University of Southern Queensland

Faculty of Health, Engineering and Sciences

Investigating what effect two face measurement has on Reflectorless EDM with respect to angle of incidence and determining the critical incident angle

A dissertation submitted by

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in fulfilment of the requirements of

ENG4111 and 4112 Research Project

towards the degree of

Bachelor of Spatial Science (Honours) (Surveying)

Abstract

This study aims to examine the effect angle of incidence has on REDM to better understand REDM technology and its capabilities. Three key areas were chosen to specifically focus on; analysing the effects of combined angle of incidence, analysing two face observations and determining if there is a critical angle of incidence. The research was justified based off the lack of previous research into combined angle of incidence and taking two face REDM observations

A testing regime was incorporated into the research testing a large variety of incident angles across three distance ranges of 10, 30 and 60 metres. At each distance range incident angles between 25 -75^0 were analysed using 5^0 increments to create trend lines and accurately model how incident angle error behaved. A reflectorless target was crucial to the research and was constructed to match the properties of Kodak grey cards reflective side.

The critical angle of incidence was found to be 60^0 and the maximum recommended angle of incidence was found to be 35^o. The second and major finding from the study was the degree of improvement two face observations make to REDM accuracy. Results showed by taking two face observations as opposed to single face observations incident angle error was almost completely removed with accuracy improving up to 5 times.

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Limitations of Use

Certification

I certify that the ideas, designs and experimental work, results, analyses and conclusions set out in this dissertation are entirely my own effort, except where otherwise indicated and acknowledged.

I further certify that the work is original and has not been previously submitted for assessment in any other course or institution, except where specifically stated.

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Student Number: [redacted]

Acknowledgments

I would like to acknowledge my beautiful partner Megan for her support during my Studies, particularly over the last eight months. I would also like to thank my father Gary and brother Dylan for their assistance with constructing equipment and fieldwork as part of the research.

Jessica Smith played a huge role in assisting my research offering guidance and feedback throughout the study. I am extremely grateful for her involvement and supervision in producing this dissertation. I would also like to thank the University of Southern Queensland staff that have directly contributed to my studies in fulfilling my degree.

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1 Introduction

Reflectorless electronic distance measurement (REDM) is a mode of distance measurement becoming more common in the surveying practice. REDM allows for measurements to be taken without the need for a reflective target that ultimately can reduce the size of a survey party from 2 to 1 and is a safer option when measuring to hazardous features such as a roof.

The technological advancements in REDM have improved the accuracy of reflectorless measurements, as the technology continues to improve the use of REDM for cadastral surveys may one day be a reality in certain circumstances. Accounting for different types of error, determining limitations of use and computing corrections for measurements make up an important part of research into improving the standards of REDM.

Angle of incidence is a major source of error that has an adverse effect on the accuracy of REDM. The angle of incidence also has an additional effect of increasing potential error caused by beam divergence and collimation error, so determining the quantitative amounts of error holds merit. The error likely to be introduced from different angles of incidence can be used to create guidelines for suitable angles of incidence to work within. Appropriate corrections can also be determined for when working outside of the suitable angle of incidence guidelines.

1.1 Previous Research

Total stations don't have the ability to record the angle of incidence of the surface being measured as the instrument cannot identify the surface, only measure the number of wavelengths and time taken to deduct a distance.

Ashraf et al (2011) performed a test measuring the angle of incidence over a range of 0^{o} to 45^{o} by increments of 5^{o}. Ashraf et al (2011) took 20 measurements at each 5^{o} increment and performed the test over 3 different distances (8.23m, 18.97m and 27.45m). The results obtained show there is a direct correlation between an increasing inclination angle and an increase in error of the calculated slope distance with a recommendation to use a small target rather than a large one to account for beam divergence.

James (2016) research on angle of inclination looked at 4 different angles of incidence (0^0, 22.5^0, 45^0 & 60^0). The results obtained showed 0^0, 22.5^0 and 45^0 had acceptable errors for the majority of short distance measurements although 60^0 resulted in error above 2mm. The 60^0 angle of incidence was also the only angle outside of the manufacturer's specifications. James (2016) also found that the error was longer than the true distance due to beam divergence. The greater the angle of incidence, the greater the beam divergence and therefore, an increase in potential error from the return signal. The distance used for analysis was 7.1 metres which is considerably smaller than typical REDM measuring applications and further analysis over larger distances would be beneficial.

Kowalczyk & Rapinski. (2014) looked at a broad range of error for REDM, with particular focus on atmospheric conditions, beam divergence, angle of incidence, colour, material instrument errors and laser rangefinders. In their research they found as the angle of incidence increases there is a direct correlation with an increase in the amount of error due to beam divergence. They stated that beam divergence and angle of incidence are closely linked.

1.2 Knowledge Gap

Previous research into REDM error has shown that there are numerous sources of error, some of which have direct correlation while others do not. Research has shown that angle of incidence does cause error as it increases although at which point this error critically increases is unknown. James J. (2016) research shows that angle of incidence error increases significantly between 45^0 and 60^0 although at which point is unknown. It is also unknown for angles of incidence greater than 60^0.

Typical REDM measurements will occur over a distance between 5 and 50 metres, although previous research has mainly looked at short distances below 20 metres. REDM measurements are particularly useful when measuring to hazardous features such as roofs, ridges, power lines or centre line of roads where access is difficult due to heights, traffic etc. Generally, these distances are greater than 7 metres. Analysing the effects of angle of incidence over distances greater than 7 metres will provide a better understanding of the expected error from beam divergence over typical distances measured in common surveying applications.

Kowalczyk & Rapinski, (2014) conducted research analysing the angle of incidence at 0^0, 22.5^0 and 45^0 and how this influenced beam divergence. Results showed little change in error between 0^0, 22.5^0 yet error significantly increased above 22.5^0, therefore, analysing angles of incidence above 22.5^0 will give a better understanding of the relationship between angle of incidence and horizontal distance error. Using 5^0 increments will allow for the error to be modelled more thoroughly and could determine if there is a critical angle of incidence where accuracy is greatly affected.

Collimation error has been identified as a source of error extensively, however, mainly in relation to observed angles rather than reflectorless distances. Similarly, to beam divergence, how collimation error is affected over small increments of angle of incidence is unknown. By observing both faces across the 25^0 - 75^0 angle of incidence range will allow for comparison between single face and two face accuracy. Particularly, whether or not two face accuracy is dramatically different at angles of incidence above 45^0.

Angle of incidence research has been focused on analysing either horizontal angle of incidence or vertical horizontal distance although a combined vertical and horizontal angle of incidence hasn't been researched. Surveying applications for reflectorless measurement rarely occur where a target is exactly perpendicular to the instrument in either the horizontal or vertical axis; making the analysis of a combined vertical and horizontal angle of incidence relatable to industry. Selecting a test site with significantly sloped terrain will allow for analysis of combined horizontal and vertical angle of incidence. Ideally, the test site should also allow for analysis over a flat surface where a zenith angle of 90^0 can be established to act as the control.

Analysing angles of incidence over small increments will deliver a better understanding on the relationship beam divergence and collimation error has on REDM. Combining this with different distances and combined angles of incidence typically seen in the surveying industry, will produce results relatable to current surveying applications of reflectorless measurement technology.

1.3 Aim

The aim of this report is to study the effect angle of incidence has on reflectorless EDM measurement and provide recommendations for the potential use of reflectorless EDM technology in surveying applications.

1.4 Objectives

The specific objectives of research:

1. Determine whether there is a critical angle of incidence that affects the accuracy of REDM.
2. Determine the maximum allowable angle of incidence for REDM to remain in the manufacturer's specifications.
3. Determine if distance affects the accuracy of REDM, particularly when the angle of incidence is greater than 25°.
4. Determine if taking face left and face right readings will reduce error in measured horizontal distances.
5. Determine if combined horizontal and vertical angle of incidence affects the accuracy of REDM in a linear or alternative pattern.

1.5 Justification

Previous work has been done analysing REDM looking at different colours, materials and angle of incidence, however, the angle of incidence research still has significant potential for further research. Analysing REDM error above 45^0 angle of incidence will help determine whether or not there is a critical angle of incidence that when reached causes a severe drop off in accuracy.

If a critical angle of incidence is found, this will provide a guideline for REDM measurement use at angles of incidence above 45 degrees. Specifically, REDM measurement to pitched roofs, road surfaces and pavement are common scenarios where the angle of incidence will be greater than 45 degrees. Analysing vertical, horizontal and combined angles of incidence will also reflect real world scenarios and determine if there is any difference or correlation between the different planes.

The majority of previous research of REDM has neglected looking at collimation error as it does not affect EDM distance observations. When angle of incidence is introduced to REDM collimation error will affect not only angle readings but also distance measurements, therefore, comparing distance error from one face and two face readings will be analysed.

1.6 Overview

Chapter 1 has focused on background information, aims and justification of the research topic. Chapter 2 will build on this by investigating further the technical information required to comprehensively assess the research. The technical and background information will then allow for a thorough methodology to be formed, which is discussed in chapter 3. Chapter 4 will present the research findings which will then be analysed in depth as part of chapter 5. Concluding remarks will be formed based off the overall report in chapter 6.

1.7 Summary

REDM provides the surveying industry with an efficient and safe option when measuring to hazardous or difficult to reach features. Analysing sources of error that are relevant to REDM will help to better understand the likely accuracy for different applications and how best to mitigate potential error sources.

Angle of incidence has the potential to affect the accuracy of REDM observations; previous research has identified incidence angles above 25^0 significantly affect accuracy. Industry generally requires measurements with greater angles of incidence than this making it important to understand the impact angle of incidence has on error. This study aims to quantify these errors and identify possibilities and limitations REDM has in surveying applications.

2 Literature Review

To refine the aims of the project and to aid in producing a detailed and appropriate methodology a thorough literature review needs to be conducted. Identifying potential sources of error relevant to the project and determining strategies that best mitigate these sources of error will ensure the project will be properly tailored to the desired outcome. As well as strategies, identifying suitable equipment will also be critical to the research.

2.1 Electronic Distance Measurement

The principle of EDM has three stages; emission, reflection and reception. A beam of energy with a known wavelength is emitted towards a reflective target, the target then reflects the beam back to the starting point where the energy is received. By measuring the time taken, a distance can be calculated based off the size of the wavelength. Total stations are commonly used in the surveying industry that use this method to deduct distances of nearby features. Figure 2.1 shows this relationship.

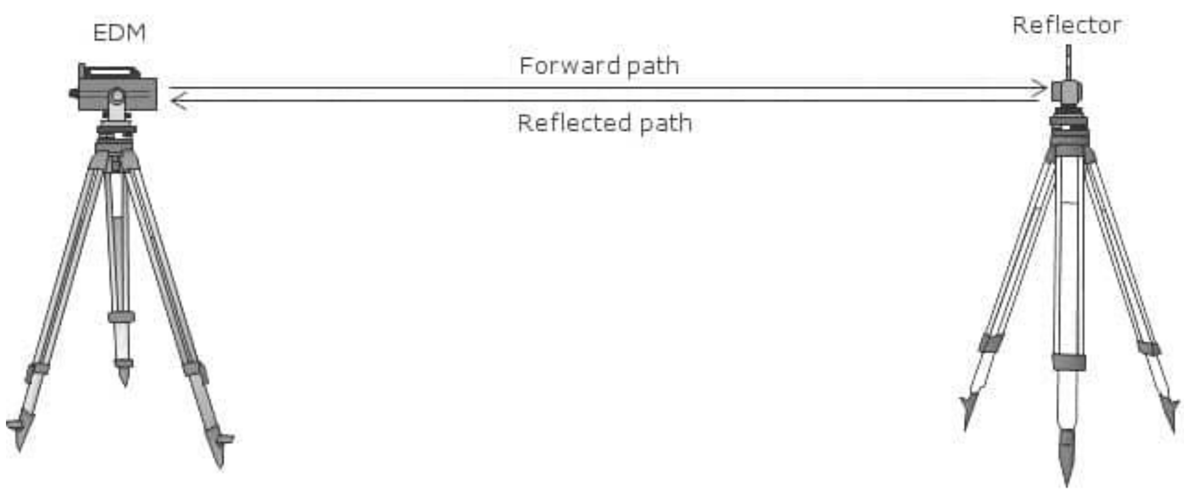


Figure 2.1 EDM Principle (N. Arjun, 2017)

There are three types of EDM commonly used by surveyors:

- Microwave EDM
- Light wave EDM
- Infrared EDM

Infrared EDM is the most common used by surveyors as the instruments are cheap and can be mounted onto theodolites. Generally, distances can be measured between 1 metre and 3 kilometres with an accuracy of +/- 10mm (Arjun, 2017).

Microwave EDM has a maximum range of 100km (Arjun, 2017), this requires two people at either end and is used for long distance measurements. It is only used for very long-distance surveys such as measuring between two mountains.

Light wave EDM has the same range as Infrared EDM of 3 kilometres although the accuracy is 0.5mm/km which is far superior to Infrared EDM. (Arjun, 2017) Light wave EDM is more expensive and is used for precise measuring on engineering projects where 1-millimetre accuracy is important over a long distance.

2.2 Reflectorless Electronic Distance Measurement

Reflectorless EDM (REDM) works from the same concept as EDM with one difference; the intensity of the beam used. EDM measurements use small amounts of energy as the reflector is designed to accurately reflect the signal straight back towards the receiver. REDM however, uses larger amounts of energy so that the signal will reflect off any surface and an acceptable portion of reflection will be received to deduct a distance. EDM typically has a signal strength of 1-7 Milliwatts compared to REDM that requires 1-20 Watts. (Key, 2005) REDM does not require a person to physically walk around to different points with a reflector as shots will reflect off any surface saving time and can decrease the survey party from two to one. Hard to reach and dangerous places such as roofs, busy roads and cliffs can be measured to with REDM making fieldwork safer and more efficient.

Types of REDM laser emission commonly used in survey instruments:

- Phase Shift
- Pulse Distance

2.2.1 Phase Shift

Phase shift is considered to be more accurate than pulse distance, although has a smaller range. Phase shift utilises a narrower beam of light, meaning when it hits a surface there is a more intense and smaller diameter of energy hitting the intended target. A narrow beam also means it is more affected by atmospherics relative to a larger beam, therefore the range phase shift laser emission can reach is less than pulse distance (Reda & Bedada, 2012). Phase shift measurement works by measuring the number of completed wavelengths, with the remainder of the final wavelength deducted. If the signal returns exactly on a completed wavelength with no remainder the signal will have no phase shift.

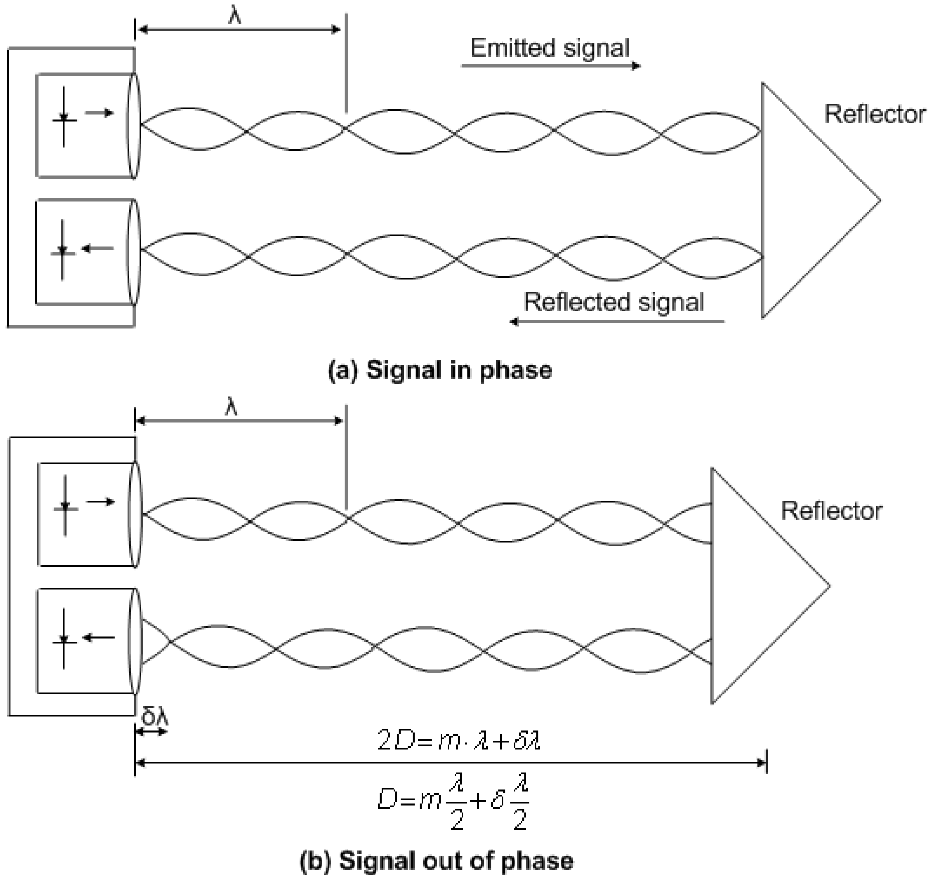


Figure 2.2 Principle of phase shift measurement (McLaughlin, 2015)

2.2.2 Pulse Distance

Pulse distance emission has an advantage of being more practical over long distances compared to phase shift as a wider, more intense beam is used. Pulse distance works by an emitter firing an intense beam that scatters once it hits the target (Reda & Bedada, 2012). The receiver then measures multiple signals that have been scattered and averages these out in a short timeframe to calculate the distance. Over very short distances pulse distance measurements are inaccurate due to the scatter not being able to diverge enough.

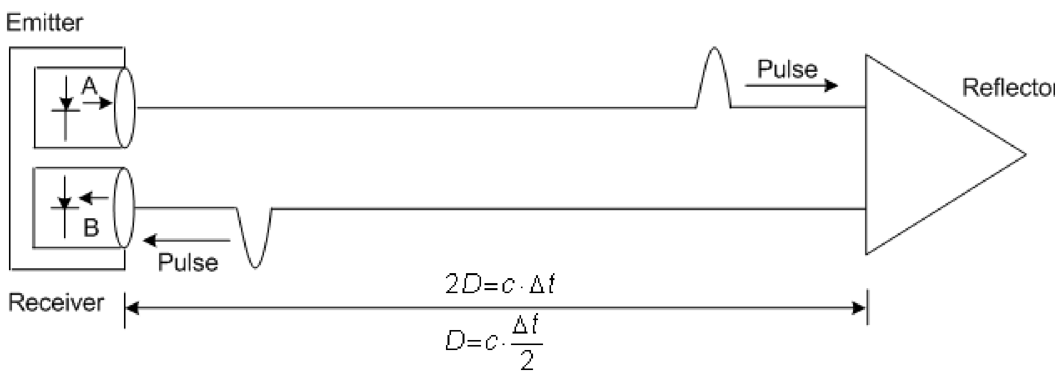


Figure 2.3 Principle of pulse measurement (McLaughlin, 2015)

2.2.3 Limitations

The accuracy of EDM is generally between 1 in 1000 to 1 in 10000 for distances between 15 and 150m and will vary depending on the manufacturer. Because REDM uses higher intensity beams to record measurements potential obstructions can be reflected off causing errors. An example of this is sighting to a building corner with a small amount of foliage in the way. Although the desired target is the building corner the foliage may cause interference and the distance to the foliage in front of the building corner may be recorded instead. The higher intensity beam for REDM also poses as a health risk to the human eye staring directly at the REDM laser being emitted should be avoided.

2.3 Electromagnetic Wavelength

Electromagnetic radiation is affected by the medium it is travelling through, which is why accurate wavelength experimental tests are performed under vacuum conditions. As surveying applications don't allow for vacuum conditions corrections need to be made for potential sources of error that will influence how electromagnetic radiation travels through the atmosphere. Electromagnetic wavelengths direction can be altered by reflection or refraction so investigating potential causes of this is important for high precision measurement. Total stations that incorporate reflectorless EDM typically use wavelengths from the visible spectrum or infrared spectrum.

