obtained analytical solutions. To find once again that the results obtained from FLAC are very accurate when compared to the analytical solutions. This could have been expected as many assumptions were made by Das to simplify the tedious analytical equations, creating more conservative solutions.

For higher density sandy soils (that is, stronger soil), a reduction was found in the total wall displacement, with decreasing wall penetration depth below the dredge line leading to a dramatic reduction of the maximum bending moment.

The effect of the water table leads to decreasing maximum bending moment values when compared to absent water table models. Due to the simplifications and assumptions made by Das, the hydrostatic pressures on either side of the wall cancel each other out. A larger soil mass with normal unit weight will produce larger lateral forces exerted on the sheet pile wall than compared to a smaller soil mass only containing an effective unit weight of the soil producing smaller pressures exerted on the pile. Hence, the increase of maximum bending moment with reduced wall penetration depth is completely acceptable.

Chapter 6: Numerical Analysis of Anchored Sheet Pile Walls

6.1 Introduction

The objective of this section of the project is to use FLAC to investigate the complex anchored sheet pile wall penetrating sand problem. The investigation includes a parametric study to examine how certain parameters may have an influence in producing more cost beneficial sheet pile walls in the engineering industry. The parameters to be investigated include:

- Adding an anchor to high cantilever sheet pile walls
- Varying depth effect

6.2 Background Information

There are several traditional design methods used to design anchored sheet pile walls like the free earth and fixed earth support methods (Das 2007). For complicated problems including the construction effect on the pile-soil system, computer programs are used to analyse the behaviour of the sheet pile. Many researchers have shown over the years that designing sheet pile walls when using the free earth support method provide stable sheet pile walls with less wall penetration depth below the dredge line required. Therefore, when using the fixed earth support methods lower wall deflections are predicted for less stable piles when compared to the free earth support methods. There are, however, multiple ways of reducing the large wall deformations obtained when using the free earth support method stated by (Erten & Bilgin 2009) to make use of multiple anchorage systems to be the most effective but also using larger pile profiles than required by the analytical design methods can also be very effective.

6.3 Problem Description

The geotechnical problem specifies a sheet pile wall penetrating a sandy soil to a 3.11 m depth below the dredge line (Figure 6-1).

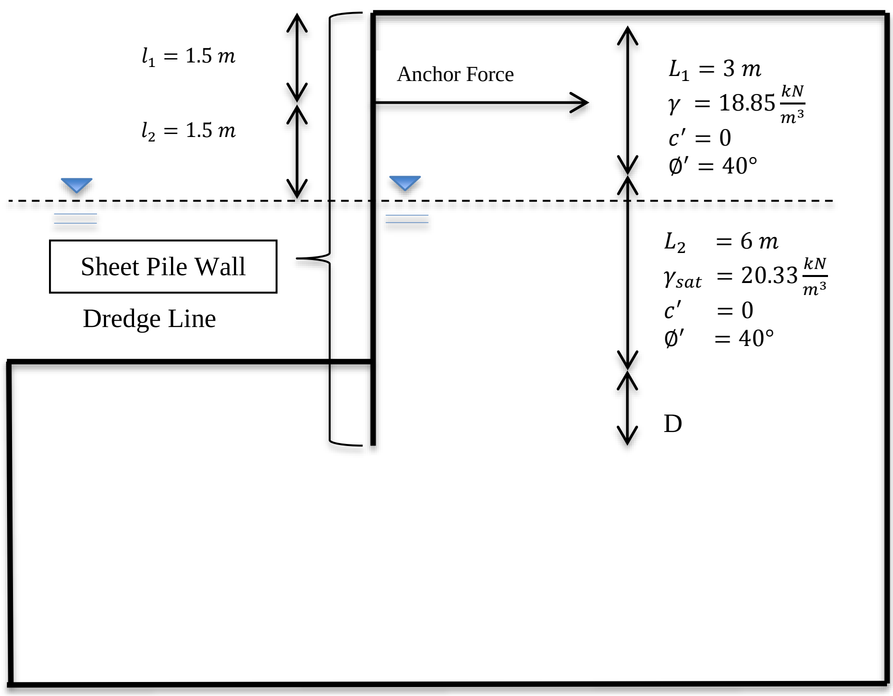

Figure 6-1: Problem to be investigated

The main parameters that will be investigated are the sheet pile wall horizontal displacement, maximum bending moment exerted on the sheet pile wall and the ground settlement.

6.4 Analysis using FLAC

The conventional analytical methods do not take into consideration the construction effect when installing the structural elements such as the sheet pile and anchor tie rod. The analytical design methods do not consider the properties of the structural elements. This leads to the development of limitations being present when using these analytical methods. To obtain more accurate sheet pile wall designs, numerical analysis in FLAC is undertaken.

6.4.1 Model Creation

The boundary of the specific sheet pile wall problem has already been established and validated in Chapter 5. Therefore, it was only necessary to install or construct the grouted tie rod anchor element to create an anchor sheet pile model in FLAC using the fine mesh grid (Figure 6-2).

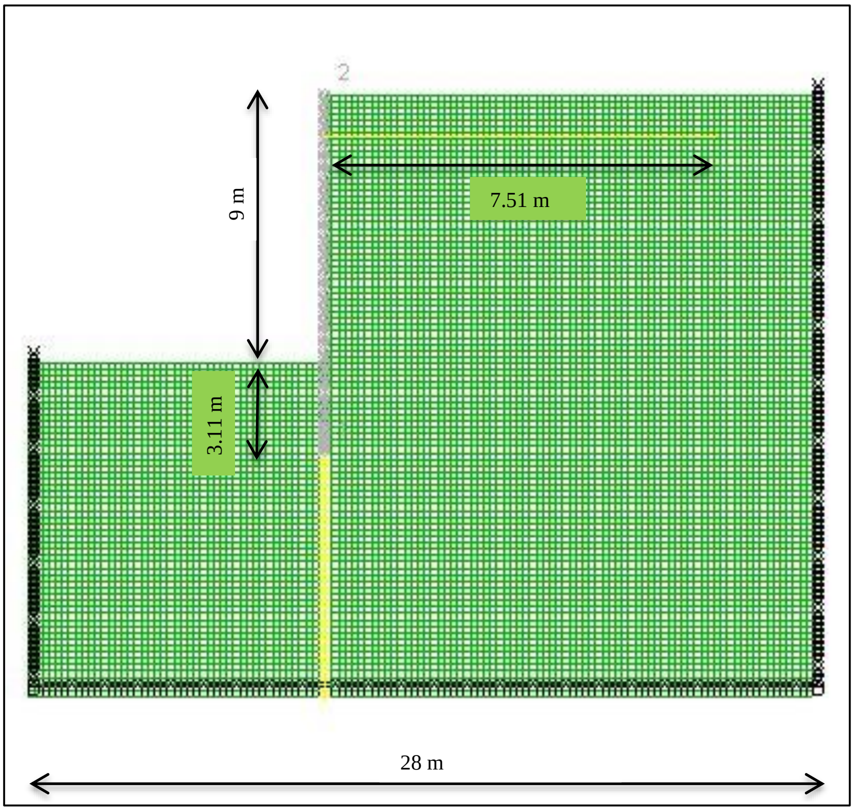

Figure 6-2: Anchor Sheet Pile Wall Model

6.4.2 Modelling of Anchor Element

Cable elements are one-dimensional axial elements that may either be anchored at a specific point in the grid (point anchored), or grouted so that the cable element develops forces along its length as the grid deforms. Cable elements can yield in tension or compression, but they cannot sustain a bending moment. If desired, cable elements may be initially pre-tensioned. Cable elements are used to model a variety of supports, including rock bolts, cable bolts and tiebacks.