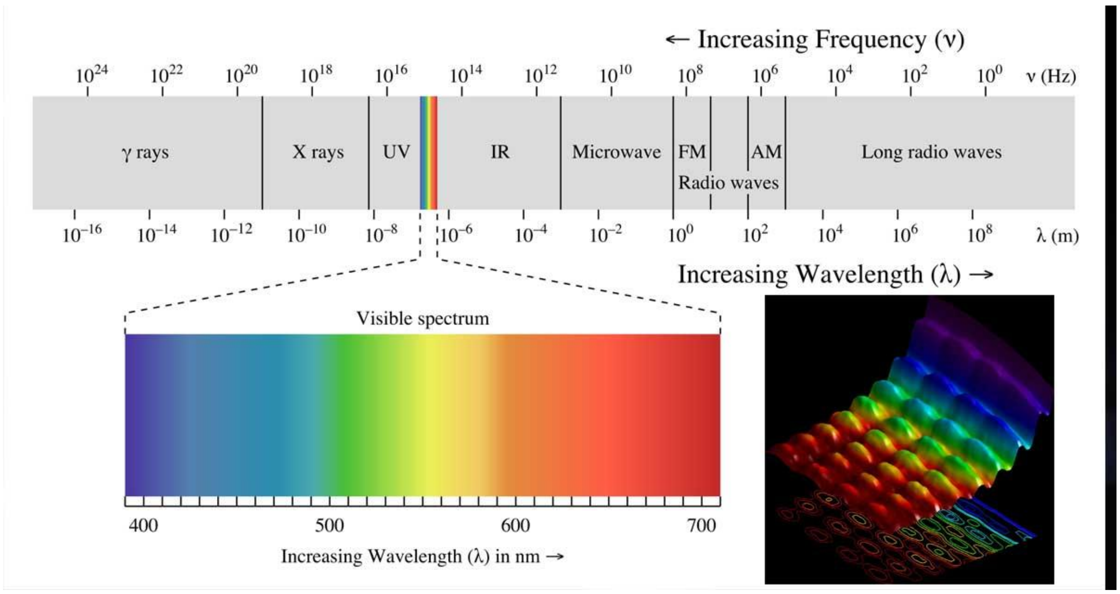


Figure 2.4 The electromagnetic spectrum (Scientifica, 2015)

2.3.1 Visible Spectrum

Visible electromagnetic radiation that is detectable by the human eye covers a small band of the spectrum between roughly 400 – 700nm as illustrated in Figure 2.4. Certain objects and features appear with different colours based off the objects ability to absorb and reflect light. When staring at the sun, it appears white, however, this is due to sunlight being made up of different colours mainly blue, red and green. Grass appears green to the human eye because plants absorb the red and blue light from the sun to use for energy and the remaining green light is reflected. As red lasers are commonly used for REDM observations, objects that absorb red light well will reflect less light which can cause error. Selecting a target that has a high reflectivity to red light will, therefore, increase accuracy.

2.3.2 Near Infrared

The near infrared (NIR) band of the spectrum covers wavelengths from 780nm to 2500nm, which means electromagnetic radiation is absorbed over this range of wavelengths. NIR radiation is emitted by anything that has a temperature and is used for vegetation analysis. The graph shown in Figure 2.5 shows that chlorophyll absorbs visible light, however, has a high reflectance for near-infrared light. Therefore, when measuring to vegetation using REDM the wavelength used is critical to the accuracy of measurement. NIR radiation is another source of possible error and any object capable of holding heat may influence the signal.

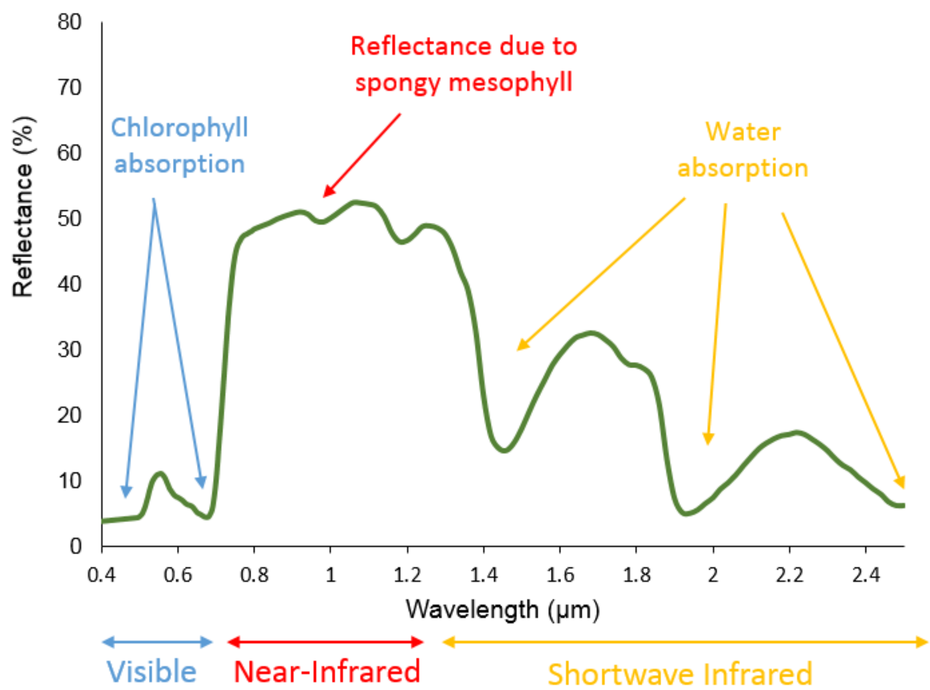


Figure 2.5 Vegetation spectral reflectance (Humboldt State University, 2018)

2.3.3 Atmospheric Refraction

As light or wavelengths pass through different mediums with varying density the result is a change in direction. The composition of the atmosphere is therefore important to realise the effect that atmospheric refraction has on REDM and to apply a relevant correction to observed distances. The refractive index is described as the ratio between the velocity of light in a vacuum to the velocity of light though a medium (Rueger, 1999).

Most modern total stations provide the option for atmospheric data to be input at the time of measurement and the refractive index is computed with the relevant adjustments made. This reduces the time spent calculating the relevant corrections and provides greater accuracy to measurements.

2.3.4 Moisture Content

To accurately address the refraction of electromagnetic radiation through a medium the refractive index of that medium must be known (Rueger, 1999). The gaseous components of the atmosphere remain relatively constant; however, the moisture content can vary considerably. Periods during or after rainfall will increase the immediate moisture content, influencing how electromagnetic radiation travels through its medium. Rain can also cause moisture to build up on the target being measured to, this may affect the reflection capability of the surface. Litchi and Harvey (2002) conducted studies on wet vs. dry surfaces and found 3mm differences between the two over a testing distance of 50m. Rueger (1999) also stated that water droplets can impede electromagnetic wavelengths direction of travel. To mitigate possible error sources from moisture it will be recommended to select an appropriate day for testing, where weather conditions are favourable and free from rain.

2.3.5 Atmospheric Corrections

EDM and REDM rely on wavelengths travelling through the atmosphere to a target. The atmospheric conditions change constantly and require corrections to minimise potential error. Atmospheric corrections are applied in parts per million (ppm) which includes corrections for temperature, air pressure and humidity.

Temperature causes the greatest error of the three, as a change in 5^{0}C can cause 1mm of error over a 100m measurement. A pressure change of 50 millibars would result in almost 1mm of error over 100m and relative humidity increase from 0-100% would result in roughly 1mm of difference over a 100m measurement. (Arseni, et al, 2015). 5 degrees C change in temperature is far more common than a 50 millibar change in pressure or an increase in humidity from 0 to 100%. Total stations allow for temperature, air pressure and humidity data to be input continuously and atmospheric corrections are automatically applied.

2.4 Error Sources

2.4.1 Angle of Incidence

Angle of incidence is defined as "the difference in angle between the ray and normal vector of the surface at the point of intersection" (Macura, 2017). This relationship can be seen in Figure 2.6. The angle that light hits a surface will affect the reflection of radiation energy off the surface. This adversely affects how much energy radiation is received by the sensor and consequently affects the deducted distance either positively or negatively.

Previous research into angle of incidences relationship with accuracy has produced differing results. Kowalczyk & Rapinski, (2014) and Lambrou and Pantazis (2010) detail that angle of incidence should be kept perpendicular to a surface or as close to this as possible to increase accuracy. Whereas, Kampouris (2011) found conflicting results, where the angle of incidence range between 30^{0} and 45^{0} produced more accurate measurements compared to zero or minimal angle of incidence. All research found that angle of incidence does effect measurements to targets making it an important element and relationship to test.

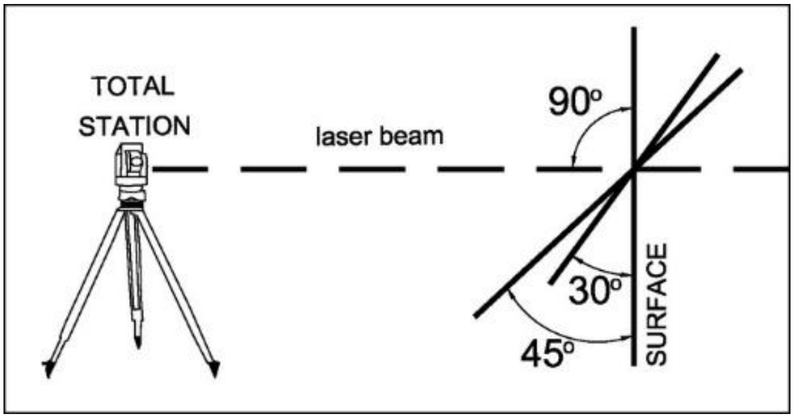


Figure 2.6 Angle of incidence (Lambrou & Pantazis, 2010)

Error can occur from the angle of incidence with the target, meaning if the target isn't perpendicular to the total station the reflection can become distorted. Angle of incidence error is similar to the concept of beam divergence. The error is not uniform and can be less or greater than the true value.

2.4.2 Beam Divergence

As a laser beam travels further and further away from its source it increases in size in the same way a flashlight beam increases in size over a greater distance (Key, 2005). This divergence creates a circular target with a measurable diameter rather than a single fixed point. As beam divergence is also a function of distance the further the target is from the instrument the greater the potential error.

Beam divergence will create minimal error when measuring a horizontal distance to a surface with a perpendicular plane, however, with the addition of angle of incidence beam divergence can cause error to distance measurements. Figure 2.7 illustrates this relationship on the following page.

As the objective is to measure to a single point rather than a large target, error can occur over long distances and significant angles of incidence, manufacturers will have different specifications for recommended maximum distances to be measured based off beam divergence. To quantify the effect of beam divergence angle of incidence up to 75° will be analysed.

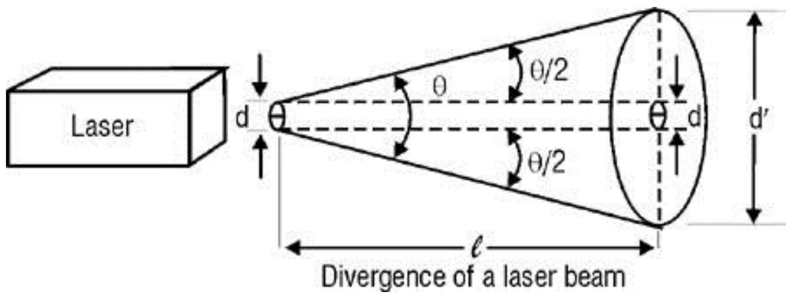


Figure 2.7 Beam Divergence (Vishnoi, 2014)

The phenomena of beam divergence brings additional sources of error into play, namely backscattering of unintended objects between the instrument and the target. Backscattering will reflect energy back to the instruments sensor and consequently affect the computed distance to the intended target. The site selection needs to provide an open area free of any obstructions such as vegetation, which could cause backscattering interference. Phase shift instruments produce a narrower beam, hence will be less effected by beam divergence compared to pulsed instruments.

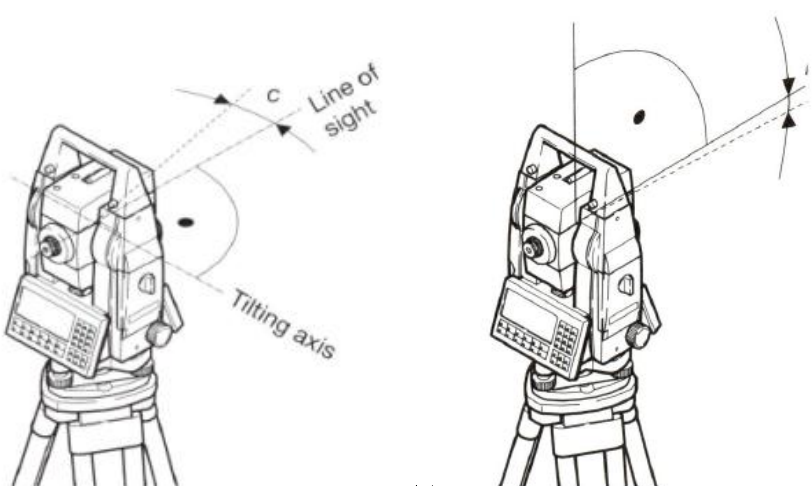
Figure 2.8 Beam divergence at different angles of incidence 0, 22.5 and 45 degrees (Kowalczyk & Rapinski, 2014)

Beam divergence is a function of incidence angle. Kowalczyk & Rapinski, (2014) found as the angle of incidence increases above 22.5^0 the effects become more severe. The beam divergence highlighted in Figure 2.8 shows little change in beam divergence between 0° and 22.5°, however, between 22.5° and 45° there is a significant change. How the divergence changes over small increments are unknown, whereas if this was known the recommendations for REDM use could be fine-tuned. To address this a major focus will be to analyse the angle of incidence range in more depth, determining if there is a critical point where accuracy is greatly affected. To do this an increment size of 5^{0} will be used with a specific focus on the range 25º - 75º.

Figure 2.8 also highlights that beam divergence is not completely circular, rather an error ellipse is produced. Kowalczyk & Rapinski's, (2014) results for an incidence angle of 45^{0} show considerable increase in divergence along the x-axis and a slight increase in divergence along the y-axis. Although only the x-axis is being rotated there is still a minor affect along the y-axis, these findings show there may be a unique relationship at play. To study this affect further, analysing the effect of combined vertical and horizontal incidence angles will be useful.

2.4.3 Collimation Error

When a total stations tilting axis is not aligned perfectly perpendicular to the telescopes line of sight it results in axial error. The tilting axis can be in error for the vertical axis, horizontal axis and the tilting axis forming three potential sources of error. To account for this both face left and face right observations can be taken with the average of the two taken to be the true value. Although this error is normally associated with recording horizontal and vertical angles it is also relevant for reflectorless measurements as the observed distance will vary depending on the reflective targets angle of incidence in reference to the measuring instrument.



(a) (b)

Figure 2.9 Horizontal (a) and Vertical (b) collimation error (gisresources, 2014)

If the target is perpendicular to the instrument there will be minimal sources of collimation error to the measured distance as seen in Figure 2.9, however, if the target is not perpendicular collimation error will affect the observed distance making it imperative to take face left and face right measurements.

Previous research into angle of incidence has generally excluded collimation error as a potential source of REDM error as it isn’t a horizontal distance error source in conventional EDM that utilises a reflective target/prism. Previous research has recommended using two face measurements, Hope (2005) recommended using both faces of the instrument for more precise reflectorless measurement, although didn’t compare single face to two face accuracy in any detail. Averaging two face measurements and comparing them to single face measurements will help clarify the effect collimation error has on REDM.

2.5 Additional REDM Error

2.5.1 Colour

The effect colour has on REDM measurements has been researched with differing results obtained. James’ (2016) research suggests colour does affect REDM measurements as different colours have different levels of spectral reflectance. The shade of colour also affects the spectral reflectance, with the colour black causing the greatest error.

The colour black absorbs more energy from the beam emitted from the total station compared to lighter colours and therefore causes scattering and interference (James, 2016). This results in error which is always longer than the true distance. Lighter more reflective colours cause less error as less beam energy is absorbed and consequently less interference making a reflective white surface the optimal choice for a reflectorless target.

2.5.2 Texture

Different textures also display different levels of reflection; materials such as glossy plastics are more reflective than sandstone brick for example. Previous research indicates that the texture does effect REDM measurements, but the amount of error caused from different textures is unclear. Research from Lambrou & Pantazis, (2010) showed a 25mm difference in measurements for paper and concrete which is quite substantial. Smooth, glossy polypropylene plastic will provide a reflective surface minimising potential error.

2.6 Summary

The literature review concludes there are numerous sources of error that can influence the accuracy of REDM observations. Angle of incidence and its effect on beam divergence has been identified as a likely source of error, particularly above 25° incidence angles. Recommendations for two face measurement have been made to reduce possible collimation error effects on reflectorless measurement, although has not been studied in any detail. Researching the effect combined vertical and horizontal angle of incidence has on REDM is unknown so determining how the beam divergence error ellipse behaves holds merit.

3 Methodology

To achieve the desired results a structured methodology in line with the project aims and previous research recommendations identified in the literature review is paramount. Minimising potential sources of error as well as selecting appropriate angles of incidence and distances will ensure reliable and relatable data is obtained. Analysing the obtained data also needs to be in line with the project aims and objectives so that relevant and valuable results are created, leading to useful conclusions.

3.1 Design Considerations

The literature review has identified multiple areas that require further research to optimise how REDM is utilised in the surveying industry. The range of incidence angle to test is of importance as it is a significant source of error that is multiplied by the phenomena of beam divergence and collimation error. A suitable site needs to be selected that allows for both horizontal collimation error and combined vertical and horizontal collimation error to be analysed.

Taking multiple face measurements to determine the collimation error also needs to be considered to mitigate potential error. Collimation error along with beam divergence are a function of distance making the range of distances to be tested important. The range of distances to analyse also must replicate the range of measurements made in typical surveying applications so that the results will be relevant.

REDM observations will require a control measurement in the form of a prism that can be compared with, this will identify any error that the reflectorless target may introduce such as incorrect alignment between the reflective target and the centre axis of the tribrach.

3.2 Preparation

3.2.1 Reflectorless Target

Critically the reflectorless target must allow for face of the target to be accurately aligned with the centre of the tribrach; to ensure reflectorless observations are to the same exact coordinates as the prism observations. The centre of the reflectorless target also needs to be 170mm above the tribrach adaptor to match the prism height of the target.

After considering the specifications for accurate measurements the reflectorless targets surface has been selected to appropriately match the specifications of reflective white Kodak Grey cards. The white side of Kodak Grey cards are 80% reflective and have a smooth surface to remove any potential error from measuring to an uneven surface. In order to adjust the angle of incidence the outline of a protractor will be attached to the base of the target so that an accurate angle of incidence can be adjusted easily in relation to the protractor.

The size of the target needs to allow for possible beam divergence and collimation error. Typically beam divergence is quantified as 2-4cm/50m, so over 60m beam divergence is equal to a maximum of 4.8cm or 48mm. To allow for this and to apply a substantial safety factor an 80mm x 80mm target will be used.

Figure 3.1 Reflectorless Target

3.2.2 Atmospheric Equipment

A Mingle BKT381 Altimeter-Barometer-Thermometer will be used to measure atmospheric pressure and temperature on site to make the relevant inputs into the total station. The Mingle BKT381 is commonly used for hiking and will serve the intended purpose as the specified temperature range is -30°C to 50°C, and pressure range 600 to 1200hPa. Relative humidity will be determined from the Bureau of meteorology website. Atmospherics will be measured at the point of emission to match the process used when the total station was calibrated.

3.2.3 Instrument Selection

The instrument selected for this study is the Topcon ES-105N. The ES-105N utilises a coaxial phase shift measuring system, with the signal source a red laser diode with a wavelength of 690nm. (Topcon, 2012) Topcon's specifications state EDM accuracy to a prism is 2mm + 2ppm and reflectorless measurement has an accuracy of 3mm + 2ppm when the reflectorless measuring range is 0.3-200m.

Figure 3.2 Topcon ES-105N Total Station (Shreeji Instruments, 2019)

3.2.4 Calibration

The ES-105N total station was calibrated at the Braeside EDM Calibration Baseline before any fieldwork was undertaken to ensure it is operating within the manufacturer's guidelines. Calibration of total stations is required for cadastral surveys annually at a minimum to maintain accuracy of equipment and distance measurements. The sequencing of measurements required for calibration is detailed in the EDM Calibration handbook. Calibration of the Topcon ES-105N showed the following errors:

Index Error = 0.91mm

Scale Error = 1.48ppm

From this the following formula should be used to adjust observed distances to a true value:

$$IC = 0.91 - 0.00148 \times L \qquad \text{(Equation 1)}$$

Where:

IC = Instrument correction (mm);

L= Distance measured (m)

Also note that these corrections should only be made to measurements to a prism, not REDM observations due to the nature of the error being a prism constant error rather than an EDM error. The calibration report can be seen in Appendix B.

Figure 3.3 - Calibration Fieldwork

3.2.5 Control

EDM observations will be taken to a prism as the control for this study. This will create the standard that REDM observations can be compared against. The same prism will be used in conjunction with the total station instrument calibration. This ensures the relationship between instrument and prism is correct and removes the possibility of any potential prism error.

A 0^0 angle of incidence and control distances will be taken each distance range (10, 30 and 60m). To remove any potential centring errors the prism will first be setup on the tribrach, then replaced with the reflectorless target by simply unlocking the tribrach and switching the targets.

Figure 3.4 Prism

EDM measurements to prisms are regarded as being more accurate than REDM which is also confirmed by the manufacturer's specifications that state the accuracy to a prism using EDM as 2mm + 2ppm, compared to REDM accuracy 3mm + 2ppm. The specifications for the prism accuracy will not be neglected and an appropriate correction to the data will be made when a 95% confidence interval is determined.

3.2.6 Site Selection

Hillview Reserve located in Dromana, Victoria was selected as a suitable testing location. The major consideration for the desired testing range was terrain that provided two differing slopes. The first testing range requires minimal slope to limit vertical angle of incidence and the second requires significant slope to maximise vertical angle. Other considerations were ease of access for the surveying equipment and occupational health and safety removing potential risk from roadways.

Figure 3.5 Test site Hillview reserve

3.3 Data Collection

3.3.1 Angles of Incidence

The angles of incidence to be analysed will be over the range 25^0 to 75^0 with 5^0 increments. There will also be a control angle of incidence of 0^0 for comparison. These angles have been chosen as previous research suggests there is minimal change in beam divergence between 0 and 25^0 and dramatic change between 25 and 60 degrees. By using increments of 5^0 this will allow for a detailed analysis to track the effect of beam divergence and determine if there is a critical point where the error significantly increases.

The first phase of research will be conducted with a zenith angle of 90^0. This will allow for an isolated focus on horizontal angle of incidence, removing any potential error from vertical angle of incidence. The second phase of research will include a vertical component of angle of incidence where the zenith angle will be significantly less than 90^0. This will allow for comparison between horizontal angle of incidence and combined horizontal and vertical angle of incidence.

3.3.2 Distance Selection

The distances selected for REDM analysis need to reflect the distances used commonly in the field with emphasis on measurements that are otherwise hazardous when using traditional EDM techniques. The width of large 2 lane roads are generally 20 metres making 30 -35 metres a common distance when the total station is setup 10 metres from the edge of the road for safety purposes. Measurements to building roofs are generally 20 – 40 metres as total stations are positioned to measure 3 of the 4 roof corners from the same setup point. Therefore, the maximum distance chosen to analyse will be 60 metres. This allows for 20 metres of play from common measurements and will produce more substantial results on the effect distance has on beam divergence. 60m also accounts for the reflectorless target size to account for possible beam divergence. The total station was calibrated over the range 7 – 919 metres so 10 metres was chosen as the minimum distance. The increments of distance to be analysed will be 10, 30 and 60 metres to cover the commonly used distances and allow for outliers.

3.3.3 Testing Procedure

Initially the instrument station and the three target stations were all setup to minimise movement and time during the testing phase. The stations were all set in a relatively straight line to mitigate any axis tilt error. Both face left and face right measurements were taken to mitigate axis tilt error and collimation error.

The three target stations were setup approximately 10, 30 and 60m from the instrument station and had their target height adjusted to ensure each target was setup at the same height as the instrument creating a zenith angle of 90^0 to the target. All observations were made in ‘fine measurement mode’ as it is stated as the most accurate by Topcon’s specifications. The following steps outline the process taken for the testing regime:

1. Position tribrach with prism at the first target station (approximately 10m from the instrument) and ensure it aligns with the line of the total station.
2. Check barometric pressure and temperature and input values into the instrument.
3. Record three observations on face left and three observations on face right to the centre of the prism (by sighting through the telescope and adjusting crosshairs to the centre of the prism).
4. Replace prism with reflectorless target.
5. Ensure the reflectorless target is aligned with 0^0 on the tribrach protractor marking, as well as aligned with the line of the total station; thus, ensuring the angle of incidence is 0^0 between the instrument and the target.
6. Record three observations on face left and three observations on face right to the centre of the target (by sighting through the telescope and adjusting crosshairs to the centre of the target outlined by a ‘+’ mark).
7. Rotate the reflectorless target horizontally to 25^0 and repeat three observations on both face left and face right
8. Rotate the reflectorless target a further 5^0 to 30^0 and repeat observations
9. Continue this process of rotating target by a further 5^0 until an angle of incidence of 75^0 is reached.
10. Replace reflectorless target with prism to ensure cross hairs are still centred on the prism.
11. Record a reflectorless measurement to the prism as a check.
12. Record a prism mode measurement to the prism as a further check.

13. Take prism and reflectorless target onto the second target station (approximately 30m from the instrument) and repeat steps 1 to 12.
14. Take prism and reflectorless target onto the third target station (approximately 60m from the instrument) and repeat steps 1 to 12.

The entire process was then repeated for significant slope. The three target stations were setup approximately 10, 30 and 60m from the instrument station and had their target height adjusted to ensure each target was setup along the same zenith angle as represented below in Figure 3.6.

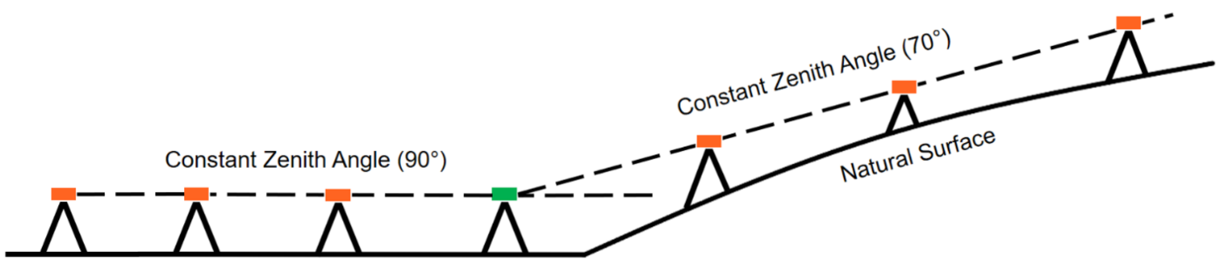


Figure 3.6 Representation of setting up targets on sloped terrain with a constant zenith angle

3.3.4 Limitations

The accuracy of the reflectorless target was not verified and although all measures were taken to ensure the highest level of accuracy was reached it can't be confirmed that reflectorless measurements are 100% comparative to their corresponding prism measurements. The angle of incidence increments were adjusted based off eyesight rather than an exact method. The results will not vary significantly from this data, however, it is something to consider.

3.4 Analysis

Microsoft excel will be used to firstly input the data obtained from fieldwork and then used to analyse the results. By graphing the measured distances over different angles of incidence compared to the true distance errors will be highlighted and trends identified for increasing angles of incidence.

Graphs will also be created for the same angle of incidence over different distances to determine trends in beam divergence and determine the optimal range for taking REDM measurements.

Overlaying particular datasets with one another will be relevant based off the results obtained. Critically analysing the data using these graphs and determining key findings will be significant in making recommendations and reaching conclusions.

3.4.1 Data Analysis

All observations will be checked for obvious blunders to verify the results are correct. Single observations will be used to form a verification plot at a 95% confidence interval to highlight any obvious trends.

The mean of the three observations for both face left (FL) and face right (FR) will be taken as the measured distance, with two face measurements being the mean from all six measurements (3 from FL and 3 from FR). The difference between prism and REDM observations were then calculated:

$$\Delta D = D_P - D_R \qquad \text{(Eq. 2)}$$

Where, ΔD = Difference between Prism and REDM distances;

D_P = Mean prism distance; and

D_R = Mean REDM distance.

ΔD was then used to determine the suitability of REDM technology. Trend lines and graphs were then produced for each face, target distance and angle of incidence to identify patterns in the datasets.

3.4.2 Statistical Analysis

Statistical analysis is critical to determine the precision of a large group of observations, to compare with manufacturer's specifications as well as identifying outliers and blunders. It is also used to highlight the confidence of observations ensuring the reliability of data. By calculating the standard deviation of the data, the variation in data from the mean can be quantified. The standard deviation can then be used to calculate the probability that an individual measurement will fall within a particular confidence interval. 1.96 standard deviations correspond to a 95% confidence interval which will be the interval calculated and utilised in the results chapter.

$$s = \sqrt{\frac{\Sigma(x-\dot{x})^2}{n-1}} \qquad \text{(Eq. 3)}$$

where s = standard deviation of the sample;

x = a value of the data set;

$\dot{x}$ = the mean value of the data set; and

n = number of values in the data set.

The standard deviation was calculated for each set of observations, however, it must be noted that only 3 individual observations were made on both face left and face right. To improve the statistical analysis a larger sample size would be ideal.

4 Results

The results for each angle of incidence, distance range and single face compared to two face measurement are outlined through this chapter. Each distance range was analysed individually to determine common trends. The trends will then be used to determine if and what effect angle of incidence and two face observations have on REDM.

4.1 Prism

The prism data will be used as a baseline for analysis as it is the most accurate form of measurement from a total station and was incorporated into the total station calibration. The manufacturer's specifications state that REDM distance accuracy as:

$$\pm(3 + 2\text{ppm} \times D)\ \text{mm},\ (0.3 - 200\text{m Range}) \qquad \text{(Equation 4)}$$

Where D = Distance (m)

This corresponds to the following:

Table 1 - Manufacturer's specifications at each target distance

Target Distance	**Manufacturer's Specifications**
10 metres	3mm + 0.02mm = 3.02mm
30 metres	3mm + 0.06mm = 3.06mm
60 metres	3mm + 0.12mm = 3.12mm

The results for the prism measurements are tabulated below in Table1. EDM was used for prism measurements to match the conditions of the total station calibration and to minimise potential error.

The prism was not subjected to angle of incidence and was kept perpendicular to the total station for all measurements to ensure accurate results. This includes the second stage of testing on the sloped terrain where the prism was vertically rotated to still be perpendicular to the total station.

The baseline true distances are shown below in Table 2. These true distances will be used as the control throughout the results section with 'error' being the distance from the true distance (mm). For simplicity the distance ranges will be designated short, medium and long range corresponding to 10m, 30m and 60m respectively.

Table 2 - Baseline True Distances and Standard Deviations

Phase 1 - Horizontal Angle of Incidence	**Phase 2 - Combined Angle of Incidence**
True Distance: 9.9952m Standard Dev: 0.00008	True Distance: 10.0243m Standard Dev: 0.00008
True Distance: 29.7776m Standard Dev: 0	True Distance: 29.8211m Standard Dev: 0.00011
True Distance: 58.9896m Standard Dev: 0.00014	True Distance: 59.4274m Standard Dev: 0.00013

4.2 Angle of Incidence

Average and exact angle of incidence trends were established for both the horizontal and combined datasets. These results were then compared against one another to determine differences and specifically identify a critical angle of incidence.

Average error was taken from all three distance ranges so that the only limiting factor was angle of incidence. This also increased the sample size, providing a more accurate representation of data by smoothing out any inconsistencies caused by outliers. Single face observations were used for this section to show an accurate representation of error trends with results from two face observations shown in the next section.

4.2.1 Horizontal Angle of Incidence

For the first stage of testing angle of incidence was isolated to horizontal only, with vertical angle of incidence kept at 0°. Vertical angle of incidence of 0º refers to the target positioned exactly perpendicular to the total station along the horizontal axis. The horizontal angle of incidence was analysed over the range 25º-75º using 5º increments across three distance ranges.

Figure 4.1 shows a clear relationship between increasing angle of incidence results and increased error. The only exception to this are the 0°-25º and 65°-70º increments. The 0°- 40° range shows minimal error averaging less than 0.4mm, this result highlights that minimal change in accuracy occurs between 0° and 40° and any of these 5 increments could produce the most accurate results.

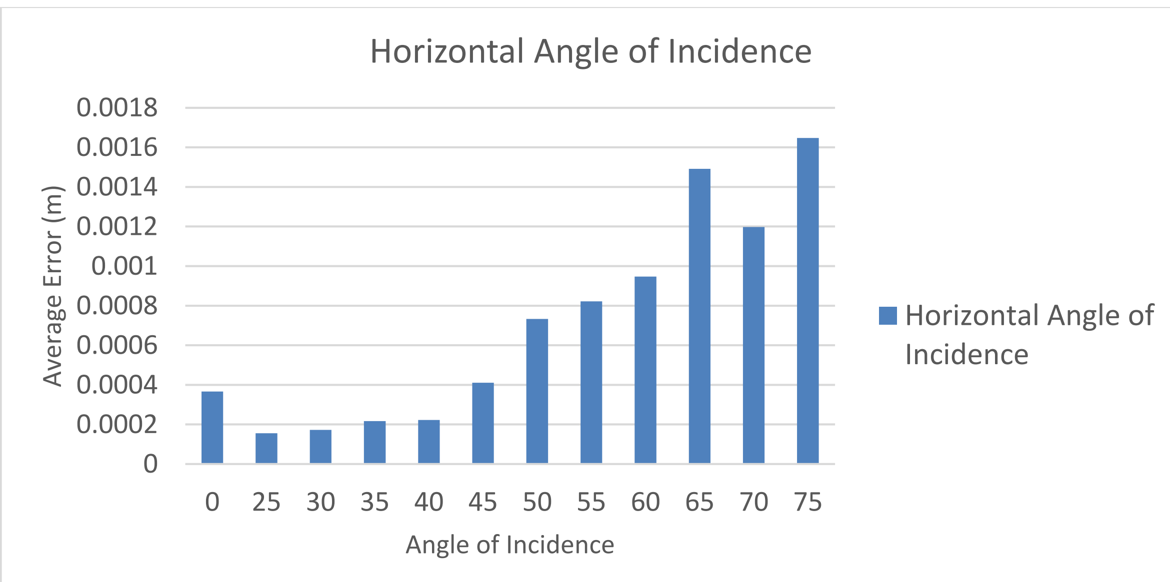


Figure 4.1 - Horizontal Angle of Incidence Error

When the angle of incidence was increased above 40° there was significant increase in error through to the maximum increment of 75°. The 65° increment showed a spike where the error was larger than the trend. This was likely caused from the extreme angle of incidence causing an outlier to significantly impact the results.

As expected, 75° angle of incidence produced the highest average error of 1.6mm for the horizontal dataset whereas 25° angle of incidence produced the lowest average error of less than 0.2mm which was unexpected. The 0° angle of incidence increment was expected to produce the best results, however, as the error was minimal for the first 4 increments the results are understandable.

4.2.2 Combined Angle of Incidence

The second stage of testing angle of incidence included a vertical angle component as well as horizontal. The vertical angle introduced was 22°30' with the results shown in figure 4.2. To keep conformity the angle of incidence was analysed over the range 25°-75° using 5° increments across three distance ranges.

Figure 4.2 shows a similar clear relationship between increasing angle of incidence results in increased error. The 0°- 35° range shows minimal error averaging less than 0.3mm, this result highlights that minimal change in accuracy occurs between 0° and 35° and any of these 4 increments could produce the most accurate results.

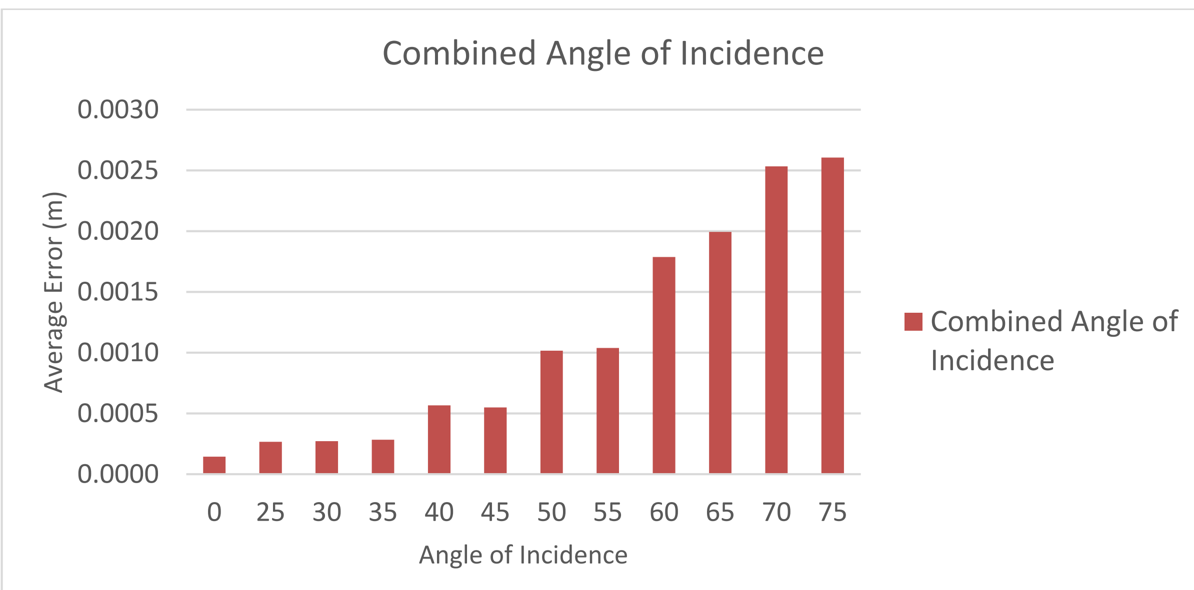


Figure 4.2 - Combined Angle of Incidence Error

When the angle of incidence was increased above 35° there was significant increase in error through to 75°. Significant increases in error occurred between 55° - 60° as well as 65° - 70°. This trend also was likely due to extreme angles of incidence causing collimation error and beam divergence to be exaggerated.

As expected, 75° angle of incidence produced the highest average error of 2.6mm for the combined dataset and 0° angle of incidence produced the lowest average error of less than 0.2mm.

4.2.3 Comparison

Comparing the two datasets shows that introducing the vertical angle of incidence increases error. Both the horizontal and combined datasets show similar trends with increasing angle of incidence resulting in increased error. The results can be broken into two halves; the first is relatively accurate measurements which occur at angles of incidence less than 45° with the second half above 45° showing increasing error. The trend for data above 45° is not linear, rather error increases at a gradient that continues to increase.

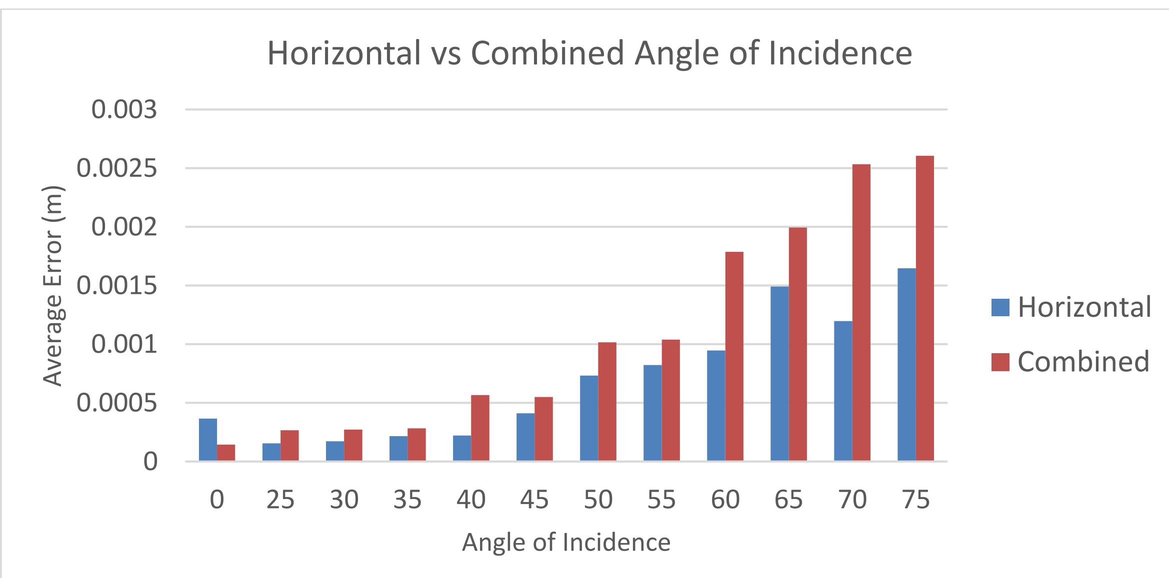


Figure 4.3 - Horizontal and Combined Angle of Incidence Error Comparison

Figure 4.3 highlights that the first significant jump in error occurs between 35 and 40° for the combined angle of incidence dataset, whereas it occurs between 45 and 50° for the horizontal dataset. This implies the critical angle of incidence will change when a combined angle of incidence is used.

The results also show that the combined dataset produced slightly more error than the horizontal dataset from 0-55°. Above 55° the combined dataset results become substantially worse than the horizontal dataset with an average increase in error of 79% across the 60-75° range. The results across this range show a dramatic increase in error which was unexpected.

The average maximum error occurred at 75° for both horizontal and combined datasets which was expected. The minimum error occurred at 0° and 25° for combined and horizontal datasets respectively.

4.3 Single Face and Two Face Accuracy

For this section the complete datasets were used to graph results. Single face data includes both face left and face right observations with only the quantity of error used rather than quantity and direction. Two face observation data was taken as the individual mean between the corresponding single face left and face right observations.

4.3.1 Single Face

Figure 4.4 shows a clear trend between increasing angle of incidence and increasing error. Every individual increment shows an increase in average error except for 0-25°. This is likely due to minimal error occurring for all increments below 35° with less than 0.25mm being the average error.

The gradient of the trend increased significantly above 45° and shows an exponential increase in error with the largest average error occurring at the 75° increment. Conversely, to the trend seen in Figure 4.1 there were no spikes in the error. This is due to both the horizontal and combined observations being used with the larger sample size smoothing over any outliers.

As expected, 75° angle of incidence produced the highest average error of 2.1mm for the dataset whereas 25° angle of incidence produced the lowest average error of less than 0.2mm. The 0° angle of incidence increment was expected to produce the best results, however, as the error was minimal for the first 4 increments the results are understandable.