A node-to-node anchor system containing of a two-node elastic spring element with constant spring stiffness (normal stiffness) is created to model typical anchor sheet pile wall applications (Figure 6-3).

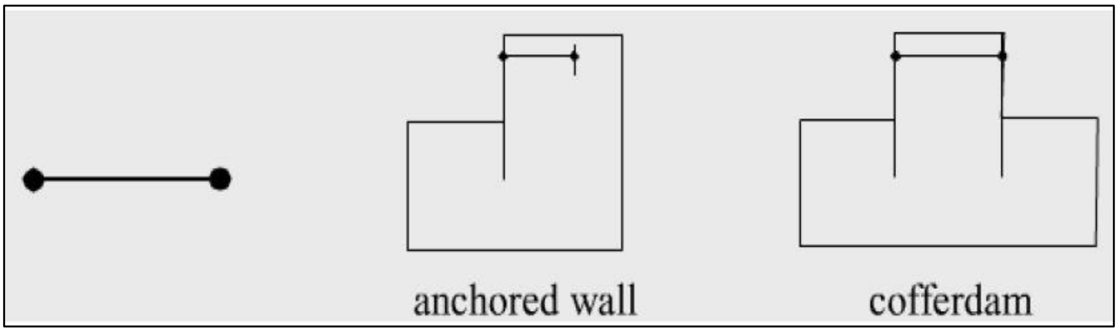

Figure 6-3: Node-to-node Anchors (Ramadan Amer 2013)

6.4.3 FLAC Results

Several output plots are investigated in this section. Outputs defined as the surface failure, plastic-elastic deformation, maximum bending moment, structure horizontal displacement and ground settlement will be inspected. Even though the cantilever model responses have been discussed in detail in Chapter 5, a new cantilever model was developed with the same penetration depth below the dredge line as required for the anchor model for the ability to compare the anchored sheet pile model results with the results obtained from the cantilever sheet pile wall model. This has been provided to establish the differences and agreements between the two types of pile models.

6.4.3.1 FLAC Model Validation

In engineering design procedures information such as the bending moment distribution exerted along the sheet pile wall is important information to consider for structural design purposes. The developed anchor sheet pile wall model has to be validated before further parametric studies can be done using this model. The anchor sheet pile wall model was validated by comparing the FLAC results with analytical solutions and observations from previous studies.

The design of anchored sheet pile walls in a sandy soil has to satisfy the following:

1. The sheet pile should be stable after the construction of the wall (ultimate limit state).
2. The displacements and deformations of the sheet pile wall should be small so that the sheet pile wall will function as intended in the design (serviceability limit state).
3. Settlements and lateral displacements caused by the installation process of the structural elements should be small so that adjacent buildings or other nearby structures are not damaged.

To validate the anchor sheet pile wall model created in the FLAC software program, it was necessary to compare the solutions obtained from the numerical modelling with the analytical solutions found from using the limit equilibrium methods (Figure 6-4). Firstly, the maximum axial anchor tie rod force between the numerical and analytical methods was distinguished.

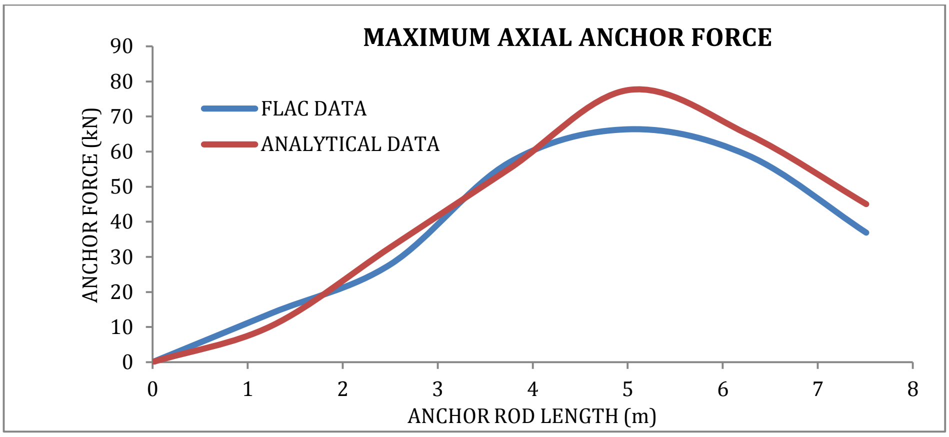

Figure 6-4: Visual Comparison of Maximum Anchor Tie Rod Force

Both the analytical and numerical data for the axial anchor tie rod force increases gradually until a maximum is reached then the force strength decreases slowly. It has been found that the maximum axial force obtained when using the numerical methods is 14.37% smaller when compared to the analytical solution for the anchor tie rod force. This indicates that the results obtained from FLAC, where the structural properties has been considered give a more realistic and accurate indication of the anchor tie rod force behaviour along the specified length.

Similarly, the bending moment distribution along the sheet pile wall for both analytical and numerical analysis has been plotted along the increasing sheet pile wall depth (Figure 6-5).

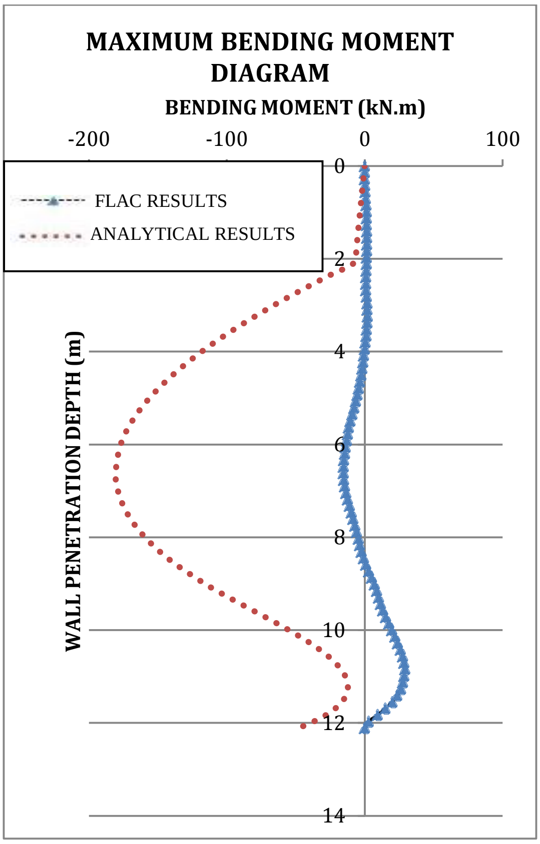

Figure 6-5: Visual Comparison of Maximum Bending Moment

After analysing the cantilever sheet pile solutions, it was assumed that similar results would be obtained for the anchor sheet pile. The visual representation gives a good understanding of how the analytical methods exaggerate the solutions when compared to the numerical solutions. This visual representation of the analytical plot of maximum bending moment is misleading as the anchor sheet pile will not necessarily behave in such a way in a 'real life' situation. Figure 6-5 indicates a reduced comparison between the analytical and numerical solutions.