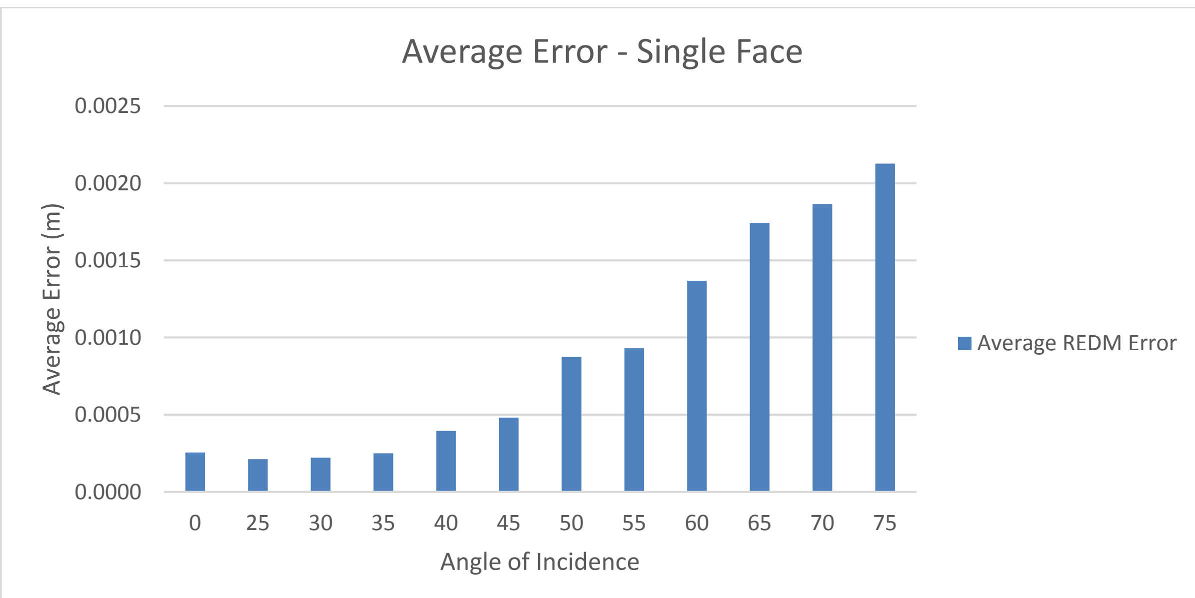


Figure 4.4 - Average error for Single Face Observations

4.3.2 Two Face

Figure 4.5 illustrates there was no obvious trend between angle of incidence and error for two face observations. This is noticeably different to the single face trend and the quantity of error has significantly improved.

The most accurate results were found to be 0-40°, similarly to single face observations. The range 45-75° showed increased error, although the degree of error was relatively minor with 75° producing average error of 0.31mm compared to 0.23mm at 0°.

The largest error was 0.51mm seen at the 65° increment, whereas the 75° increment which was expected to produce the largest error had an average error of only 0.3mm. The smallest error was seen at the 30° increment having an average of 0.15mm and all increments below 45° had minimal average error below 0.25mm.

The results were not expected and suggest if two face observations are taken the effects of incident angle error are almost completely removed. Incident angle is still causing error as 0-40° all produced more accurate results compared to 45-75°, however, the quantity of error across the entire two face dataset is minimal.

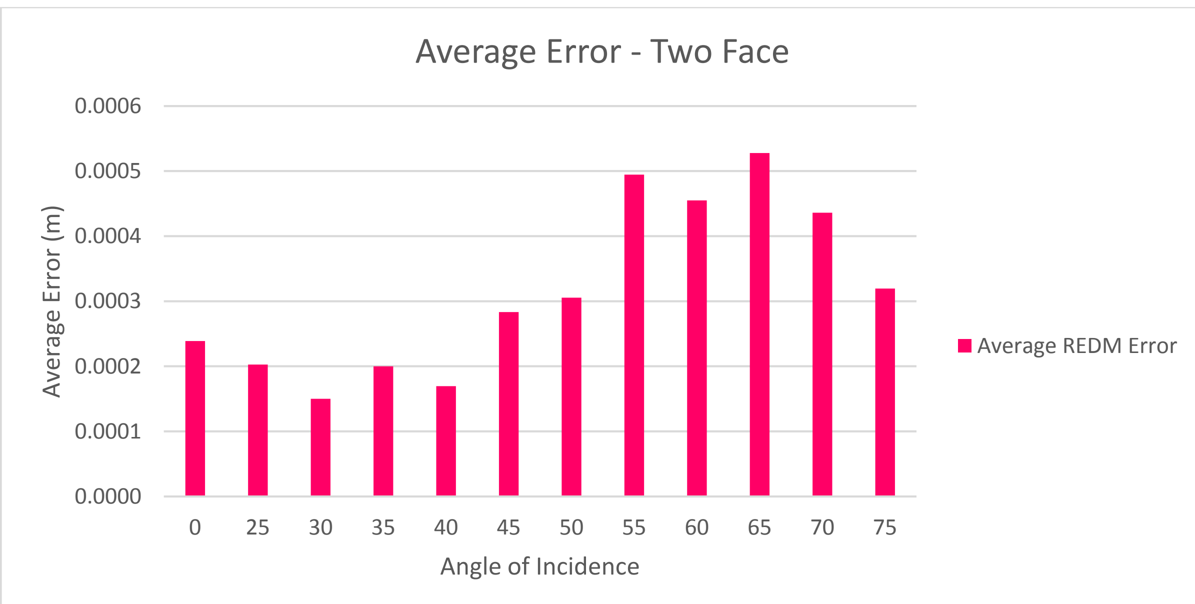


Figure 4.5 - Average Error Two Face Observations

4.3.3 Comparison

Comparing single face to two face measurements clearly shows a dramatic decrease in error when two face measurements are taken. Figure 4.6 shows the two datasets have different trends, particularly at increments above 45°. Between the 45 and 75° increment range two face measurements have less than half the error of single face measurements. When the 65-75° range is isolated we see two face measurements improve the error by 3 to 5 times. This relationship shows as angle of incidence increases it becomes more important to take two face measurements compared to single face only.

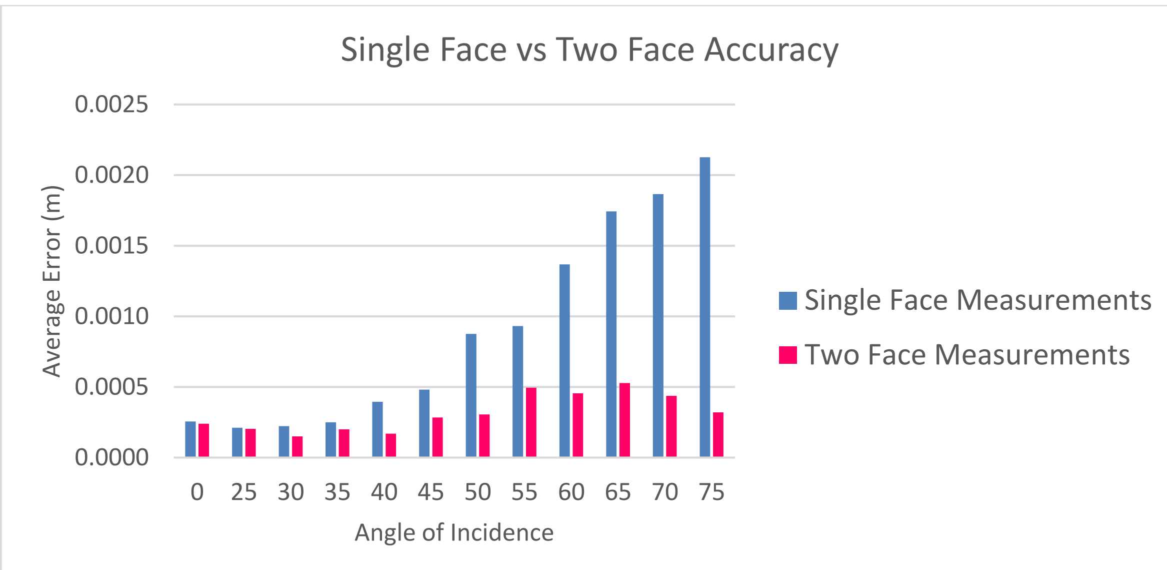


Figure 4.6 - Average Error Comparison of Single Face and Two Face Observations

Results show taking two face observations almost completely remove the angle of incidence error source for REDM. The single face and two face comparison can also be used to calculate the collimation error of the total station used. As previously stated, the error is minimal between the 0-45° range so by isolating the gradient across the 50-75° range for the single face measurements the likely collimation error was calculated as 3-4”.

4.4 Accuracy

The terms accuracy and error have been used loosely throughout this report for simplicity when comparing datasets. Prism measurements were taken as the true distance with REDM accuracy and error defined as the deviation from the prism measurements. With this said it must be noted that possible error in prism observations should also be considered. The acceptable range for an appropriate confidence interval can be calculated from:

$$-z \cdot \sigma_{\Delta D} \leq \Delta D \geq z \cdot \sigma_{\Delta D} \qquad \text{(Eq. 5)}$$

where z = Applicable confidence level constant; and

$$\sigma_{\Delta D} = \sqrt{\sigma^2{}_P + \sigma^2{}_R} \qquad \text{(Eq. 6)}$$

where σ_P = EDM error to prism accordingly to the manufacturer; and

σ_R = REDM error according to the manufacturer.

The Topcon ES-105N total station used has an accuracy of 2mm + 2ppm when measuring to prism using EDM, and 3mm + 2ppm to a reflectorless target using REDM. To calculate the 95% confidence interval in accordance with standard surveying practices, the z value used was 1.96. Table 3 below shows the standard deviations and 95% confidence interval values for the three distance ranges analysed.

Table 3 - Standard Deviations and 95% Confidence Interval Values

	10m	30m	60m
Prism Error	2.02mm	2.06mm	2.12mm
Reflectorless Error	3.02mm	3.06mm	3.12mm
σΔD	3.63mm	3.69mm	3.77mm
σΔD 95%	7.12mm	7.23mm	7.39mm

4.5 Verification Plots

To statistically analyse the datasets obtained from the fieldwork verification plots were calculated as outlined in Chapter 3.4.2. The statistical analysis took a z value of 1.96 to produce a 95% confidence interval of the manufacturer's specifications. The verification plot was firstly applied to the overall dataset and then broken down to a small angle of incidence range (0-45°) and a large angle of incidence range (50-75°). These values were chosen based off the results obtained in Chapters 4.2 and 4.3.

4.5.1 Overall

Figure 4.7 below shows the 95% confidence interval verification plot for the entire set of data. The results show there are 5 individual observations that are outside the confidence interval. This corresponds to 1.16% of individual observations being outside the 95% confidence interval from the manufacturer's specifications, with all outliers occurring at the longest target distance of 60m.

Figure 4.7 also shows a clear trend between target distance and the quantity of error. It also shows a sporadic nature of results increases with target distance, both of which were expected due to error caused by beam divergence.

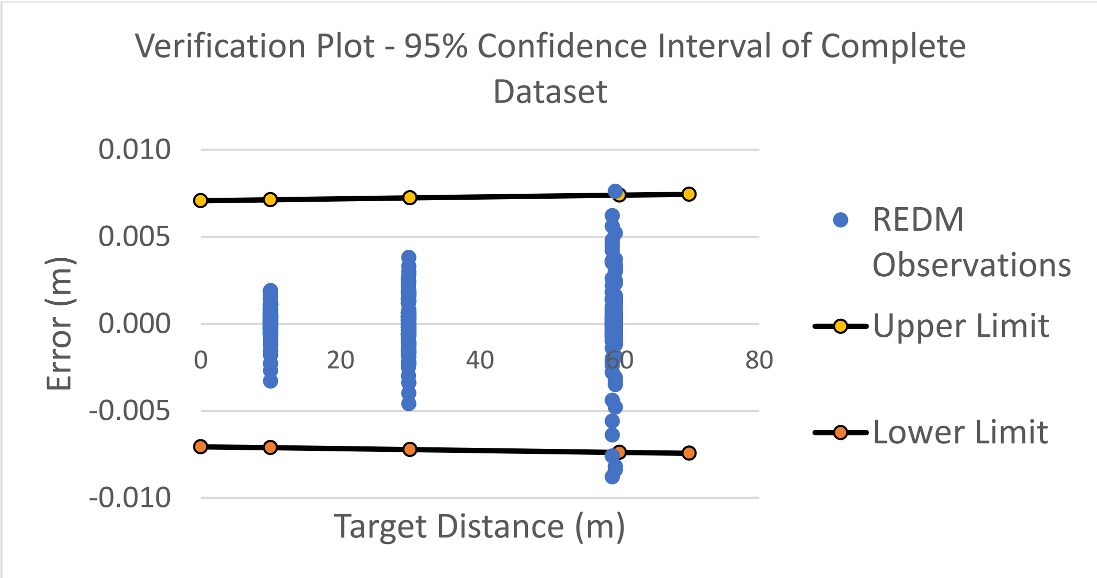


Figure 4.7 - Verification Plot at 95% Confidence Interval for Complete Dataset

4.5.2 Small Angle of Incidence

The small angle of incidence verification plot shown below in Figure 4.8 isolated observations taken to the reflectorless target when the horizontal angle of incidence was 0-45° based off previous results. The results show a dramatic increase in accuracy and precision compared to the overall verification plot. There are no 95% confidence interval outliers and the maximum error occurring across all observations was slightly above 2mm which occurred at the 30m range. These results highlight that angles of incidence greater than 45° significantly increase the potential for distance error when using REDM.

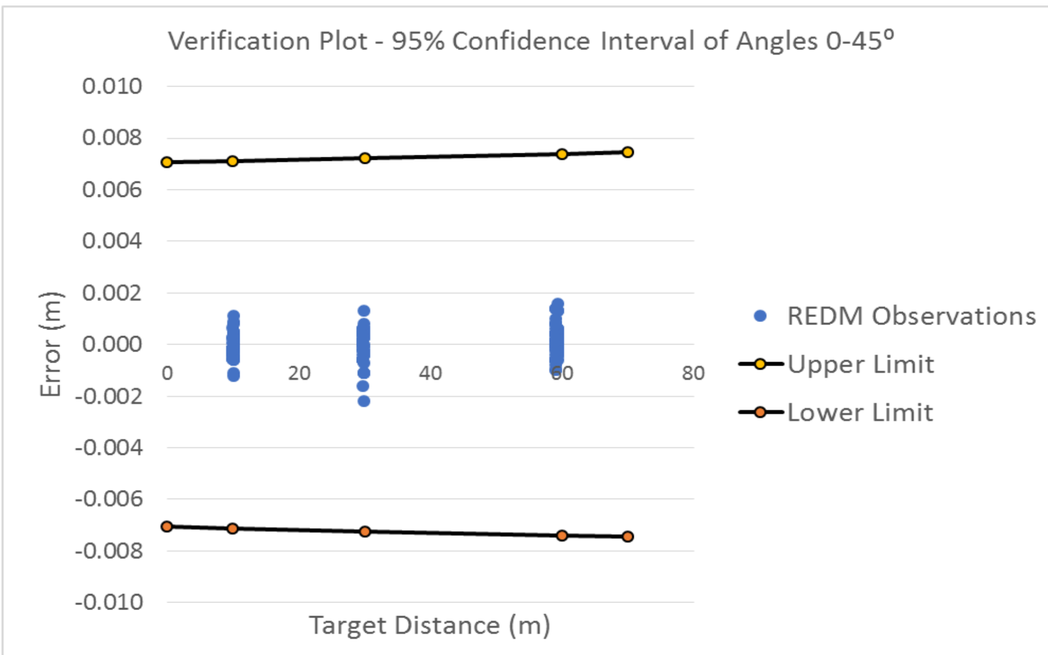


Figure 4.8 - Verification Plot at 95% Confidence Interval for Angles 0-45°

4.5.3 Large Angle of Incidence

Figure 4.9 shows the verification plot for observations taken to the reflectorless target when the horizontal angle of incidence was in excess of 45°.

If the results were isolated to observations taken at 60m with a horizontal angle of incidence between 65-75° there is a significant drop in accuracy with 14% of observations failing the 95% confidence interval. Therefore, target distances in excess of 50m and extreme angles of incidence should be avoided.

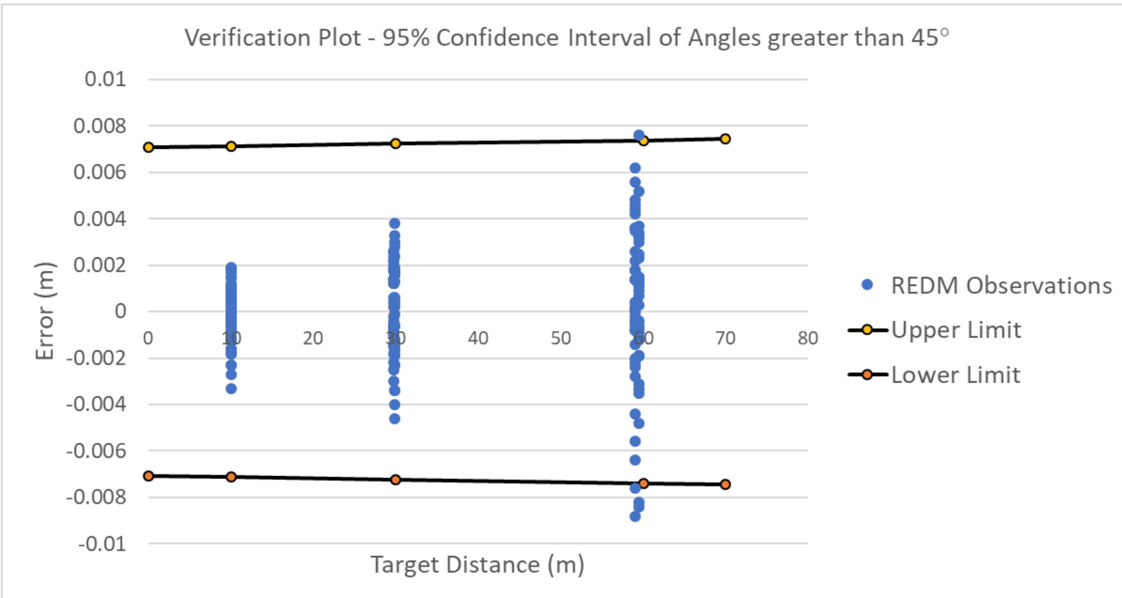


Figure 4.9 - Verification Plot at 95% Confidence Interval for Angles greater than 45°

4.5.4 Conclusion

Breaking down the overall verification plot into the two sections based off previous results showed angles of incidence greatly affects error. Angles of incidence in excess of 45° were shown to be significantly less precise and accurate compared to angles of incidence between 0-45°. The verification plot also highlighted that severe angles of incidence (65-75°) produced the least accurate results with all outliers occurring in this section.

This statistical analysis allows for key areas to focus on and identifying where outliers occur in the next stage of results where angle of incidence and single face observation trends are described.

4.6 Horizontal Angle of Incidence Trends

4.6.1 Short Range

Figure 4.10 shows the results from testing at a range of 10 metres. All measurements were within the manufacturer's specifications with the largest error of 1.3mm which occurred at 75° face left measurement. There is no obvious trend between the datasets and the results are sporadic. Face left observations begin to increase in error above 35° and above 65° there is a significant jump in error. Face right observations had less error than face left with sporadic results above 55°.

The direction of error can also be determined from Figure 4.10, which shows face left measurements being longer than the true distance and face right measurements being shorter than the true distance. Two face observations at 10m were longer than the true distance above the 45° increment and shorter below the 45° increment.

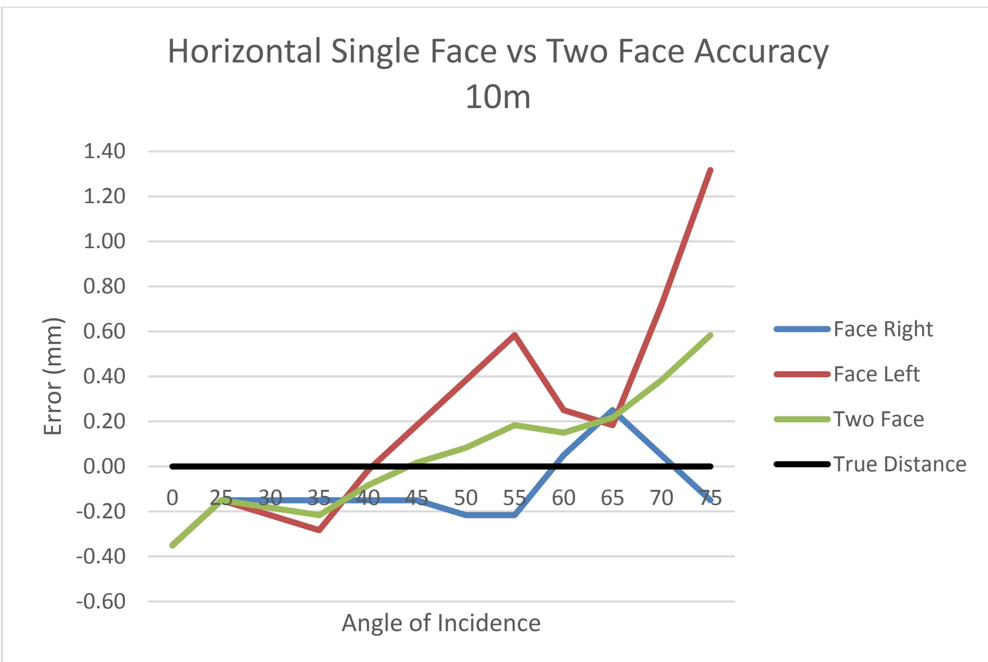


Figure 4.10 - Horizontal Angle of Incidence Short Range Trend lines

4.6.2 Medium Range

Figure 4.11 shows the results from testing at a range of 30 metres. All measurements were within the manufacturer's specifications with the largest error of -2.4mm which occurred at the 75° face right measurement. The results show a trend between face left and face right observations mirroring each other from the true distance. The face left and face right observations agree with each other relatively well from 0-45° then begin to separate away from each other with a maximum separation of 4.6mm at 75°.

Face left observations begin to increase in error above 45° and above 60° there is a significant jump in error. Face right observations had less error than face left particularly from 0-60°, although above 60° the error increased exponentially.

Figure 4.11 also shows face left measurements being longer than the true distance and face right measurements being shorter than the true distance. Two face observations at 30m were longer than the true distance across 75% of the increments analysed.

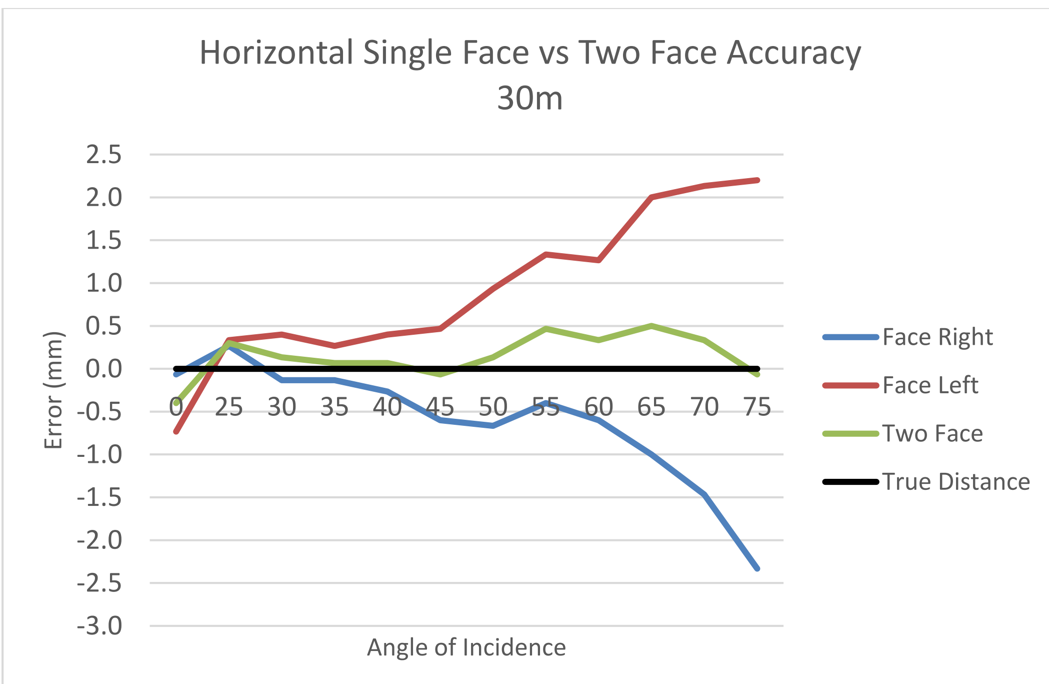


Figure 4.11 - Horizontal Angle of Incidence Medium Range Trend lines

4.6.3 Long Range

Figure 4.12 shows the results from testing at a range of 60 metres. All measurements below 55° angle of incidence are within the manufacturer's specifications, whereas angles 55°, 60° and 65° produced observations outside of the manufacturer's specifications on single face measurements. Combined or two face observations were all within the manufacturer's specifications. There is no obvious trend occurring as the angle of incidence is increased. Observations become more erratic as angle of incidence increases which is highlighted by the difference in measurements between face left and face right. At 60° there is 3.4mm difference between face left and face right and 5.4mm at 65°. Although the combined two face measurement at 75° shows no error from the true distance, the face left is 4mm different to the face right measurement. 58% of two face measurements were longer than the true distance.

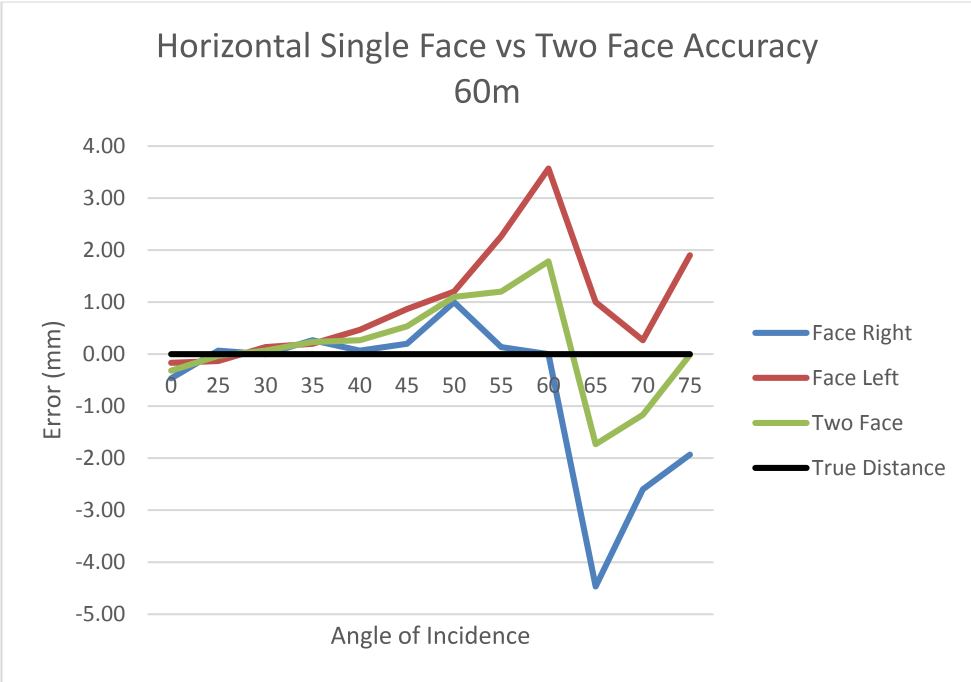


Figure 4.12 - Horizontal Angle of Incidence Long Range Trend lines

4.6.4 Range Comparison

Across the three distance ranges there was an obvious relationship between target distance and error as expected. As the target distance increased so did the error due to beam divergence and collimation error being a function of distance. The three distance ranges showed similarities regarding face left measurements being longer than the true distance and face right measurements being shorter than the true distance.

The 30m range showed a clean trend of increasing error with increasing angle of incidence whereas the 10m and 60m ranges produced sporadic results. This could be due to the 10m range being too close for beam divergence and collimation error to have an adverse effect and the 60m range being too long where beam divergence was the leading factor for error rather than collimation error. The 30m ranges clean upward trend could be due to collimation error being the leading factor for error as beam divergence doesn't have enough distance to significantly distort results.

4.7 Combined Angle of Incidence Trends

4.7.1 Short Range

Figure 4.13 shows the results from testing at a range of 10 metres. All measurements were within the manufacturer's specifications at a 95% CI with the largest error of -1.7mm which occurred at 75° face right measurement. The results show a trend between increasing angle of incidence and error, particularly above 45°. Face left observations begin to increase in error above 30° and then again above 45°. Face right observations had more error than face left with error increasing substantially above 45°.

Figure 4.13 also highlights the direction of error with face left measurements being longer than the true distance and face right measurements being shorter than the true distance. Two face observations at 10m were relatively accurate with no increment producing error greater than 0.5mm.

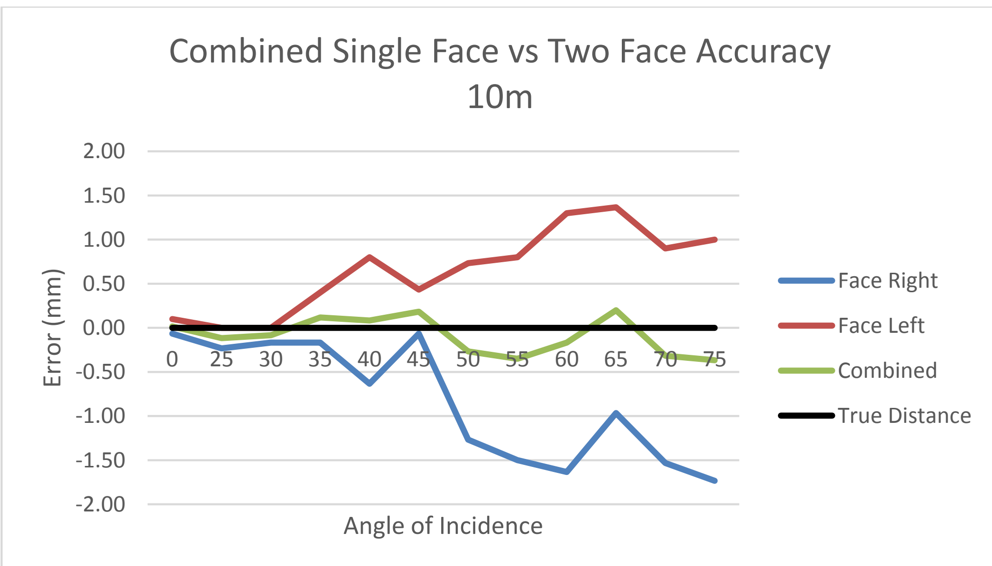


Figure 4.13 - Combined Angle of Incidence Short Range Trend lines

4.7.2 Medium Range

Figure 4.14 shows the results from testing at a range of 30 metres. All measurements were within the manufacturer's specifications at a 95% CI with the largest error of -3.7mm occurring at the 70° face right measurement. The results show a trend between face left and face right observations mirroring each other from the true distance. The face left and face right observations agree with each other relatively well from 0-35°, reasonably well from 35-55°, and poorly above 55°. The maximum separation between the two faces is 6.4mm at 70°.

Face left observations begin to increase in error above 55° where a significant jump in error occurs. Face right observations had less error than face left apart from the 70° increment. Figure 4.14 also shows face left measurements being longer than the true distance and face right measurements being shorter than the true distance. Two face observations at 30m were accurate with all increments producing error less than 0.5mm.

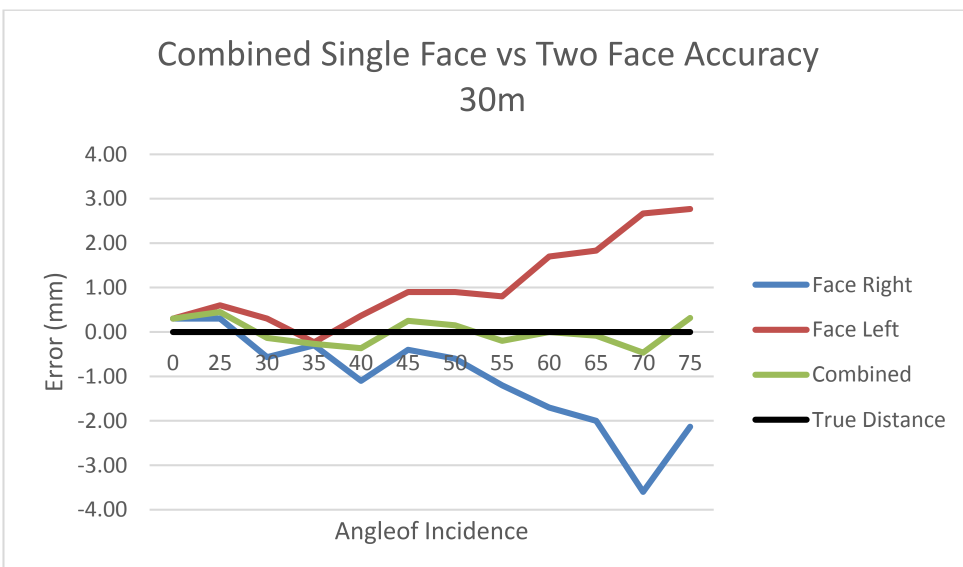


Figure 4.14 - Combined Angle of Incidence Medium Range Trend lines

4.7.3 Long Range

Figure 4.15 shows the results from testing at a range of 60 metres. All measurements were within th manufacturer's specifications at a 95% CI. All two face observations were also well within the manufacturer's specifications. The largest error was recorded as -4.7mm which occurred at the 75° face right observation.

The results show a trend between angle of incidence increasing and error increasing with face left measurements longer than the true distance and face right measurements shorter than the true distance. Observations above 55° showed a significant increase in error compared to 0-50°. At 75° there is 8.0mm difference between face left and face right observations which is a significant amount. Two face measurements showed accurate results with 45° and 55° being the only increments to have error in excess of 0.5mm, with the majority of two face measurements being longer than the true distance.

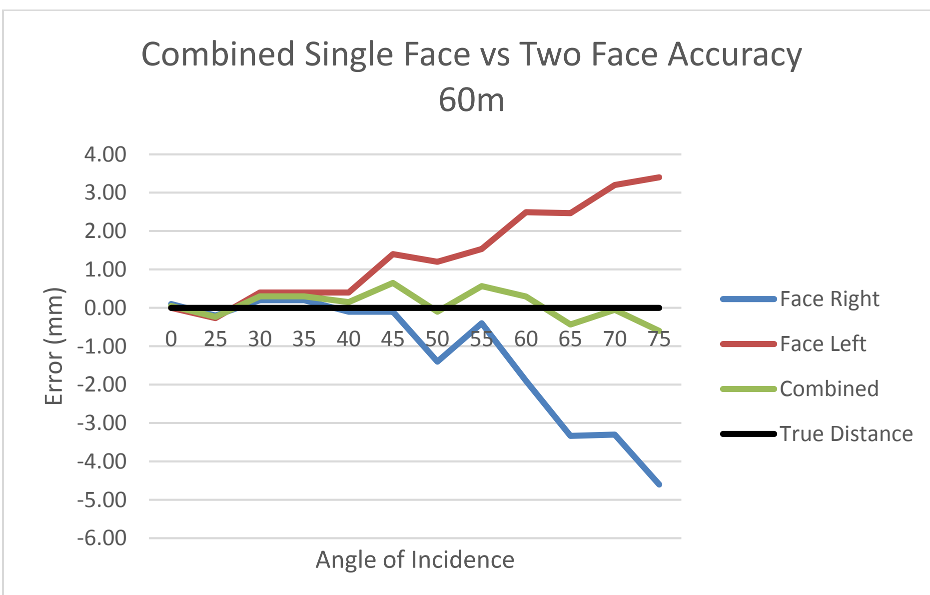


Figure 4.15 - Combined Angle of Incidence Long Range Trend lines

4.7.4 Range Comparison

Across the three distance ranges there was an obvious relationship between increasing target distance and error as expected. As the target distance increased so did the error due to beam divergence and collimation error being a function of distance. The three distance ranges showed similar trends between error increasing with angle of incidence with a constant gradient from 0-45° and an increasing gradient from 45-75°. All distance ranges produced results showing face left measurements being longer than the true distance and face right measurements being shorter than the true distance.

The trends for the combined angle of incidence graphs showed more consistency with shape compared to the horizontal angle of incidence graphs. The quantity of error was larger for the combined angle of incidence datasets across all distance ranges.

All datasets showed an overwhelming majority of face left observations being longer than the true distance and face right observations being less than the true distance, indicating collimation error is likely causing error in a similar manner for all observations. Across both datasets a slight majority of two face measurements were greater than the true distance with 51% of observations being greater, 47% less and 2% equal to the true distance.

4.8 Critical Angle of Incidence

Both trend line and tabulated data results showing the error in measurement across the angle of incidence range will be used to determine if there is a critical angle of incidence. Firstly the complete dataset of error will be generated, then secondly separated to identify specific trends.

Separating the complete dataset into horizontal and combined angle of incidence data we can gage if the introduction of the vertical angle of incidence component has any bearing on the critical angle of incidence. It will also allow for comparison on the differences or similarities in sporadic results based off the literature suggesting beam divergence creates an error ellipse rather than circular error.

4.8.1 Complete Dataset

To help determine if there is a critical angle of incidence all observation datasets were overlayed with one another highlighted in Figure 4.16 below. This set of data shows a clear upward trend as expected with significant error occurring at angles of incidence in excess of 60°.

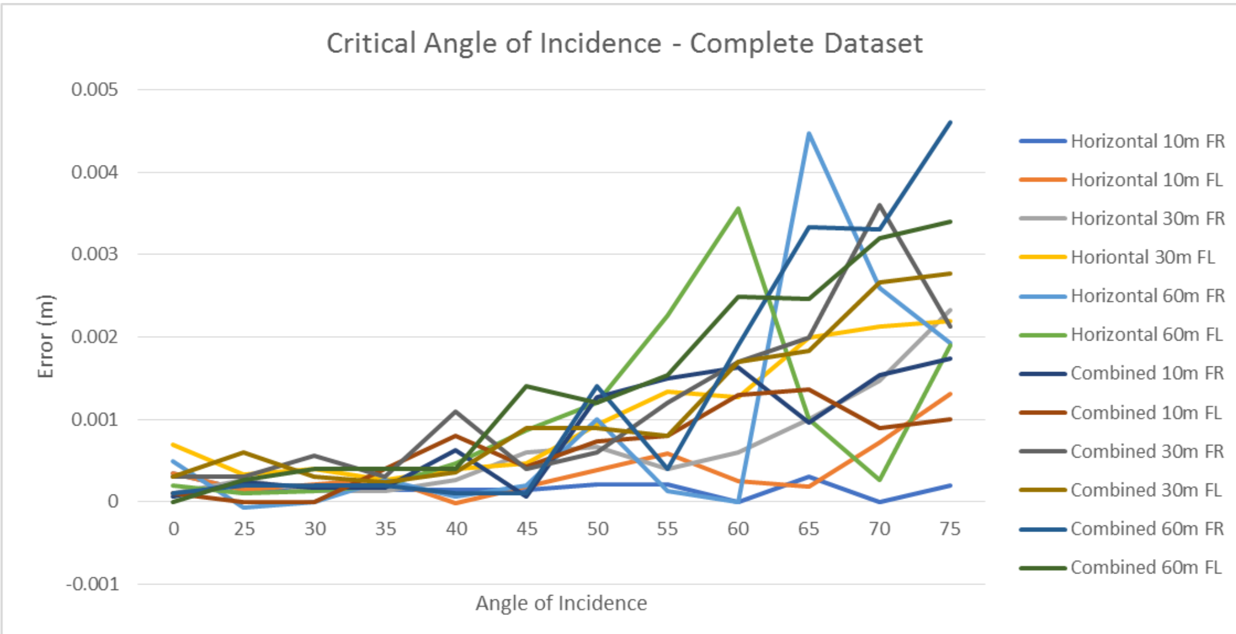


Figure 4.16 - Angle of Incidence Trends complete dataset

The following table will be used to help quantify the error relationships between the horizontal and combined datasets to determine a critical angle of incidence.

Table 4 - Accuracy and precision guidelines

	Accurate Precise	Somewhat Accurate Somewhat Precise	Not Accurate Not Precise
Range	0-1mm	1.1-3.0mm	> 3.1mm
Mean	+/- 1mm	+/- 1.1-3.0mm	>+/-3.1mm

Figure 4.16 shows over the incidence angle range 0° - 35° the results are precise and accurate as no datasets exceed 1mm error. The incidence angle range 25° - 50° shows relatively precise and accurate results as no datasets exceed 1.5mm error. Above 50° angle of incidence the results are not precise or accurate with more than half of the datasets exceeding 2mm error. Overall the results are very sporadic, particularly at angles of incidence above 50°.