The numerical solutions indicate that the analytical methods are very conservative when compared to the numerical solutions. FLAC is able to produce more accurate results of the soil-structure system behaviour, ensuring more accurate sheet pile wall designs in the engineering industry (Table 6-1).

Table 6-1: Comparison of Maximum Bending Moment

Wall Total Depth (m)	Cantilever Sheet Pile	Anchored Sheet Pile		Percentage	
	Numerical Solution	Analytical Solution	Numerical Solution	Reduction (%) Numerical	Reduction (%) Design Methods
12.11	45.16	174.26	29.36	34.98	83.15

The discovery in Chapter 5 provided the maximum bending moment obtained from the numerical methods to be very accurate compared to the analytical methods. In order to be able to compare solutions between cantilever and anchor models, it was necessary to create a cantilever model with the same penetrating depth below the dredge line as for the established anchor model to establish the effect of the installation of an anchor to a cantilever sheet pile.

It was found that the maximum bending moment decreased with 76.48% after the installation of the anchor tie rod to the cantilever sheet pile wall. The presence of the anchor force as also changed the effect of the bending moment on the sheet pile for a depth of 1.5 m below the surface from a negative to a positive bending moment, showing a decrease of 34.98% at 1.5 m. The installation of anchors proves to have sufficient effect on the structural integrity of the sheet pile wall system.

Comparing the maximum bending moment solution obtained from the analytical methods with the numerical FLAC software, the maximum bending moment decreases 83.15%. This is a very large decrease, which indicates that the results obtained do not compare well with each other. This is due to the assumption that no pivot point exists for the static system when using the free earth support method (Das 1990). The results indicate that assumption have major impacts on the outputs and solutions. The assumption of the absence of the pivot point was not considered when analysing the maximum bending moment exerted on the anchored sheet pile in FLAC.

FLAC, however, analyses the anchor sheet pile problem as a ‘real life’ situation, where different lateral soil pressures may lead to the existence of a pivot point on the sheet pile. Although the analytical and numerical results do not compare well, the net bending moment behaviour along the sheet pile is proven acceptable by comparing the numerical results obtained in FLAC, with finite element analysis results (Woods 2003).

Failure Surface

The failure surface will be analysed more in depth for this particular chapter as the basic understanding of obtaining and examining the failure surface plots has been established in Chapter 5 for a cantilever sheet pile wall problem. Due to specimen weakness or imperfect boundary conditions, inhomogeneous deformations occur and strains thus become concentrated into narrow zones, also known as ‘shear bands’.

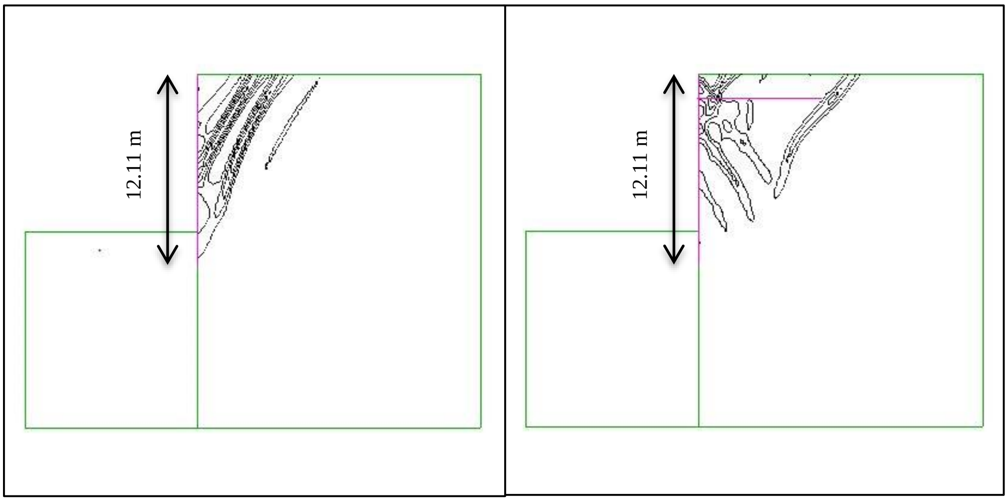

Figure 6-6: Failure Surface Plots; for (a) Cantilever Pile, (b) Anchored Pile

The failure surface within the soil mass for both cantilever and anchored sheet pile wall models similarly consists of distinct parallel slip surfaces. However, there is a slight change between these two figures. The shear bands obtained from the cantilever sheet pile model indicates one directional shear plane behaviour, whereas the anchor sheet pile model obtains multidirectional shear band occurrence. The occurrence of these shear bands consisting of thin multilayers are bounded by two material discontinuity surfaces of a velocity gradient.

This is due to the Mohr-Coulomb criterion that indicates the friction angle of the soil defines the maximum ratio of the shear stress to normal stress than can be mobilised in cohesion less soil (Sadrekarimi & Olson 2010). A specific shear band will form in the direction of the plane on which this friction angle is mobilised. At failure, this plane is inclined at an angle of $(45 - \frac{\emptyset}{2})$ with respect to the direction of σ_1 and is termed the Coulomb rapture plane (Sadrekarimi & Olson 2010).

The shear bands specified for both the cantilever and anchored sheet pile wall models are much larger than the failure point of 25°, thus the shear bands do not represent failure of occurring for the above-developed models. The cantilever sheet pile wall surface failure plot contains shear bands with smaller band thickness.

The reason for this occurrence is due to the large soil mass acting laterally towards the cantilever sheet pile, whereas this action is counter balanced by the anchor force in the anchor sheet pile wall failure surface plot, resulting in the thicker shear bands. It was found that the width of the shear bands is not affected by any geometrical dimensions of a soil body other than its grain size (Vardoulakis 1987). This is an important phenomenon when analysing the progressive failure in granular soils.

Soil Mass Failure

As mentioned briefly in Section 5.4.6 FLAC Results, the failure of the soil mass can be examined by plotting the elastic-plastic deformation plot. The fundamental assumption of the Elastic-Plastic Soil Mechanism is that strains can be separated into two main components, a recoverable elastic strain component and an irrecoverable plastic strain component. In notation form this is written as $\varepsilon_v = \varepsilon_v^e + \varepsilon_v^p$ for volume strain and, similarly, for shear strain as $\varepsilon_s = \varepsilon_s^e + \varepsilon_s^p$.

Comparing the elastic-plastic plots for cantilevered sheet pile walls with the anchored sheet pile wall model, it can be seen that the red elements, which indicate elements undergoing plastic deformation is connecting the ground surface on one side of the sheet pile wall to the ground surface on the other side of the sheet pile wall (Figure 6-7). This indicates that the soil mass is failing due to the mechanism called shallow shear failure.