Table 5 - Angle of Incidence Error to determine Critical Angle of Incidence

	10m												
	Horizontal Angle of Incidence							**Combined Angle of Incidence**					
0	-0.0004	-0.0002	-0.0006	-0.0002	-0.0004	-0.0006		-0.0002	0.0000	0.0000	0.0001	0.0000	0.0002
25	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002		0.0005	-0.0006	-0.0006	-0.0002	0.0000	0.0002
30	-0.0002	-0.0002	-0.0002	-0.0004	-0.0002	-0.0002		0.0005	-0.0005	-0.0005	-0.0001	-0.0001	0.0002
35	-0.0002	-0.0002	-0.0002	-0.0006	-0.0002	-0.0002		0.0004	-0.0005	-0.0004	0.0004	0.0004	0.0004
40	0.0002	-0.0004	-0.0004	0.0002	-0.0002	-0.0002		0.0004	-0.0012	-0.0011	0.0004	0.0009	0.0011
45	-0.0002	-0.0002	-0.0002	0.0000	0.0006	-0.0002		0.0003	-0.0003	-0.0002	0.0008	0.0003	0.0002
50	-0.0002	-0.0004	-0.0002	0.0000	0.0006	0.0004		-0.0018	-0.0018	-0.0002	0.0011	0.0000	0.0011
55	-0.0002	-0.0004	-0.0002	0.0002	0.0006	0.0008		-0.0016	-0.0013	-0.0016	0.0004	0.0008	0.0012
60	0.0000	-0.0002	0.0002	-0.0004	0.0002	0.0008		-0.0003	-0.0023	-0.0023	0.0017	0.0011	0.0011
65	-0.0002	0.0000	0.0008	0.0000	0.0004	0.0000		-0.0009	-0.0011	-0.0009	0.0011	0.0019	0.0011
70	-0.0002	-0.0006	0.0008	0.0000	0.0006	0.0015		-0.0012	-0.0027	-0.0007	0.0011	0.0005	0.0011
75	-0.0006	-0.0004	0.0004	0.0002	0.0018	0.0018		-0.0012	-0.0033	-0.0007	0.0011	0.0004	0.0015
	30m												
	Horizontal Angle of Incidence							**Combined Angle of Incidence**					
0	0.0000	-0.0004	0.0002	-0.0016	0.0000	-0.0006		0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
25	0.0004	0.0002	0.0002	0.0004	0.0002	0.0004		0.0003	0.0003	0.0003	0.0006	0.0006	0.0006
30	-0.0006	0.0004	-0.0002	0.0002	0.0004	0.0006		0.0005	0.0000	-0.0022	-0.0001	0.0005	0.0005
35	-0.0002	0.0004	-0.0006	0.0004	0.0004	0.0000		-0.0003	-0.0003	-0.0003	-0.0007	-0.0003	0.0003
40	-0.0002	0.0000	-0.0006	0.0006	0.0004	0.0002		-0.0011	-0.0011	-0.0011	0.0005	0.0003	0.0003
45	-0.0006	-0.0006	-0.0006	0.0006	0.0004	0.0004		-0.0004	-0.0004	-0.0004	0.0013	0.0008	0.0006
50	-0.0006	-0.0006	-0.0008	0.0012	0.0012	0.0004		-0.0006	-0.0006	-0.0006	0.0017	0.0006	0.0004
55	0.0002	-0.0006	-0.0008	0.0014	0.0014	0.0012		-0.0012	-0.0012	-0.0012	0.0018	0.0004	0.0002
60	-0.0006	-0.0002	-0.0010	0.0018	0.0006	0.0014		-0.0016	-0.0019	-0.0016	0.0016	0.0016	0.0019
65	-0.0006	-0.0014	-0.0010	0.0022	0.0014	0.0024		-0.0014	-0.0023	-0.0023	0.0024	0.0018	0.0013
70	-0.0022	-0.0004	-0.0018	0.0020	0.0018	0.0026		-0.0034	-0.0040	-0.0034	0.0030	0.0033	0.0017
75	-0.0030	-0.0025	-0.0015	0.0022	0.0018	0.0026		-0.0046	-0.0017	-0.0001	0.0038	0.0017	0.0028
	60m												
	Horizontal Angle of Incidence							**Combined Angle of Incidence**					
0	0.0000	-0.0008	-0.0006	-0.0001	0.0002	-0.0006		0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
25	0.0002	-0.0002	0.0002	0.0000	0.0002	-0.0006		0.0000	-0.0003	-0.0003	-0.0006	-0.0003	0.0001
30	0.0004	0.0000	-0.0004	0.0006	0.0004	-0.0006		0.0003	-0.0001	0.0004	0.0002	0.0006	0.0004
35	0.0004	0.0004	0.0000	0.0008	0.0008	-0.0010		0.0002	0.0005	-0.0001	0.0004	0.0004	0.0004
40	0.0004	-0.0006	0.0004	0.0014	0.0010	-0.0010		-0.0004	-0.0002	0.0003	0.0005	0.0006	0.0001
45	0.0008	-0.0006	0.0004	0.0014	0.0014	-0.0002		-0.0002	-0.0004	0.0003	0.0013	0.0013	0.0016
50	0.0022	-0.0006	0.0014	0.0026	0.0014	-0.0004		-0.0019	-0.0011	-0.0012	0.0013	0.0011	0.0012
55	-0.0002	-0.0008	0.0014	0.0046	0.0018	0.0004		0.0003	-0.0009	-0.0006	0.0009	0.0023	0.0014
60	0.0002	-0.0020	0.0018	0.0036	0.0035	0.0036		-0.0019	-0.0019	-0.0019	0.0013	0.0037	0.0025
65	-0.0024	-0.0088	-0.0022	-0.0014	-0.0004	0.0048		-0.0084	-0.0004	-0.0012	0.0007	0.0015	0.0052
70	0.0042	-0.0076	-0.0044	0.0044	-0.0008	-0.0028		-0.0033	-0.0035	-0.0031	0.0034	0.0030	0.0032
75	0.0062	-0.0064	-0.0056	0.0056	0.0001	0.0000		-0.0082	-0.0048	-0.0008	0.0076	-0.0007	0.0033

The datasets plotted are the average error across the six observations, with the average error of every dataset falling within the manufacturer's specifications at a 95% confidence interval. These results are pleasing as although a small percentage of individual observations do fall outside the manufacturer's specifications, even at extreme angles of incidence as high as 75° the average trend is still within the 95% confidence interval. All individual outliers occur at angles of incidence in excess of 60° as shown in Table 5.

4.8.2 Horizontal Angle of Incidence

The isolated horizontal datasets show a similar trend to that of the complete dataset. The general shape of the trend lines and quantity of error are still in proportion although there are two small differences compared to the complete dataset.

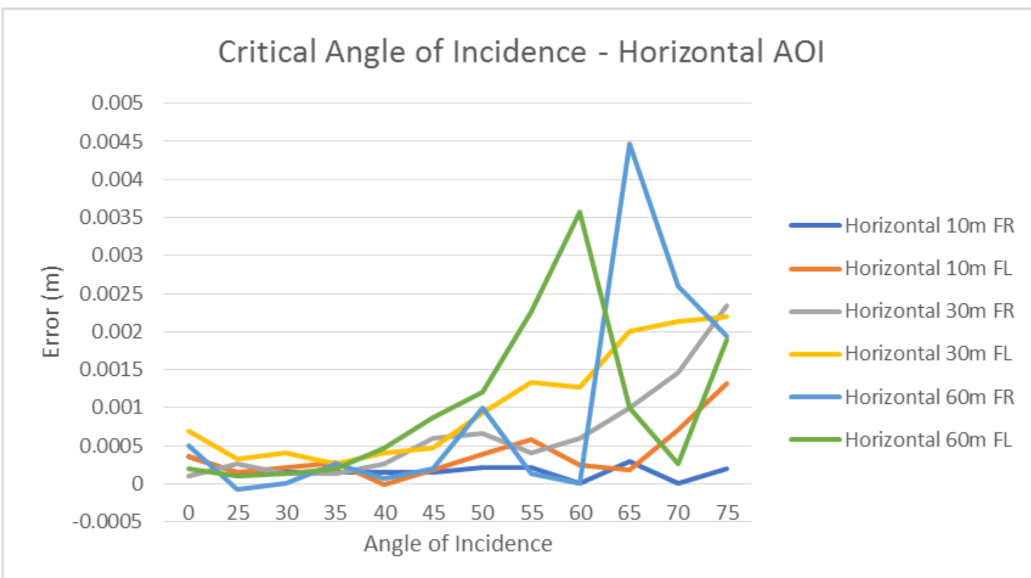


Figure 4.17 - Horizontal Angle of Incidence Trends

Figure 4.17 shows 65° had the worst performing value occurring at the 60m face right interval. This reinforces the fact that angles in excess of 60° fall outside the manufacturer's specifications. 60° produced the second worst result of 3.5mm average error, also occurring at the 60m barrier. When these two trend lines represented by green and light blue are compared with one another it can be seen at 60° the face left observation (Green) shows 3.5mm of error whereas the face right observation (light blue) shows 0mm of error. Conversely, at the 65° increment the face left observation shows 1mm of error compared to the face right observation that shows 4.5mm of error. These sporadic results indicate that beam divergence error is significantly influencing REDM distance.

Figure 4.17 also shows over the incidence angle range 0° - 45° the results are precise and accurate as no datasets exceed 1mm error. The graph appears to be broken into two halves with minimal error at incidence angles below 45° and significant error occurring at incidence angles above 45°. Incident angle range 50° - 55° shows relatively precise and accurate results as no datasets exceed 3mm error. Above 60° angle of incidence the results are not precise or accurate with 4 of 6 datasets exceeding 3mm error.

4.8.3 Combined Angle of Incidence

The isolated combined datasets show a similar trend to that of the complete dataset. The general shape of the trend lines and quantity of error are still in proportion.

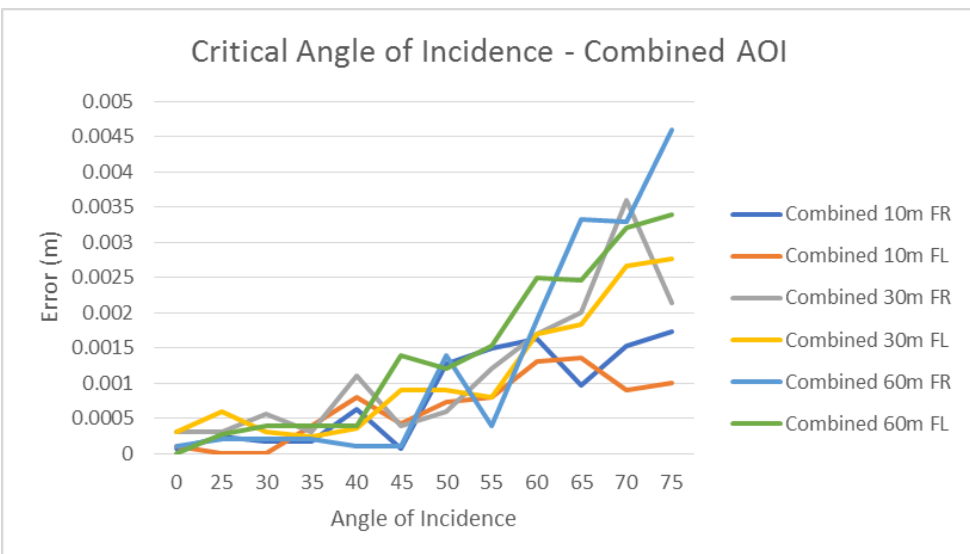


Figure 4.18 - Combined Angle of Incidence Trends

Figure 4.18 shows over the incident angle range 0° - 35° the results are precise and accurate as no datasets exceed 1mm error. The incidence angle range 40° - 60° shows relatively precise and accurate results as no datasets exceed 3mm error. Above 65° angle of incidence the results are not precise or accurate with 4 of 6 datasets exceeding 3mm error. Overall the graph shows sporadic results, particularly at angles of incidence above 50°.

When these results are compared with the isolated horizontal datasets small differences between the two are identified. Particularly, the range of precise and accurate results is wider for the horizontal datasets compared to the combined datasets.

Table 6 shows a summary of the results and indicates that 0-35° is the recommended incident angle range and 65°+ is the critical range where observations begin to fall outside of the manufacturer's specifications at a 95% CI. Typical REDM surveying applications are exposed to both horizontal and vertical incident angles which is why the lesser value of 0-35° was taken to be the recommended incident angle range.

Table 6 - Accuracy and Precision Results

	Accurate Precise	Somewhat Accurate Somewhat Precise	Not Accurate Not Precise
Horizontal datasets	0 - 45°	50 - 55°	60+°
Combined datasets	0 - 35°	40 - 60°	65+°

5 Discussion

This chapter will discuss and evaluate the results and justify the findings based off the literature. Angle of incidence, distance, two face measurement and combined angle of incidence will be analysed which will then form the basis for determining the critical angle of incidence and possible applications for reflectorless EDM. The results chapter highlighted that minimal changes in error occurred over the range of 0-35°, as such the discussion chapter will predominantly focus on angles of incidence above this range.

5.1 Angle of Incidence

For the testing procedure 12 angles were chosen to analyse; 0°, 25° - 75° increasing with 5° increments. It was expected that as angle of incidence increased so would error which was the case for the majority of the findings. Results showed that there was a definite trend between angle of incidence and error for REDM distance observations although the 0-35° range did have exceptions. As the error over the 0-35° range was relatively small any of these increments had the potential to produce the most accurate results. This range of minimal error was larger range than expected as it was believed 30° angle of incidence would produce less accurate results than 0°. These findings may explain why Khalil (2015) also found an angle of incidence of 30° produced better results than a target perpendicular to the total station. °. It must be noted however, that Khalil (2015) did utilise a longer distance range of 100m which would increase potential error from both collimation error and beam divergence.

Results for angle of incidence trends above 35° strongly supported findings from Kowalczyk & Rapinski, (2014) as well as Lambrou and Pantazis (2010), which stated targets close to perpendicular will facilitate higher accuracy. As the angle of incidence increased above 35° so did the quantity of error from the true distance. As discussed in section 2.4.2 and 2.4.3 beam divergence and collimation error are likely sources of this error.

5.1.1 Best Performing Angle

Overall the different sets of data 25^{o} was the best performing angle as it produced the best results for measurements being closest to the true value. All angles of incidence below 40^{0} performed well with the maximum error of 0.7mm occurring across all datasets. All datasets below 40^{0} angle of incidence were well within the manufacturer's specification set for the ES-105N REDM mode. It would have been expected that an angle of 0^{o} would produce the best results as the potential error from collimation error and beam divergence were at their lowest. The results partially confirmed this as although 0^{0} wasn't the best performing angle, the four smallest angles 0^{0}, 25^{o}, 30^{0} and 35^{0} performed the best across all datasets. With the mean values across 0^{0}, 25^{0}, 30^{0} and 35^{0} being so close it is reasonable to assume that if the testing was to be conducted again, one of these four angles could be the best performing angle.

5.1.2 Worst Performing Angle

The worst performing angle was 75^{o} as it produced the worst results for measurements being closest to the true value. On average there is 2.2mm of error when measuring to targets with an angle of incidence set at 75^{o}. The average error is within the manufacturer's specifications for all three distance ranges analysed. As the angle of incidence increased there was an obvious trend of increasing error with the only exception being the jump from 0^{o} to 25^{o}. This upward trend and the fact 75^{0} was the worst performing angle was expected and confirms the hypothesis that collimation error affects the distance measurement of REDM.

The results show a clear trend when the average error is taken shown in Figure 4.4, whereas when the individual datasets are graphed against each other Figure 4.16 there is a sporadic nature, particularly at angles of incidence above 40^{o}. This sporadic nature is most likely put down to error caused by beam divergence, meaning both collimation error and beam divergence are affecting the measured distance separately.

5.1.3 Conclusion

The results show there is a clear upward trend between increasing angle of incidence and measured distance error. Angles of incidence less than 35° all show minimal error and therefore any angle increment across 0-35° has the potential to produce the most accurate results for the three distance ranges analysed. The best performing angle was 25° and the worst performing angle was 75°. In every instance as incidence angle increased by a 5° increment so did the average error with the only exception occurring at the first increment from 0 to 25°. Collimation error and beam divergence are both evidently affected by angle of incidence due the constant increase in error.

5.2 Combined Angle of Incidence

Previous research analysing angle of incidence has focused on vertical or horizontal angle in isolation to establish a reliable control, however, when reflectorless EDM is actually used in a real-world situation there is almost always a horizontal and vertical component of angle of incidence.

It was expected that there would be an increase in error when a vertical component of angle of incidence was added although to what degree was unknown. The results showed two distinct findings; the combined dataset showed significantly greater error than the horizontal dataset at angles of incidence above 55^0 and the range of precise and accurate measurements was greater for the horizontal dataset.

At angles of incidence in excess of 55^0 the combined dataset results become substantially worse than the horizontal dataset with an average increase in error of 79%. Although previous research has not been conducted on combined angle of incidence, we can still rationalise these findings based off relevant information. Kowalczyk & Rapinski, (2014) showed beam divergence is not a circular error, rather it represents an error ellipse. Figure 5.1 illustrates this relationship with the 45° range shown in isolation.

Figure 5.1 - Beam Divergence Ellipse at 45° (Kowalczyk & Rapinski, 2014)

The results obtained follow this trend as when the angle of incidence became severe the relative error between the horizontal and combined datasets didn't behave in a linear fashion. The error for the combined dataset increased exponentially relative to the horizontal dataset with a maximum error increase of 110% occurring at the 75° increment. This follows the trend of an offset ellipse of beam divergence rather than a circular error. When analysing the sporadic nature of the results it was found that the horizontal dataset was in fact more sporadic than the combined dataset. This was not expected and highlights the random nature of beam divergence. Therefore, the average of the observations was used to account for potential significant outliers.

The results also found that the first significant jump in error occurs between 35 and 40° for the combined angle of incidence dataset, whereas it occurs between 45 and 50° for the horizontal dataset. This implies the critical angle of incidence is affected by the combined nature of angle of incidence rather than being isolated to only horizontal or vertical angle of incidence. As previously stated in the real-world situation reflectorless EDM is almost always subject to both horizontal and vertical angles of incidence so therefore the combined angle of incidence should be used with more weight when determining a critical angle of incidence as part of any recommendation.

5.3 Critical Angle of Incidence

The critical angle of incidence can be defined as the maximum allowable angle of incidence to stay within the manufacturer's specifications at a 95% confidence interval. Figure 4.16 and Table 5 show that the critical angle of incidence is 60^{o} for both horizontal and combined datasets.

In terms of recommended angles of incidence, the recommended allowable angle of incidence can be defined as the maximum increment before error significantly starts to increase. The summary table (Table 6) illustrated the recommended angles of incidence to be $0\text{-}45^{o}$ for the horizontal dataset and $0\text{-}35^{o}$ for the combined dataset. The $0\text{-}35^{o}$ recommended range will therefore be adopted to cover all angles of incidence present in standard surveying practice. By staying within the recommended angle of incidence range will REDM observations will be significantly more accurate and precise. This is particularly important for surveys that require a high level of accuracy and precision such as monitoring surveys.

These results expand on Khalil's (2015) findings that the range $0 - 20^{o}$ is recommended. By utilising a smaller angle increment of 5^{o} this was found to be $0\text{-}35^{o}$. Khalil (2015) analysed 0^{o}, 20^{o} and 45^{o} so this research is more specific and thorough in its findings for recommended angles of incidence.

5.4 Distance

During testing three different distance ranges were tested to determine what effect distance has on REDM. The three distance ranges tested were 10m, 30m and 60m. Based off the averaged data obtained there were no constant trends for the three distances, in fact each distance range produced a unique trend. The 10m range produced sporadic results regardless of the angle of incidence used, whereas the 30m and 60m ranges showed trends on increasing error with angle of incidence. 30m distance range data shows an almost linear relationship between angle of incidence and error.

The 60m distance range data produced a non-linear trend between increasing angle of incidence and error. When the angle of incidence reached 40^{o} there was a sharp increase in error. This trend was unique to the 60m range and suggests there could be a critical angle of incidence which is dependent on the target distance.

5.4.1 Best Performing Distance

The best performing distance was the 10m range. The 10m range showed an average error of 0.5mm from the true distance with a maximum error of 1.7mm. The 1.7mm error occurred at the maximum angle of incidence. The 10m range was expected to produce the most accurate results as there is the least possibility for atmospherics, collimation error and beam divergence to affect the distance measurements. The results confirm this as does the sporadic nature of the results at the 10m range.

5.4.2 Worst Performing Distance

The worst performing distance was the 60m range. The 60m range showed an average error of 1.2mm from the true distance with a maximum error of 4.6mm. The 4.6mm error occurred at the maximum angle of incidence. The 60m range was expected to produce the most accurate results as there is the maximum potential for atmospherics, collimation error and beam divergence to affect the distance measurements. The results confirmed this as there was an obvious trend between target distance and increasing error.

5.4.3 Conclusion

As the target distance increased the emitted laser had a longer distance to travel from the total station to the target and back. This creates more potential for temperature, pressure and humidity to affect the lasers path as well as any angular error from collimation to be exaggerated. Beam divergence is also a function of distance which allows for a greater potential for error as the target distance increases.

5.5 Single Face Measurement

The results show 91% of face left observations had a positive error meaning the distance measured was greater than the true distance. Conversely, 87% of face right observations had a negative error less than the true distance. These results indicate there is a direct relationship between face left and face right observations caused from collimation error. Combining these 51% of all observations were longer than the true distance and 47% were shorter with the excess 2% being equal to the true distance. Lambrou and Pantazis (2010) and Khalil (2015) found similar results where observations were longer than the true distance. This may be due to previous research neglecting two face observations with collimation error causing measurements to be slightly greater than the true value. Analysing the beam geometry of the laser may also explain this anomaly.

Single face results were accurate at incident angles below 35°, whereas error began to significantly increase above this incident angle. These findings agree with previous research from Kowalczyk & Rapinski, (2014) who found smaller incident angles produced more accurate REDM results. Angles in excess of 35° can therefore be considered as non-reliable for high accuracy work.