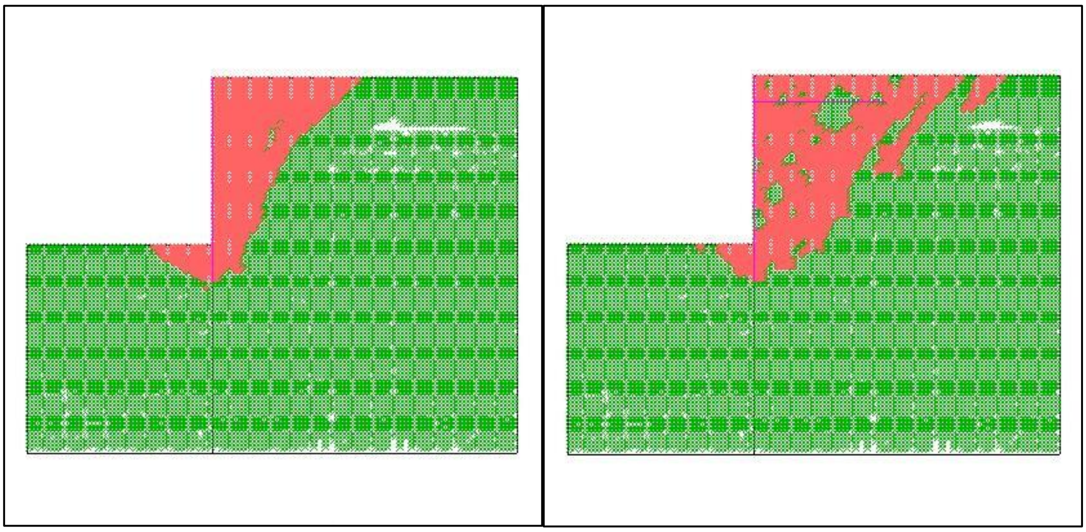

Figure 6-7: Plasticity Indicators; (a) Cantilever Model, (b) Anchor Model

Similarly, the same is occurring for the anchored sheet pile wall model; however the failure incident is due to possible rotational failure. The combination of both plastic and elastic elements occurring at the same time indicates that the applied stress is greater than the current yielding surface, predicted that the strains will have both elastic and plastic components. When a combination of stresses greater than the yield surface is applied, plastic strains develop in response to this stress and enlarge the yield surface to include the current stress state. The strength law describes this scenario as the stress state where the material fails. The reason for this failure occurring is assumed to be due to the very small penetration depth of the sheet piles beneath the dredge line. It was therefore found interesting to undertake a parametric study of varying the depth of the sheet pile below the dredge line, to establish whether this was the main cause of failure.

It can also be noted when comparing the size of the surface area existing of the plastically deformed elements for the cantilever and anchor elastic-plastic plots that a larger plastically deformed surface area is obtained for the cantilever sheet pile wall than for the anchored sheet pile wall model. This indicates that the anchor tie rod force has decreased the surface area of the plasticity, thus reducing the stress.

It is important to note for both sheet pile models that there is no indication of plastically deformed elements extending far below the tip of the sheet pile. Thus the sheet pile wall is not undergoing structural failure (Jardine et al. 1986).

Parametric Study

A large number of factors affect the behaviour of anchored sheet pile walls. In this dissertation, experience with sheet pile walls primarily penetrating sandy soil have been reviewed. FLAC has been used to determine the stability of an anchored sheet pile wall under large excavation. The sheet pile penetration depth will be increased four times to analyse the effect on the sheet pile response and determine methods for increasing the stability of anchored sheet pile walls undergoing large excavations.

The parametric study results were performed to investigate the effect of increasing the wall penetration depth on the anchored sheet pile wall behaviour in a sandy soil. The analysis results in terms of maximum total wall displacement, maximum bending moment, maximum ground settlement and anchor forces with increasing wall depth (Table 6-2).

Table 6-2: Maximum Bending Moment, Maximum Wall Displacement, Maximum Ground Settlement, and Anchor Forces with Increasing Wall Penetration Depths

Normalised Penetration Depth (D/H)	Wall Penetration Depth, D (m)	Maximum Wall Displacement (m)	Maximum Ground Settlement (m)	Maximum Wall Bending Moment (kN.m)	Anchor Tie Rod Force (kN)
0.34	3.11	0.297	0.500	38.26	64.57
0.44	4.00	0.235	0.386	37.56	64.36
0.56	5.00	0.226	0.321	36.97	64.25
0.67	6.00	0.188	0.305	32.54	64.18
0.77	7.00	0.173	0.289	29.95	63.82

The results indicate that as the penetration depth of the sheet pile wall below the dredge line increases the maximum horizontal wall displacement decreases. There is a very good relation between the values of maximum wall displacement. This can be expected, as the position of the anchor tie rod force did not change. The ground settlement also indicated an indirect behaviour of decrease with increasing pile penetration depth. These results are similar when compared to the solutions obtained from the cantilever sheet pile wall models.

The maximum bending moment for the anchored sheet pile wall, decrease with an increase of sheet pile penetration depth. This indicated that the system became more stable as the penetration depth increased. Therefore, the initial assumption of both the cantilever and anchor sheet piles undergoing material failure as indicated on the plasticity plots was due to the insufficient sheet pile wall penetration depth. The anchor axial force results indicate a slight decrease of anchor force with respect to an increasing penetration depth. The direct relationship between the decreasing maximum bending moment and anchor force is expected, as the position of the anchor force was not altered.

6.4.4 Chapter Summary

The effect of increasing the wall penetration depth showed a significant effect on the behaviour of the anchor sheet pile wall maximum bending moment and the failure of the entire pile-soil system. However increasing the sheet pile penetrating depth will increase the cost of installing sheet pile walls.

To conclude this chapter it was found that the anchor sheet pile required a sheet pile wall length increase beneath the dredge line in order to obtain a safe pile-soil system. Unfortunately, due to time constraints, further parametric studies analysing the effect of placing the anchors at an inclined angle and varying the depth of the anchor force placement position to find a cost-effective solution for sheet pile wall designs in the industry could not be undertaken.

However, the anchor sheet pile wall model that has been created in this dissertation proves sufficient due to delivering accurate results when compared to the analytical method and cantilever sheet pile wall model solutions, as fewer assumptions were made when undertaking the numerical anchored sheet pile problem. The numerical model developed in FLAC was validated by comparing the sheet pile wall behaviour with solutions obtained by (Woods 2003).

Chapter 7: Conclusions

This chapter summarises the outcomes of this research project and discusses the achievements of the project and recommendations for areas of future work.

7.1 Spread Sheet Development for Sheet Pile Wall Design

Designing sheet pile walls using the limit equilibrium methods and finding solutions by means of hand calculations is time consuming and prone to human error. In the engineering industry, time is money, so the development of an automatic sheet pile design tool that could solve the tedious analytical equations accurately and create visual solutions, all in a fraction of the time and with only the necessity of inputting known data, would be highly valuable.

The solutions obtained using the Excel spread sheet are similar to the solutions derived from the analytical methods. Thus, the Excel spread sheet has proven both accurate and successful.

However, these analytical methods of design have been found very conservative due to making several simplifications and assumptions. For example, important information such as ground settlement and possible surface failures cannot be obtained from the analytical methods. Thus, it is proposed to use available industrial software such as FLAC to attain critical information and provide greater accuracy of results.

7.2 Cantilever Sheet Pile Wall

The cantilever sheet pile wall was successfully researched, including methods of solution and design. The wall behaviour was investigated through the wall displacements, bending moments and ground settlement. FLAC was used to analyse this sheet pile wall behaviour, investigate the effect of changing specific parameters and undertake a parametric study to investigate the behaviour of the pile in certain situations.

Many valuable conclusions have been drawn. For example, using a very fine mesh when examining the numerical models leads to very accurate results. Examining the results correctly when modelling geotechnical problems in FLAC using course mesh grids is quite challenging, as the results may be misleading. Therefore, numerical models using course mesh grids should be avoided. This emphasises the importance of having a skilled FLAC user undertake the numerical modelling and evaluate the outputs. A parametric study was implemented to investigate the effect of a change in soil density and the presence of a water table below the ground surface. Such parametric studies illustrate possible 'real life' situations. Testing models in this way indicates the possibility of using these numerical models in the engineering industry to find optimum designs and save engineering companies unnecessary expense by providing an alternative to the design of conservative sheet pile walls.

7.3 Anchored Sheet Pile Wall

The anchored sheet pile wall was successfully researched, designed and analysed according to design requirements. However, as no information was obtained for previous numerical design analysis in FLAC of this topic, very good background knowledge was required for this design analysis to quantify the results. Firstly, comparing the solutions to analytical solutions validated the anchored sheet pile wall model created in FLAC. Comparing the solutions for the anchored sheet pile wall to the cantilevered sheet pile wall indicated that applying an anchor tie rod force leads to a decrease in the maximum bending moment exerted on the sheet pile wall, maximum wall deflection and ground settlement.

The system was initially found to be unstable due to the lack of sheet pile wall penetration depth below the dredge line and the occurrence of a very large excavation. Several methods can be suggested to increase the stability of high-excavated sheet pile walls. These include increasing the sheet pile penetration depth below the dredge line, varying the placement of the anchor tie rod force and applying the anchor tie rod force at an angle with respect to the sheet pile wall.

After undertaking a parametric study to evaluate the effect of increasing the penetration depth of the wall below the dredge line, the system was found to be stable and the results acceptable. However, increasing the sheet pile penetrating depth will increase the cost of installing the sheet pile wall, which is unfavourable in the engineering industry, as cost should always be optimised provided structural stability is maintained. Due to time constraints, it was not possible to undertake further parametric studies to analyse the effect of placing the anchors at an inclined angle and varying the depth of the anchor force placement position to find a cost-effective solution for sheet pile wall designs in the industry.

The models created confirm that FLAC is a valuable tool for analysing the behaviour of sheet pile walls. While the limit equilibrium methods are limited to the calculation of the required depth of embedment and the maximum bending moment exerted on the sheet pile wall, FLAC could be used to study the maximum wall deformation, ground settlement, bending distribution and possible failure surfaces.

While the limit equilibrium methods certainly provided a basic understanding of the wall-soil system, the hypothesis on which these methods are based is very conservative. The development of numerical models, undertaking of parametric studies and validation of the solutions obtained with analytical methods are definitely topics worth pursuing, as they will lead to more effective sheet pile wall designs in the engineering industry.

7.4 Future Work

This project is very broad, leading to the identification of many areas of future work.

In regard to the automated design tool developed in Excel, it is recommended to implement multiple sheet pile wall designs in one spread sheet. Specific code should also be written in Excel, making use of ‘for loops’ to solve the tedious analytical equations in a back program rather than in a specific cell.

Areas of additional work on the cantilever sheet pile wall model problem include an investigation into the behaviour of cohesive soils, applying the ground water table at different levels behind the sheet pile wall to analyse the effect of hydrostatic pressure development on the sheet pile wall, investigating the performance of a rough sheet pile wall, and undertaking parametric studies focussing on varying the section properties of the sheet pile wall material and soil properties.

Further work on the anchored sheet pile wall problem could be undertaken through investigating the influence of different placements of the anchor tie rod force to investigate the possibility of improved structural stability. A parametric study applying the anchor tie rod force connecting at an inclined angle would also be useful. This could lead to a reduction of the required length of the anchor tie rod. Further work could also investigate even more advanced numerical models to develop a script to facilitate the study of installing multiple anchors by increasing the efficiency.

In addition, feasibility studies would be very useful to perform comparisons between the cost of increasing wall penetration depth with the benefits resulting from this increase and the long-term effect on the structure. It would also be valuable to perform some field monitoring to accompany this study and confirm some of the findings of this research.

7.5 Achievement of Objectives

The aims and objectives as specified in Appendix A for this particular research project were:

1. Research and understand background information on the design procedures, construction considerations and methodologies for sheet pile walls.
    - Previous research completed.
    - Available analytical methods were identified.
    - The effects of construction considerations were closely examined.
    - Available solutions were obtained.
2. Use the developed knowledge of designing cantilever sheet pile walls to design more advanced anchored sheet pile walls.

- Designed cantilevered and anchored sheet pile walls using the limit equilibrium methods to obtain hand calculation solutions.
- Developed a design tool in Excel that can automatically solve any sheet pile wall problems.
- Simplified the iterative hand calculations.
- Used this to validate numerical model solutions.

3. Prepare advanced numerical models using FLAC.
    - Gained sufficient knowledge of the program to create the necessary models.
    - Developed a cantilever sheet pile wall model.
    - Completed a detailed parametric study to examine the behaviour on the sheet pile wall.
    - Developed an anchor sheet pile wall model.
    - Analysed the effect of increasing wall penetration depth on the stability of the system.
4. Evaluate, compare and discuss the results and findings for both theoretical and numerical solutions.
    - Verified both numerical models through critical examination and comparison to available analytical solutions.

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Appendices

Appendix A: Project Specification

University of Southern Queensland

Faculty of Health, Engineering & Sciences

ENG4111/ENG4112 Research Project

Project Specification

For: Chane Brits

Topic: Developing a Numerical Model for the Design of Sheet Pile Walls

Supervisor: Dr Jim Shiau

Project Aim: To develop numerical models in FLAC to analyse various geotechnical problems. The investigation will higlight modelling techniques that can be used to enable predicting engineerings to model real structures more accurately.

Programme: Issue A, 17th March 2014

1. Reseach backround information on the application of analyitcal and numerical analysis for geotechnical design.
2. Using the analytical methods to develop an excel spreadsheet that aids as a design tool for sheet pile walls.
3. Gain sufficient knowledge of the program FLAC to create the necessary models by writing a script code.
4. Prepare an advanced numerical models using the computer software FLAC that represents 'real life' problems including cantilever and anchored sheet piles.
5. Verifying the models through critical examination and comparison to available solutions.