5.6 Two Face Measurement

During testing both face left and face right observations were taken to each target to determine if collimation alignment would have an effect on REDM distance measurements. By averaging the face left and face right observations at each interval this was used to create a new dataset called ‘two face’. Two face measurements were then compared against the original single face measurements to determine if the error was reduced. The results showed there was a definitive improvement in every case with two face measurements. This relationship was expected although the degree of improvement was significantly higher than first thought.

Key (2005) recommended taking two face observations for REDM rather than only taking a single face observation. Her recommendation although not based off any research proved to be correct as the results showed two face observations significantly improved accuracy. The degree of improvement across all incident angles suggests that taking two face observations almost completely removes error caused from angle of incidence. These findings are extremely relevant to the surveying profession and strongly indicate that two face observations should be taken whenever a high level of accuracy is required using REDM.

As the angle of incidence increased, so did the amount of improvement when comparing single face with two face measurements. Isolating data with an incident angle above 60^0 showed 0% of two face observations were outside of the manufacturer's specifications 95% CI compared to 13% of single face observations. It can therefore be concluded that collimation error has a significant effect on REDM distance measurements, and two face measurements should be taken when a high level of accuracy is required.

5.6.1 Beam Geometry

To further understand the mathematics behind the results the lasers beam geometry was investigated. 53% or the two face observations were longer than the true distance. These results agree with previous findings from Lambrou and Pantazis (2010) and Khalil (2015), where most measurements were longer than the true distance. By analysing the beam geometry of both the face left and face right observations we can see a possible reason for this.

Figure 5.2 is a direct representation of the results from the 30m barrier taken for the horizontal dataset with the 4 seconds of collimation error calculated based off the average error across the dataset. The face left observation beam has to travel a further 2.1mm to reach the target than the face right observation. Although this doesn't sound like much, if this representation was taken for an observation with 20 second collimation error to a target distance of 100m utilised by Khalil (2015) this would result in the face left observation having to travel a further 4cm. As the beam also must return to the receiver as well this actually corresponds to the face left beam having to travel 8cm further.

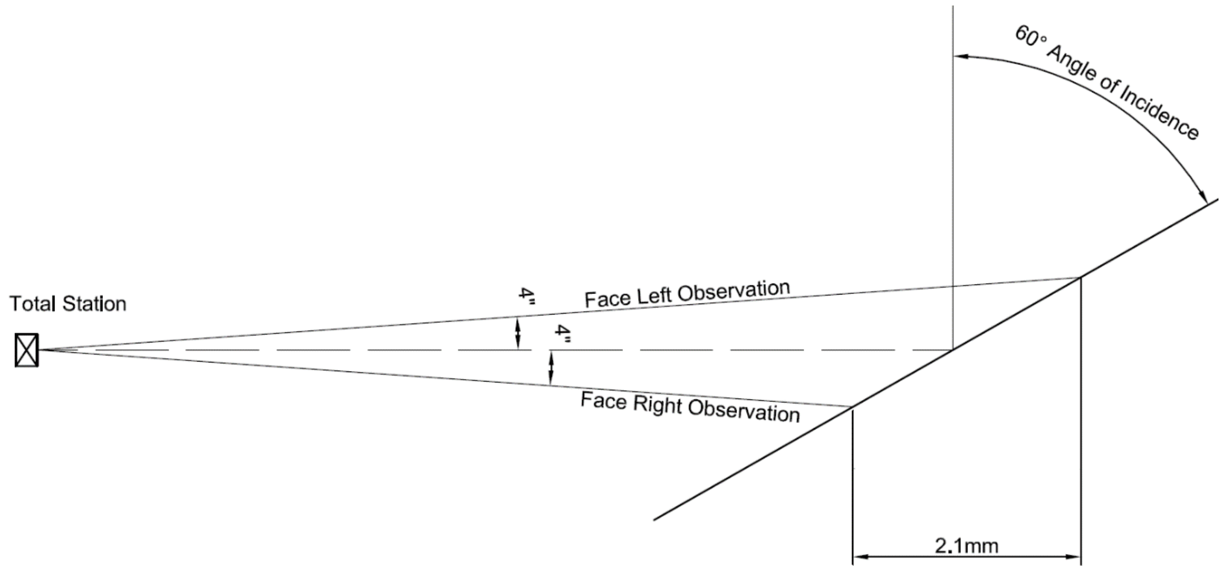


Figure 5.2 - Beam Geometry

In this example the angle of incidence is also 8 seconds greater for the face left observation compared to face right. As this report has established at extreme angle of incidence small increases begin to produce significant error. The extra 8 seconds of angle of incidence may also be a possible reason why most observations are longer than the true distance. The total station records the distance based off the returned laser energy, which is adversely affected by backscattering. Backscattering effects are more severe at extreme angles of incidence highlighted in Figure 5.3.

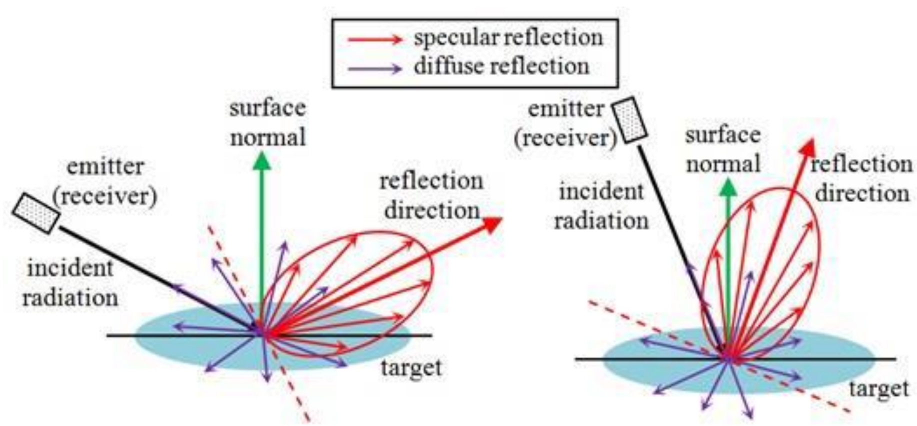


Figure 5.3 - Representation of specular and diffuse reflection with respect to incident ray (Tan & Cheng, 2017)

5.7 Applications

Currently REDM technology has a wide range of uses in the surveying industry although current legislation prohibits its use for cadastral surveys. Based off the findings it is a possibility REDM will one day be used for cadastral surveys, especially if a 3D cadastre is developed in the future.

Currently REDM is predominantly used for detail surveys as well as engineering surveys. Engineering surveys include monitoring, as-built and set-out surveys which all require a high level of accuracy. The findings from this dissertation will help to improve surveying practice for these specific surveys, in particular two face observations should be recorded rather than single face and angles of incidence in excess of 35° should be avoided

In Victoria the Surveying (Cadastral Surveys) Regulations 2015 – Regulation 7 states that a licensed surveyor must ensure “all lengths are measured or determined to an accuracy of 10 millimetres + 60 parts per million”. (Victorian Consolidated Regulations, 2015) 100% of REDM observations taken were well within this level of uncertainty and the manufacturer’s specifications for REDM convincingly meet this level of accuracy. As REDM technology also continues to advance it may become a reality in the future where legislation is changed to allow REDM technology to be used for cadastral surveying.

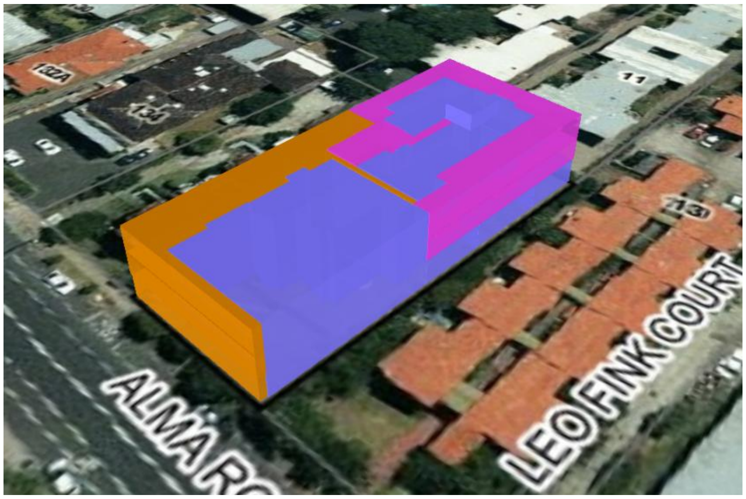


Figure 5.4- 3D Cadastre Prototype(SPEAR, 2018)

A Victorian company SPEAR is currently building a 3D cadastre prototype for a block of units outlining where each unit legally starts and ends illustrated in Figure 5.4. Measuring to boundaries between stories would be difficult and hazardous using traditional EDM, whereas REDM would be a safer and more efficient option. If a 3D cadastre does become a reality in Australia this may be the steppingstone for legislation to be changed and allow REDM technology to be used in cadastral surveying.

6 Conclusion

The research project set out with the aims of examining the effect angle of incidence has on REDM observations, specifically analysing combined angle of incidence, two face observations and determining if there is a critical angle of incidence. The research was justified off the lack of previous research into combined angle of incidence and two face REDM observations.

A testing regime was incorporated into the research testing a large variety of incident angles across three distance ranges of 10, 30 and 60 metres. The total station used for the research was a TOPCON ES-105N that utilises a coaxial phase shift red laser diode with a wavelength of 690nm. At each distance range incident angles 0, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70 & 75^{o} were analysed to accurately model how error behaved over small increments. The reflectorless target was crucial to the research and was constructed to match the properties of Kodak grey cards reflective side. This was done to match the conditions used in the manufacturer's specifications for the TOPCON ES-105N. The reflectorless target also required a paper protractor to be attached to alter the angle of incidence progressively and accurately.

Angle of incidence was found to be a significant influencer on REDM error as expected. Combined angle of incidence followed a non-liner increase in error which was in line with the literature suggesting beam divergence produces an error ellipse rather than a circular error. Two face measurement results also highlighted that accuracy can be dramatically increased by taking two face observations compared to isolated single face observations. As expected, it was also found increasing the target distance does increase the potential of REDM error.

A key finding from the research was determining the critical angle of incidence. The critical angle of incidence was found to be 60^{o} as 100% of observations at 60^{o} or below were within the manufacturer's specifications using a 95% confidence interval. Conversely incident angles above 60^{o} had a 13% chance of falling outside the 95% confidence interval. Recommended angles of incidence were also determined with 35^{o} being the maximum recommended angle of incidence. This was based off the combined dataset results to match a real world REDM use scenario.

The second key finding from the research project was the degree of improvement two face observations make to REDM accuracy. Two face observations almost completely remove error associated with angle of incidence. At severe angles of incidence error was reduced by 3 to 5 times, hence two face observations should always be utilised when high levels of accuracy are required. This is particularly useful for monitoring surveys where traditional EDM cannot be used due to safety or access constraints such as working on a cliff face.

Overall, all objectives outlined in the dissertation were achieved. A detailed methodology guided by the literature provided the backbone for analysis and justification of errors associated with angle of incidence. Ultimately REDM technology was found to be reliable within the tested scope utilised in the research. Benefits to accuracy, safety and efficiency towards the surveying profession can be taken from this research as the possibilities of REDM technology continue to be unearthed.

6.1 Recommendations

The findings from this dissertation indicate that REDM technology can meet the manufacturer's specifications at a 95% confidence interval, with the only exception being extreme angles of incidence in excess of 60°. As the research conducted was only focused on a small section off possible scenarios found in field environments, these results should not be taken beyond the scope given. To ensure accurate results, REDM observations should:

- Record two face observations with the average distance taken rather than using single face observations in isolation;
- Avoid angles of incidence above 35°; and
- A maximum range limit of 60m

Permitted applications of REDM technology include monitoring, as-constructed, detail and set-out surveys. Although REDM is currently not applicable for cadastral surveys as technology advancements continue and the possibility of a 3D cadastre coming online in Australia that may one day change.

Angles of incidence above 35° should be avoided for high accuracy work and 65° is the critical angle of incidence for the Topcon ES-105N, where measurements begin to consistently fall outside of the manufacturer's specifications.

6.2 Further Research

This research was only conducted over a relatively short baseline of 60m, there is potential for further research over longer distances to better understand the relationship between critical angle of incidence and distance. Beam divergence and collimation error may also influence reflectorless targets differently across longer baselines.

Further research could also test different materials rather than the Kodak grey card. This would expand the understanding and knowledge base of critical angle of incidence for surveyors. REDM is generally used when measuring to buildings so focusing on a range of building materials to test would be valuable.

Another avenue for further research would be to analyse two face observations and combined angle of incidence with a different total station that utilises pulse distance technology rather than phase shift. Different total stations will also have differing specifications for the laser diodes they utilise for REDM. More modern total stations could also be analysed as technological advancements may have significantly improved error sources from angle of incidence.

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Appendix A

Project Specification

ENG4111/4112 Research Project

Project Specification

For: Brent Martin

Title: Effect angle of incidence has on reflector-less EDM measurement and determining if there is a critical point

Major: Land Surveying

Supervisor: Jessica Smith

Sponsorship: N/A

Enrolment: ENG4111 – ONL S1, 2019

ENG4112 – ONL S2, 2019

Project Aim: To investigate the effect angles of incidence has on reflector-less EDM measurements and ultimately determine if there is a critical point where the accuracy of measurement decreases at a rapid rate.

Programme: Version 1, 12th March 2019

1. Research the background information relating to reflector-less EDM measurement, in particular the effect angle of incidence has on accuracy.
2. Perform a site calibration on TOPCON total station prior to fieldwork
3. Take EDM measurements with total station, both prism and non-prism measurements. The same measurement will be taken numerous times with the target set at different vertical angles between 0^0 and 90^0 .
4. Instrument height will be matched to target height so that the vertical angle is exactly 90^0 , then the angle of incidence adjusted by angling the target in 5^0 increments progressively.
5. The results will be analysed in detail with the different angles of incidence compared with one another.
6. Graph/model the results to determine if the increasing angle of incidence affects the accuracy in a linear, exponential or other fashion.
7. Evaluate the results and provide guidelines for acceptable angles of incidence for reflector-less measurements.

If time permits:

8. Repeat the process with a different type of material as the target and compare results.

Appendix B

Calibration Report

Job Identification: Calibration 2019

EDM Calibration Certificate

This report has been generated by program Baseline Version 6.0.0.5, developed by the Western Australian Land Information Agency.

Use of this application elsewhere should rely on baseline distances certified by the relevant authority.

Observation Date: 12/04/2019
Instrument Operator: Brent Martin
Computation Date: 6/05/2019
Computation Time: 4:23:54 PM

Equipment Details

Instrument Owner: Gary Martin
Owner Address: Shop 12, 143 Point Nepean Rd Dromana
EDM Instrument Make: Topcon
EDM Instrument Model: ES 105N
EDM Serial Number: GZ5309

Reflector Make: Topcon
Reflector Model: Topcon
Serial Number: HMF04
Reflector Constant: 0 mm

Baseline Details

Name Braeside
Authority: Geodetic Survey - SGV
Authority Address Level 11, 2 Lonsdale Street, Melbourne 3000
Location: Braeside Park, Lower Dandenong Road, Braeside
Last calibration Date: 29/11/2018

This baseline consists of known lengths, which are the certified distances between the pillars of the baseline. All certified distances are on the same horizontal plane and on the same vertical plane running through the first and last stations.

The baseline distances are traceable to the national standard for length in accordance with the scope of the Surveyor-General Victoria NATA accreditation.

Instrument Correction (IC) in mm (to be added to the instrument reading)

IC = **0.91 - 0.00148 L**

Where L = distance in metres

The reflector constant has been entered into the instrument

CYCLIC ERRORS ARE INSIGNIFICANT

Calibration Parameters	Value	Uncertainty(95%)
Index	0.91 mm	± 0.61 mm
Scale	(-1.48×10^{-3} L) mm	±(1.74×10^{-3} L) mm
	where L = length in metres	

The instrument correction has been determined from measurements in the range of 7 to 919 metres

Job Identification: Calibration 2019

EDM Calibration Certificate

This report has been generated by program Baseline Version 6.0.0.5, developed by the Western Australian Land Information Agency.

Use of this application elsewhere should rely on baseline distances certified by the relevant authority.

Uncertainty of the Instrument Correction

Minimum standard for the uncertainty of calibration of an EDM instrument is ±(4.00 + 20.00 x 10^{-3} L) mm as described in terms of Recommendation No.8 of the Working Party of the National Measurement Institute on the calibration of EDM Equipment of 1 February, 1983. All uncertainties are specified at the 95 % confidence level. A coverage factor of 2 has been used for the uncertainty computations.

Uncertainty of instrument correction: ±(0.61 + 1.74 x 10^{-3} L) mm where L = length in metres

Distance (metres)	Instrument Uncertainty (mm)	Minimum Standard (mm)	Comparison Test
50	±0.70	±5.00	PASS
100	±0.79	±6.00	PASS
300	±1.14	±10.00	PASS
500	±1.49	±14.00	PASS
700	±1.83	±18.00	PASS
900	±2.18	±22.00	PASS

This instrument satisfies the National Measurement Institute standards.

First Velocity Correction (Atmospheric Correction)

The atmospheric correction dial of the EDM instrument was set for all observations. Therefore the observed distances have already been corrected for atmospheric effects.

To obtain a regulation 13 Certificate for the purpose of legal traceability to the Australian standard of length contact the Verifying Authority responsible for length measurements in your State or Territory.

The calibration of the EDM Instrument has been carried out according to QP-0061 Calibration and Standardisation of Electro-optical Distance Meters (EDM) maintained by the Office of Surveyor-General Victoria.

Data entry by: BRENT MARTIN	Results checked by: GARY MARTIN
Position: SURVEY ASSISTANT	Position: PRINCIPAL
Signature: [signature]	Approved Signatory: [signature]
Date: 6/5/2019	Date: 6-5-2019

Appendix C

Summary of Data Analysis

Horizontal Angle of Incidence

Face Left Observations at 10m range							
	TOPCON	ES-105N					
	Prism =	9.9952					Average
Angle	Dist	Mean (ẋ)	x-ẋ	(x-ẋ)²	Std Dev	S at 95%	Diff to prism
	9.9950		0.0002	4.0E-08	0.00014	0.00028	
0	9.9948	9.9948	0.0000	0.0E+00	0.00000	0.00000	-0.0004
	9.9946		-0.0002	4.0E-08	0.00014	0.00028	
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
25	9.9950	9.9950	0.0000	0.0E+00	0.00000	0.00000	-0.0002
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
	9.9948		-0.0001	1.8E-08	0.00009	0.00018	
30	9.9950	9.9949	0.0001	4.4E-09	0.00005	0.00009	-0.0002
	9.9950		0.0001	4.4E-09	0.00005	0.00009	
	9.9946		-0.0003	7.1E-08	0.00019	0.00037	
35	9.9950	9.9949	0.0001	1.8E-08	0.00009	0.00018	-0.0003
	9.9950		0.0001	1.8E-08	0.00009	0.00018	
	9.9954		0.0003	7.1E-08	0.00019	0.00037	
40	9.9950	9.9951	-0.0001	1.8E-08	0.00009	0.00018	0.0000
	9.9950		-0.0001	1.8E-08	0.00009	0.00018	
	9.9952		-0.0001	1.8E-08	0.00009	0.00018	
45	9.9958	9.9953	0.0005	2.2E-07	0.00033	0.00065	0.0002
	9.9950		-0.0003	1.1E-07	0.00024	0.00046	
	9.9952		-0.0003	1.1E-07	0.00024	0.00046	
50	9.9958	9.9955	0.0003	7.1E-08	0.00019	0.00037	0.0004
	9.9956		0.0001	4.4E-09	0.00005	0.00009	
	9.9954		-0.0003	1.1E-07	0.00024	0.00046	
55	9.9958	9.9957	0.0001	4.4E-09	0.00005	0.00009	0.0006
	9.9960		0.0003	7.1E-08	0.00019	0.00037	
	9.9948		-0.0006	3.6E-07	0.00042	0.00083	
60	9.9954	9.9954	0.0000	0.0E+00	0.00000	0.00000	0.0003
	9.9960		0.0006	3.6E-07	0.00042	0.00083	
	9.9952		-0.0001	1.8E-08	0.00009	0.00018	
65	9.9956	9.9953	0.0003	7.1E-08	0.00019	0.00037	0.0002
	9.9952		-0.0001	1.8E-08	0.00009	0.00018	
	9.9952		-0.0007	4.4E-07	0.00047	0.00092	
70	9.9958	9.9959	-0.0001	4.4E-09	0.00005	0.00009	0.0007
	9.9966		0.0007	5.4E-07	0.00052	0.00102	
	9.9954		-0.0011	1.1E-06	0.00075	0.00148	
75	9.9970	9.9965	0.0005	2.8E-07	0.00038	0.00074	0.0013
	9.9970		0.0005	2.8E-07	0.00038	0.00074	