Agreed:

Student Name:	Chane Brits
Supervisor Name:	Dr Jim Shiau
Examiner:	Chris Snook
Date:	17th March 2014

Appendix B: Cantilever Sheet Pile Wall-Limit State Method Calculations

The calculations to determine the depth of embedment and maximum bending moment with reference made to Figure B1.

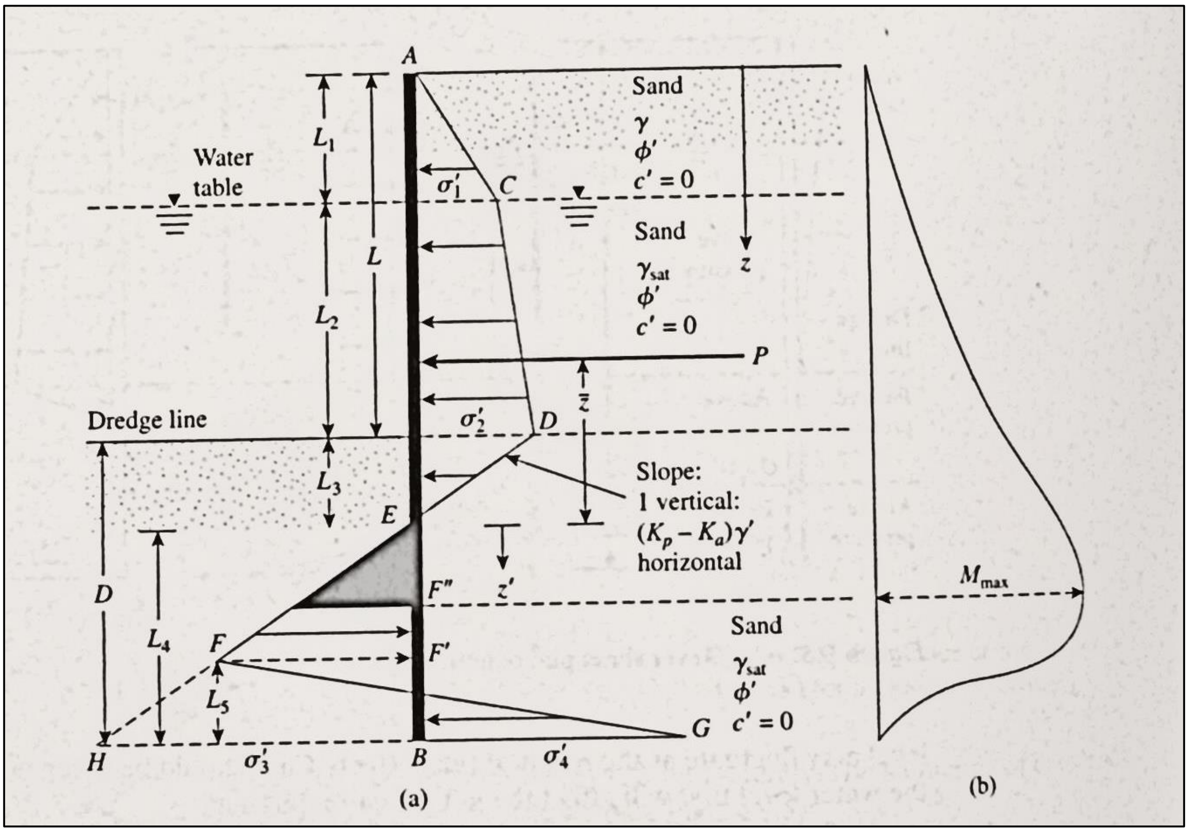

Figure B1: Cantilever sheet pile wall in sand; (a) Net pressure, (b) Moment diagram (Das 1990)

<u>Step 1</u>: Calculate K_a and K_p

$$K_a = \tan^2\left(45 - \frac{\emptyset}{2}\right)$$
$$= \tan^2\left(45 - \frac{40}{2}\right)$$
$$= 0.217$$

$$K_p = \tan^2\left(45 + \frac{\emptyset}{2}\right)$$
$$= \tan^2\left(45 + \frac{40}{2}\right)$$
$$= 4.598$$

Step 2: Calculate σ'_1 and σ'_2

$$\sigma'_1 = \gamma L_1 K_a$$

$$= 18.85 \times 3 \times 0.217$$

$$= 12.27 \text{ KPa}$$

$$\sigma'_2 = (\gamma L_{1+}\gamma' L_2) \times K_a$$

$$= [(18.85 \times 3 + (20.33 - 9.81) \times 6]0.217$$

$$= 25.96 \text{ KPa}$$

Step 3: Calculate L_3, where the net pressure is zero

$$L_3 = \frac{\sigma'_2}{\gamma'(K_p - K_a)}$$

$$= \frac{25.96}{9.81(4.598-0.217)}$$

$$= 0.604 \text{ m}$$

Step 4: Calculate P, by summing all the horizontal forces

$$P = 0.5\,\sigma'_1 L_1 + \sigma'_1 L_2 + 0.5(\sigma'_2 - \sigma'_1)L_2 + 0.5\sigma'_2 L_3$$

$$= (0.5 \times 12.27 \times 3) + (12.27 \times 6) + 0.5(25.96 - 12.27)6 + (0.5 \times 25.96 \times 0.604)$$

$$= (18.405) + (73.62) + 40.26 + (7.83992)$$

$$= 140.12\ kN/m$$

Step 5: Calculate $\bar{z}$, i.e., the centre of the pressure for the area ACDE by taking the moments about point E

$$\bar{z} = \left[\begin{array}{c} 0.5\sigma_1' L_1 * \left(\frac{L_1}{3} + L_2 + L_3\right) + \\ \sigma_1' L_2 * \left(L_3 + \frac{L_2}{2}\right) + \\ 0.5(\sigma_2' - \sigma_1') L_2 * \left(L_3 + \frac{L_2}{3}\right) + \\ 0.5\sigma_2' L_3 * \frac{L_3}{3} \end{array}\right] \Big/ P$$

$$\bar{z} = \left[\begin{array}{c} (0.5 \times 12.27 \times 3) \times \left(\frac{3}{3} + 6 + 0.604\right) + \\ (12.27 \times 6) \times \left(0.604 + \frac{6}{2}\right) + \\ 0.5(25.96 - 12.27)6 \times \left(0.604 + \frac{6}{3}\right) + \\ 0.5 \times 25.96 \times 0.604 * \frac{0.604}{3} \end{array}\right] \Big/ 140.12$$

$$\bar{z} = 3.66\, m$$

Step 6: Calculate σ_5', the net lateral pressure at the bottom of the sheet pile on the left hand side.

$$\sigma_5' = (\gamma L_{1+} \gamma' L_2) \times K_p + \gamma' L_3 (K_p - K_a)$$

$$\sigma_5' = (18.85 \times 3 + 10.52 \times 6) \times 4.598 + 10.52 \times 0.604(4.598 - 0.217)$$

$$\sigma_5' = 578.0\, KPa$$

Step 7: Calculate the constants A_1, A_2, A_3 and A_4 according to given equations as follows:

$$A_1 = \frac{\sigma_5'}{\gamma'(K_p - K_a)}$$

$$A_1 = \frac{578.0}{10.52 \times (4.598 - 0.217)}$$

$$A_1 = 12.54$$

$$A_2 = \frac{8P}{\gamma'(K_p - K_a)}$$

$$A_2 = \frac{8\times(140.12)}{10.52(4.598-0.217)}$$

$$A_2 = 24.32$$

$$A_3 = \frac{6P[2\bar{z}\gamma'(K_p - K_a) + p_5]}{\gamma'^{\,2}\,(K_p - K_a)^2}$$

$$A_3 = \frac{6\times 140.12[2\times 3.66\times 10.52\times(4.598-0.217)+578.0]}{10.52^{\,2}\,(4.598-0.217)^2}$$

$$A_3 = 362.29$$

$$A_4 = \frac{P[6\bar{z}p_5 + 4P]}{\gamma'^{\,2}\,(K_p - K_a)^2}$$

$$A_4 = \frac{140.12[6\times 3.66\times 578 + 4\times 140.12]}{10.52^{\,2}\,(4.598-0.217)^2}$$

$$A_4 = 874.27$$

<u>Step 8</u>: Calculate unknown length L_4 by using a trial and error method to solve the 4th exponential equation.

$L_4^4 + A_1L_4^3 - A_2L_4^2 - A_3L_4^1 - A_4 = 0$ Substitute L_4 = 5.945

$$5.945^4 + 12.54 \times 5.945^3 - 24.32 \times 5.945^2 - 5.945^1 - 874.27 = 0$$

$$L_4 = 5.945\ m$$

Step 9: Calculate σ_4' the net pressure at the bottom of the sheet pile on the right side of the pile:

$$\sigma_4' = \sigma_5' + \gamma' L_4 (K_p - K_a)$$

$$\sigma_4' = 578 + 10.52 \times 5.945(4.598 - 0.217)$$

$$\sigma_4' = 578 + 10.52 \times 5.945(4.598 - 0.217)$$

$$\sigma_4' = 851.99 \text{ KPa}$$

Step 10: Calculate σ_3' in relation to the length L_4

$$\sigma_3' = \gamma' L_4 (K_p - K_a)$$

$$\sigma_3' = 10.52 \times 5.945(4.598 - 0.217)$$

$$\sigma_3' = 273.99 \text{ KPa}$$

Step 11: Obtain L_5,

$$L_5 = {\sigma_3' L_4 - 2P}\Big/{\sigma_3' + \sigma_4'}$$

$$L_5 = {273.99 \times 5.945 - 2 \times 140.12}\big/{273.99 + 851.99}$$

$$L_5 = 1.2\ m$$

Step 12: Obtain the theoretical depth of penetration as $L_3 + L_4$.

$$D_{theoretical} = L_3 + L_4$$

$$D_{theoretical} = 0.604 + 5.945$$

$$D_{theoretical} = 6.549\ m$$

Step 13: Increase the theoretical depth with a factor of safety of 1.4

$$D_{actual} = D_{theoretical} \times 1.4$$

$$D_{actual} = 6.549 \times 1.4$$

$$D_{actual} = 9.17\ m$$

Step 14: Calculate the point where maximum shear force occurs by making a cut on the beam and summing the moments about that specific point, z'

$$z' = \sqrt{\frac{2P}{(K_p - K_a)\gamma'}}$$

$$z' = \sqrt{\frac{2 \times 140.12}{(4.598 - 0.217)10.52}}$$

$$z' = 2.47\ \mathrm{m}$$

Step 15: Determine the maximum bending moment by summing all the moments about the point where zero shear force occurs:

$$\mathrm{Mmax} = P(\bar{z} + z') - [0.5\gamma' z'^2 (K_p - K_a)](\frac{Z'}{3})$$

$$\mathrm{Mmax} = 140.12(3.66 + 2.47) - [0.5 \times 10.52 \times 2.47^2 (4.598 - 0.217)](\frac{2.47}{3})$$

$$\mathrm{Mmax} = 743.18\ \mathrm{kN.m}$$

Appendix C: Anchored Sheet Pile Wall-Limit State Method Calculations

The calculations to determine the depth of embedment, anchor tie rod force and maximum bending moment with reference made to Figure C1.

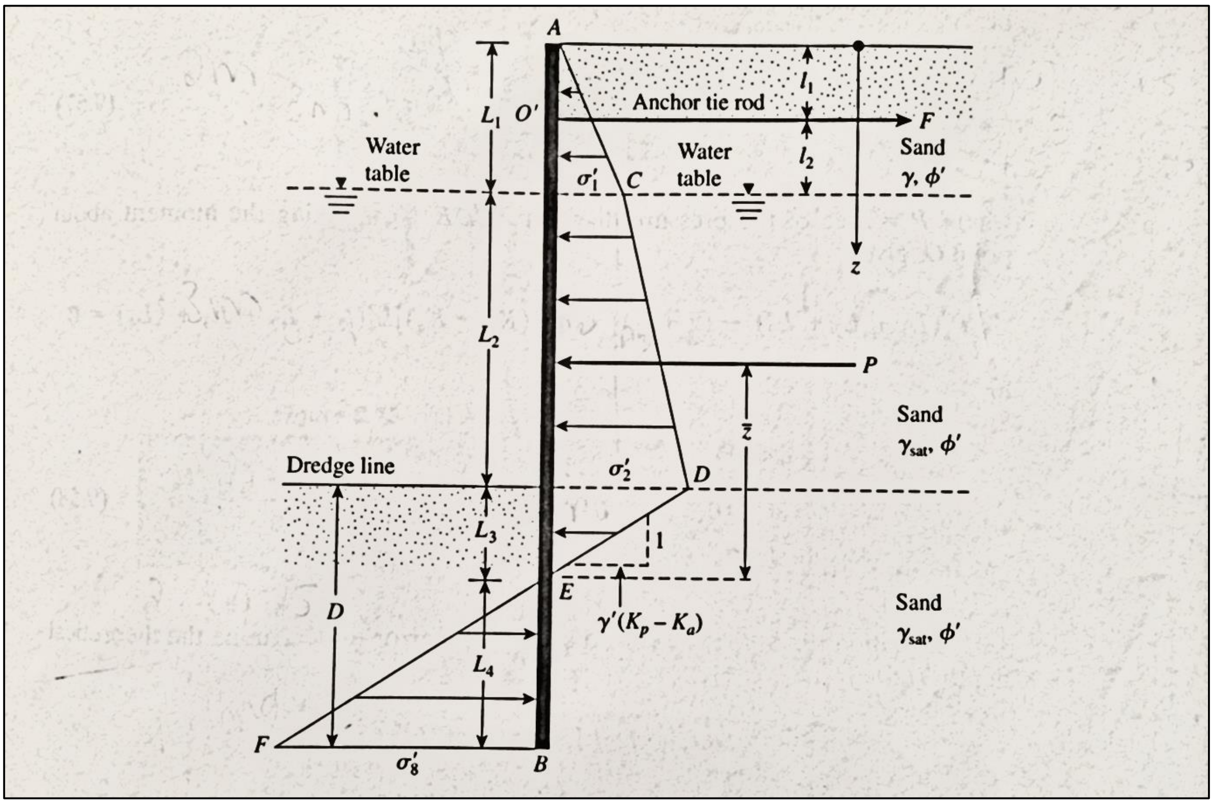

Figure C1: Net pressure distribution for the Anchor sheet pile wall in sand

The diagram above the dredge line is similar to Figure B1.

Where $z = L_1$

$$\sigma'_1 = \gamma L_1 K_a$$

Where $z = L_1 + L_2$

$$\sigma'_2 = (\gamma L_1 + \gamma' L_2) \times K_a$$

Below the dredge line, the net pressure will be zero at:

$$z = L_1 + L_2 + L_3$$

Step 1: Calculate K_a and K_p

$$K_a = \tan^2\left(45 - \emptyset/_2\right)$$
$$= \tan^2(45 - 40/_2)$$
$$= 0.217$$

$$K_p = \tan^2\left(45 + \emptyset/_2\right)$$
$$= \tan^2(45 + 40/_2)$$
$$= 4.598$$

Step 2: Calculate $\sigma_1^{'}$ and $\sigma_2^{'}$

$$\sigma_1^{'} = \gamma L_1 K_a$$

$$= 18.85 \times 3 \times 0.217$$

$$= 12.27 \text{ KPa}$$

$$\sigma_2^{'} = (\gamma L_{1+} \gamma' L_2) \times K_a$$

$$= [(18.85 \times 3 + (20.33 - 9.81) \times 6]0.217$$

$$= 25.96 \text{ KPa}$$

Step 3: Determine L_3, by using the relation given of 1 vertical to $\gamma^{'}(K_p - K_a)$ in the horizontal

$$L_3 = \sigma_2^{'} / \gamma^{'}(K_p - K_a)$$

$$L_3 = 25.96 / 10.52(4.598 - 0.217)$$

$$L_3 = 0.563 \text{ m}$$

Step 4: Calculate P, by summing all the horizontal forces

$$P = 0.5\,\sigma_1^{'} L_1 + \sigma_1^{'} L_2 + 0.5(\sigma_2^{'} - \sigma_1^{'}) L_2 + 0.5\sigma_2^{'} L_3$$

$$= (0.5 \times 12.27 \times 3) + (12.27 \times 6) + 0.5(25.96 - 12.27)6 + (0.5 \times 25.96 \times 0.604)$$

$$= (18.405) + (73.62) + 40.26 + (7.83992)$$

$= 140.12\ kN/m$

<u>Step 5</u>: Calculate $\bar{z}$, i.e., the centre of the pressure for the area ACDE by taking the moments about point E

$$\bar{z} = \left[\begin{matrix} 0.5\mathrm{p}_1\mathrm{L}_1 \ * \left(\frac{L_1}{3} + L_2 + L_3\right) + \\ \mathrm{p}_1\mathrm{L}_2 * \left(L_3 + \frac{L_2}{2}\right) + \\ 0.5(\mathrm{p}_2 - \mathrm{p}_1)\mathrm{L}_2 * \left(L_3 + \frac{L_2}{3}\right) + \\ 0.5\mathrm{p}_2\mathrm{L}_3 * \frac{L_3}{3} \end{matrix} \right] \Big/ P$$

$$\bar{z} = \left[\begin{matrix} (0.5 \times 12.27 \times 3) \ \times \left(\frac{3}{3} + 6 + 0.604\right) + \\ (12.27 \times 6) \times \left(0.604 + \frac{6}{2}\right) + \\ 0.5(25.96 - 12.27)6 \times \left(0.604 + \frac{6}{3}\right) + \\ 0.5 \times 25.96 \times 0.604 * \frac{0.604}{3} \end{matrix} \right] \Big/ 140.12$$

$\bar{z} = \ 3.66\ m$

<u>Step 6</u>: Calculate the unknown length L_4 by means of trial and error:

$\mathrm{L}_3^4 + 1.5 \times \mathrm{L}_4^2(\mathrm{l}_2 + \mathrm{L}_2 + \mathrm{L}_3) - \frac{3\mathrm{P}[(\mathrm{L}_1+\mathrm{L}_2+\mathrm{L}_3)-(\bar{\mathrm{z}}+\mathrm{l}_1)]}{\gamma'(K_p - K_a)} = 0$ Substitute $L_4 =$ 1.66

$$0.563^4 + 1.5 \times 1.66^2(1.5 + 6 + 3) - \frac{3 \times 140.12[(3+6+0.563)-(3.66+1.5)]}{10.52(4.598-0.217)} = 0$$

$L_4 = 1.66$ m

<u>Step 7</u>: Obtain the theoretical depth of penetration as $\mathrm{L}_3 + \mathrm{L}_4$.

$D_{theoretical} = L_3 + L_4$

$D_{theoretical} = 0.563 + 1.66$

$D_{theoretical} = 2.22\ m$

Step 8: Increase the theoretical depth with a factor of safety of 1.4

$D_{actual} = D_{theoretical} \times 1.4$

$D_{actual} = 2.22 \times 1.4$

$D_{actual} = 3.11\ m$

Step 9: Determine the anchor force F, by summing all the horizontal forces,

$F = P - 0.5 \times [\gamma'(K_p - K_a)] \times {L_4}^2$

$F = 140.12 - 0.5 \times [10.52 \times (4.598 - 0.217)] \times 1.66^2$

$F = 76.62\ kN$

Step 10: Calculate the point where maximum shear force occurs by making a cut on the beam and summing the moments about that specific point z, it has been assumed that this point of zero shear will occur where:

$L_1 + L_2 < z < L_1$

Therefore obtaining the following equation after summing the forces around the point where the cut was made on the beam:

$0.5 \times \sigma'_1 L_1 - F + \sigma'_1(z - L_1) + 0.5 \times K_a\gamma'(z - L_1)^2$ Substitute $z =$ 6.599

$0.5 \times 12.27 \times 3 - 76.62 + 12.27(5.77 - 3) + 0.5 \times 0.217 \times 10.52(5.77 - 3)^2$

$z = 6.599\ m$

And to make calculations simpler:

$x = 6.599 - 3$

$x = 3.599\ m$

Step 11: Determine the maximum bending moment by summing all the moments about the point where zero shear force occurs:

$$M_{max} = -(0.5 \times \sigma'_1 L_1) \times \left[x + \left({L_1}/{3}\right)\right] + F(x+1) - \sigma'_1 x \times \left(\frac{x}{2}\right) - 0.5 K_a \gamma' (x^2) \frac{x}{3}$$

$$M_{max} = -(0.5 \times 12.27 \times 3) \times \left[3.599 + \left({3}/{3}\right)\right] + 76.62(3.599 + 1) - 12.27 \times 3.599 \times \left(\frac{3.599}{2}\right) - 0.5 \times 0.217 \times 10.52(3.599^2)\frac{3.599}{3}$$

$$M_{max} = 174.26 \text{ kN.m}$$