Face Right Observations at 10m range							
	TOPCON	ES-105N					
	Prism =	9.9952					Average
Angle	Dist	Mean (ẋ)	x-ẋ	(x-ẋ)²	Std Dev	S at 95%	Diff to prism
0	9.9948	9.9948	0.0000	0.0E+00	0.00000	0.00000	-0.0004
	9.9950		0.0002	4.0E-08	0.00014	0.00028	
	9.9946		-0.0002	4.0E-08	0.00014	0.00028	
25	9.9950	9.9950	0.0000	0.0E+00	0.00000	0.00000	-0.0002
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
30	9.9950	9.9950	0.0000	0.0E+00	0.00000	0.00000	-0.0002
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
35	9.9950	9.9950	0.0000	0.0E+00	0.00000	0.00000	-0.0002
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
40	9.9954	9.9950	0.0004	1.6E-07	0.00028	0.00055	-0.0002
	9.9948		-0.0002	4.0E-08	0.00014	0.00028	
	9.9948		-0.0002	4.0E-08	0.00014	0.00028	
45	9.9950	9.9950	0.0000	0.0E+00	0.00000	0.00000	-0.0002
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
	9.9950		0.0000	0.0E+00	0.00000	0.00000	
50	9.9950	9.9949	0.0001	4.4E-09	0.00005	0.00009	-0.0002
	9.9948		-0.0001	1.8E-08	0.00009	0.00018	
	9.9950		0.0001	4.4E-09	0.00005	0.00009	
55	9.9950	9.9949	0.0001	4.4E-09	0.00005	0.00009	-0.0002
	9.9948		-0.0001	1.8E-08	0.00009	0.00018	
	9.9950		0.0001	4.4E-09	0.00005	0.00009	
60	9.9952	9.9952	0.0000	0.0E+00	0.00000	0.00000	0.0000
	9.9950		-0.0002	4.0E-08	0.00014	0.00028	
	9.9954		0.0002	4.0E-08	0.00014	0.00028	
65	9.9950	9.9954	-0.0004	1.6E-07	0.00028	0.00055	0.0003
	9.9952		-0.0002	4.0E-08	0.00014	0.00028	
	9.9960		0.0006	3.6E-07	0.00042	0.00083	
70	9.9950	9.9952	-0.0002	4.0E-08	0.00014	0.00028	0.0000
	9.9946		-0.0006	3.6E-07	0.00042	0.00083	
	9.9960		0.0008	6.4E-07	0.00057	0.00111	
75	9.9946	9.9950	-0.0004	1.6E-07	0.00028	0.00055	-0.0002
	9.9948		-0.0002	4.0E-08	0.00014	0.00028	
	9.9956		0.0006	3.6E-07	0.00042	0.00083	

Face Left Observations at 30m range							
	TOPCON	ES-105N					
	Prism =	29.7776					Average
Angle	**Dist**	**Mean ($\dot{x}$)**	**$x-\dot{x}$**	**$(x-\dot{x})^2$**	**Std Dev**	**S at 95%**	**Diff to prism**
	29.7760		-0.0009	7.5E-07	0.00061	0.00120	
0	29.7776	29.7769	0.0007	5.4E-07	0.00052	0.00102	-0.0007
	29.7770		0.0001	1.8E-08	0.00009	0.00018	
	29.7780		0.0001	4.4E-09	0.00005	0.00009	
25	29.7778	29.7779	-0.0001	1.8E-08	0.00009	0.00018	0.0003
	29.7780		0.0001	4.4E-09	0.00005	0.00009	
	29.7778		-0.0002	4.0E-08	0.00014	0.00028	
30	29.7780	29.7780	0.0000	0.0E+00	0.00000	0.00000	0.0004
	29.7782		0.0002	4.0E-08	0.00014	0.00028	
	29.7780		0.0001	1.8E-08	0.00009	0.00018	
35	29.7780	29.7779	0.0001	1.8E-08	0.00009	0.00018	0.0003
	29.7776		-0.0003	7.1E-08	0.00019	0.00037	
	29.7782		0.0002	4.0E-08	0.00014	0.00028	
40	29.7780	29.7780	0.0000	0.0E+00	0.00000	0.00000	0.0004
	29.7778		-0.0002	4.0E-08	0.00014	0.00028	
	29.7782		0.0001	1.8E-08	0.00009	0.00018	
45	29.7780	29.7781	-0.0001	4.4E-09	0.00005	0.00009	0.0005
	29.7780		-0.0001	4.4E-09	0.00005	0.00009	
	29.7788		0.0003	7.1E-08	0.00019	0.00037	
50	29.7788	29.7785	0.0003	7.1E-08	0.00019	0.00037	0.0009
	29.7780		-0.0005	2.8E-07	0.00038	0.00074	
	29.7790		0.0001	4.4E-09	0.00005	0.00009	
55	29.7790	29.7789	0.0001	4.4E-09	0.00005	0.00009	0.0013
	29.7788		-0.0001	1.8E-08	0.00009	0.00018	
	29.7794		0.0005	2.8E-07	0.00038	0.00074	
60	29.7782	29.7789	-0.0007	4.4E-07	0.00047	0.00092	0.0013
	29.7790		0.0001	1.8E-08	0.00009	0.00018	
	29.7798		0.0002	4.0E-08	0.00014	0.00028	
65	29.7790	29.7796	-0.0006	3.6E-07	0.00042	0.00083	0.0020
	29.7800		0.0004	1.6E-07	0.00028	0.00055	
	29.7796		-0.0001	1.8E-08	0.00009	0.00018	
70	29.7794	29.7797	-0.0003	1.1E-07	0.00024	0.00046	0.0021
	29.7802		0.0005	2.2E-07	0.00033	0.00065	
	29.7798		0.0000	0.0E+00	0.00000	0.00000	
75	29.7794	29.7798	-0.0004	1.6E-07	0.00028	0.00055	0.0022
	29.7802		0.0004	1.6E-07	0.00028	0.00055	

Face Right Observations at 30m range							
	TOPCON	ES-105N					
	Prism =	29.7776					Average
Angle	Dist	Mean (ẋ)	x-ẋ	$(x-\dot{x})^2$	Std Dev	S at 95%	Diff to prism
	29.7776		0.0001	4.4E-09	0.00005	0.00009	
0	29.7772	29.7775	-0.0003	1.1E-07	0.00024	0.00046	-0.0001
	29.7778		0.0003	7.1E-08	0.00019	0.00037	
	29.7780		0.0001	1.8E-08	0.00009	0.00018	
25	29.7778	29.7779	-0.0001	4.4E-09	0.00005	0.00009	0.0003
	29.7778		-0.0001	4.4E-09	0.00005	0.00009	
	29.7770		-0.0005	2.2E-07	0.00033	0.00065	
30	29.7780	29.7775	0.0005	2.8E-07	0.00038	0.00074	-0.0001
	29.7774		-0.0001	4.4E-09	0.00005	0.00009	
	29.7774		-0.0001	4.4E-09	0.00005	0.00009	
35	29.7780	29.7775	0.0005	2.8E-07	0.00038	0.00074	-0.0001
	29.7770		-0.0005	2.2E-07	0.00033	0.00065	
	29.7774		0.0001	4.4E-09	0.00005	0.00009	
40	29.7776	29.7773	0.0003	7.1E-08	0.00019	0.00037	-0.0003
	29.7770		-0.0003	1.1E-07	0.00024	0.00046	
	29.7770		0.0000	0.0E+00	0.00000	0.00000	
45	29.7770	29.7770	0.0000	0.0E+00	0.00000	0.00000	-0.0006
	29.7770		0.0000	0.0E+00	0.00000	0.00000	
	29.7770		0.0001	4.4E-09	0.00005	0.00009	
50	29.7770	29.7769	0.0001	4.4E-09	0.00005	0.00009	-0.0007
	29.7768		-0.0001	1.8E-08	0.00009	0.00018	
	29.7778		0.0006	3.6E-07	0.00042	0.00083	
55	29.7770	29.7772	-0.0002	4.0E-08	0.00014	0.00028	-0.0004
	29.7768		-0.0004	1.6E-07	0.00028	0.00055	
	29.7770		0.0000	0.0E+00	0.00000	0.00000	
60	29.7774	29.7770	0.0004	1.6E-07	0.00028	0.00055	-0.0006
	29.7766		-0.0004	1.6E-07	0.00028	0.00055	
	29.7770		0.0004	1.6E-07	0.00028	0.00055	
65	29.7762	29.7766	-0.0004	1.6E-07	0.00028	0.00055	-0.0010
	29.7766		0.0000	0.0E+00	0.00000	0.00000	
	29.7754		-0.0007	5.4E-07	0.00052	0.00102	
70	29.7772	29.7761	0.0011	1.1E-06	0.00075	0.00148	-0.0015
	29.7758		-0.0003	1.1E-07	0.00024	0.00046	
	29.7746		-0.0007	4.4E-07	0.00047	0.00092	
75	29.7751	29.7753	-0.0002	2.8E-08	0.00012	0.00023	-0.0023
	29.7761		0.0008	6.9E-07	0.00059	0.00115	

Face Left Observations at 60m range							
	TOPCON	ES-105N					
	Prism =	58.9896					Average
Angle	Dist	Mean (ẋ)	x-ẋ	(x-ẋ)²	Std Dev	S at 95%	Diff to prism
0	58.9895	58.9894	0.0001	4.4E-09	0.00005	0.00009	-0.0002
	58.9898		0.0004	1.3E-07	0.00026	0.00051	
	58.9890		-0.0004	1.9E-07	0.00031	0.00060	
25	58.9896	58.9895	0.0001	1.8E-08	0.00009	0.00018	-0.0001
	58.9898		0.0003	1.1E-07	0.00024	0.00046	
	58.9890		-0.0005	2.2E-07	0.00033	0.00065	
30	58.9902	58.9897	0.0005	2.2E-07	0.00033	0.00065	0.0001
	58.9900		0.0003	7.1E-08	0.00019	0.00037	
	58.9890		-0.0007	5.4E-07	0.00052	0.00102	
35	58.9904	58.9898	0.0006	3.6E-07	0.00042	0.00083	0.0002
	58.9904		0.0006	3.6E-07	0.00042	0.00083	
	58.9886		-0.0012	1.4E-06	0.00085	0.00166	
40	58.9910	58.9901	0.0009	8.7E-07	0.00066	0.00129	0.0005
	58.9906		0.0005	2.8E-07	0.00038	0.00074	
	58.9886		-0.0015	2.2E-06	0.00104	0.00203	
45	58.9910	58.9905	0.0005	2.8E-07	0.00038	0.00074	0.0009
	58.9910		0.0005	2.8E-07	0.00038	0.00074	
	58.9894		-0.0011	1.1E-06	0.00075	0.00148	
50	58.9922	58.9908	0.0014	2.0E-06	0.00099	0.00194	0.0012
	58.9910		0.0002	4.0E-08	0.00014	0.00028	
	58.9892		-0.0016	2.6E-06	0.00113	0.00222	
55	58.9942	58.9919	0.0023	5.4E-06	0.00165	0.00323	0.0023
	58.9914		-0.0005	2.2E-07	0.00033	0.00065	
	58.9900		-0.0019	3.5E-06	0.00132	0.00259	
60	58.9932	58.9932	0.0000	1.1E-09	0.00002	0.00005	0.0036
	58.9931		-0.0001	4.4E-09	0.00005	0.00009	
	58.9932		0.0000	1.1E-09	0.00002	0.00005	
65	58.9882	58.9906	-0.0024	5.8E-06	0.00170	0.00333	0.0010
	58.9892		-0.0014	2.0E-06	0.00099	0.00194	
	58.9944		0.0038	1.4E-05	0.00269	0.00527	
70	58.9940	58.9899	0.0041	1.7E-05	0.00292	0.00573	0.0003
	58.9888		-0.0011	1.1E-06	0.00075	0.00148	
	58.9868		-0.0031	9.4E-06	0.00217	0.00425	
75	58.9952	58.9915	0.0037	1.4E-05	0.00262	0.00513	0.0019
	58.9897		-0.0018	3.2E-06	0.00127	0.00249	
	58.9896		-0.0019	3.6E-06	0.00134	0.00263	

Face Right Observations at 60m range							
	TOPCON	ES-105N					
	Prism =	58.9896					Average
Angle	Dist	Mean (ẋ)	x-ẋ	(x-ẋ)²	Std Dev	S at 95%	Diff to prism
0	58.9896	58.9891	0.0005	2.2E-07	0.00033	0.00065	-0.0005
	58.9888		-0.0003	1.1E-07	0.00024	0.00046	
	58.9890		-0.0001	1.8E-08	0.00009	0.00018	
25	58.9898	58.9897	0.0001	1.8E-08	0.00009	0.00018	0.0001
	58.9894		-0.0003	7.1E-08	0.00019	0.00037	
	58.9898		0.0001	1.8E-08	0.00009	0.00018	
30	58.9900	58.9896	0.0004	1.6E-07	0.00028	0.00055	0.0000
	58.9896		0.0000	0.0E+00	0.00000	0.00000	
	58.9892		-0.0004	1.6E-07	0.00028	0.00055	
35	58.9900	58.9899	0.0001	1.8E-08	0.00009	0.00018	0.0003
	58.9900		0.0001	1.8E-08	0.00009	0.00018	
	58.9896		-0.0003	7.1E-08	0.00019	0.00037	
40	58.9900	58.9897	0.0003	1.1E-07	0.00024	0.00046	0.0001
	58.9890		-0.0007	4.4E-07	0.00047	0.00092	
	58.9900		0.0003	1.1E-07	0.00024	0.00046	
45	58.9904	58.9898	0.0006	3.6E-07	0.00042	0.00083	0.0002
	58.9890		-0.0008	6.4E-07	0.00057	0.00111	
	58.9900		0.0002	4.0E-08	0.00014	0.00028	
50	58.9918	58.9906	0.0012	1.4E-06	0.00085	0.00166	0.0010
	58.9890		-0.0016	2.6E-06	0.00113	0.00222	
	58.9910		0.0004	1.6E-07	0.00028	0.00055	
55	58.9894	58.9897	-0.0003	1.1E-07	0.00024	0.00046	0.0001
	58.9888		-0.0009	8.7E-07	0.00066	0.00129	
	58.9910		0.0013	1.6E-06	0.00090	0.00176	
60	58.9898	58.9896	0.0002	4.0E-08	0.00014	0.00028	0.0000
	58.9876		-0.0020	4.0E-06	0.00141	0.00277	
	58.9914		0.0018	3.2E-06	0.00127	0.00249	
65	58.9872	58.9851	0.0021	4.3E-06	0.00146	0.00286	-0.0045
	58.9808		-0.0043	1.9E-05	0.00306	0.00601	
	58.9874		0.0023	5.1E-06	0.00160	0.00314	
70	58.9938	58.9870	0.0068	4.6E-05	0.00481	0.00942	-0.0026
	58.9820		-0.0050	2.5E-05	0.00354	0.00693	
	58.9852		-0.0018	3.2E-06	0.00127	0.00249	
75	58.9958	58.9877	0.0081	6.6E-05	0.00575	0.01127	-0.0019
	58.9832		-0.0045	2.0E-05	0.00316	0.00619	
	58.9840		-0.0037	1.3E-05	0.00259	0.00508	

Combined Angle of Incidence

Face Left Observations at 10m range							
	TOPCON	ES-105N					
	Prism =	10.0243					Average
Angle	**Dist**	**Mean (ẋ)**	**x-ẋ**	**(x-ẋ)²**	**Std Dev**	**S at 95%**	**Diff to prism**
	10.0244		0.0000	0.0E+00	0.00000	0.00000	
0	10.0243	10.0244	-0.0001	1.0E-08	0.00007	0.00014	0.0001
	10.0245		0.0001	1.0E-08	0.00007	0.00014	
	10.0241		-0.0002	4.0E-08	0.00014	0.00028	
25	10.0243	10.0243	0.0000	0.0E+00	0.00000	0.00000	0.0000
	10.0245		0.0002	4.0E-08	0.00014	0.00028	
	10.0242		-0.0001	1.0E-08	0.00007	0.00014	
30	10.0242	10.0243	-0.0001	1.0E-08	0.00007	0.00014	0.0000
	10.0245		0.0002	4.0E-08	0.00014	0.00028	
	10.0247		0.0000	0.0E+00	0.00000	0.00000	
35	10.0247	10.0247	0.0000	0.0E+00	0.00000	0.00000	0.0004
	10.0247		0.0000	0.0E+00	0.00000	0.00000	
	10.0247		-0.0004	1.6E-07	0.00028	0.00055	
40	10.0252	10.0251	0.0001	1.0E-08	0.00007	0.00014	0.0008
	10.0254		0.0003	9.0E-08	0.00021	0.00042	
	10.0251		0.0004	1.3E-07	0.00026	0.00051	
45	10.0246	10.0247	-0.0001	1.8E-08	0.00009	0.00018	0.0004
	10.0245		-0.0002	5.4E-08	0.00016	0.00032	
	10.0254		0.0004	1.3E-07	0.00026	0.00051	
50	10.0243	10.0250	-0.0007	5.4E-07	0.00052	0.00102	0.0007
	10.0254		0.0004	1.3E-07	0.00026	0.00051	
	10.0247		-0.0004	1.6E-07	0.00028	0.00055	
55	10.0251	10.0251	0.0000	0.0E+00	0.00000	0.00000	0.0008
	10.0255		0.0004	1.6E-07	0.00028	0.00055	
	10.0260		0.0004	1.6E-07	0.00028	0.00055	
60	10.0254	10.0256	-0.0002	4.0E-08	0.00014	0.00028	0.0013
	10.0254		-0.0002	4.0E-08	0.00014	0.00028	
	10.0254		-0.0003	7.1E-08	0.00019	0.00037	
65	10.0262	10.0257	0.0005	2.8E-07	0.00038	0.00074	0.0014
	10.0254		-0.0003	7.1E-08	0.00019	0.00037	
	10.0254		0.0002	4.0E-08	0.00014	0.00028	
70	10.0248	10.0252	-0.0004	1.6E-07	0.00028	0.00055	0.0009
	10.0254		0.0002	4.0E-08	0.00014	0.00028	
	10.0254		0.0001	1.0E-08	0.00007	0.00014	
75	10.0247	10.0253	-0.0006	3.6E-07	0.00042	0.00083	0.0010
	10.0258		0.0005	2.5E-07	0.00035	0.00069	

Face Right Observations at 10m range							
	TOPCON	ES-105N					
	Prism =	10.0243					Average
Angle	**Dist**	**Mean (ẋ)**	**x-ẋ**	**(x-ẋ)²**	**Std Dev**	**S at 95%**	**Diff to prism**
0	10.0241	10.0242	-0.0001	1.8E-08	0.00009	0.00018	-0.0001
	10.0243		0.0001	4.4E-09	0.00005	0.00009	
	10.0243		0.0001	4.4E-09	0.00005	0.00009	
25	10.0248	10.0241	0.0007	5.4E-07	0.00052	0.00102	-0.0002
	10.0237		-0.0004	1.3E-07	0.00026	0.00051	
	10.0237		-0.0004	1.3E-07	0.00026	0.00051	
30	10.0248	10.0241	0.0007	4.4E-07	0.00047	0.00092	-0.0002
	10.0238		-0.0003	1.1E-07	0.00024	0.00046	
	10.0238		-0.0003	1.1E-07	0.00024	0.00046	
35	10.0247	10.0241	0.0006	3.2E-07	0.00040	0.00079	-0.0002
	10.0238		-0.0003	1.1E-07	0.00024	0.00046	
	10.0239		-0.0002	5.4E-08	0.00016	0.00032	
40	10.0247	10.0237	0.0010	1.1E-06	0.00073	0.00143	-0.0006
	10.0231		-0.0006	3.2E-07	0.00040	0.00079	
	10.0232		-0.0005	2.2E-07	0.00033	0.00065	
45	10.0246	10.0242	0.0004	1.3E-07	0.00026	0.00051	-0.0001
	10.0240		-0.0002	5.4E-08	0.00016	0.00032	
	10.0241		-0.0001	1.8E-08	0.00009	0.00018	
50	10.0225	10.0230	-0.0005	2.8E-07	0.00038	0.00074	-0.0013
	10.0225		-0.0005	2.8E-07	0.00038	0.00074	
	10.0241		0.0011	1.1E-06	0.00075	0.00148	
55	10.0227	10.0228	-0.0001	1.0E-08	0.00007	0.00014	-0.0015
	10.0230		0.0002	4.0E-08	0.00014	0.00028	
	10.0227		-0.0001	1.0E-08	0.00007	0.00014	
60	10.0240	10.0227	0.0013	1.8E-06	0.00094	0.00185	-0.0016
	10.0220		-0.0007	4.4E-07	0.00047	0.00092	
	10.0220		-0.0007	4.4E-07	0.00047	0.00092	
65	10.0234	10.0233	0.0001	4.4E-09	0.00005	0.00009	-0.0010
	10.0232		-0.0001	1.8E-08	0.00009	0.00018	
	10.0234		0.0001	4.4E-09	0.00005	0.00009	
70	10.0231	10.0228	0.0003	1.1E-07	0.00024	0.00046	-0.0015
	10.0216		-0.0012	1.4E-06	0.00082	0.00162	
	10.0236		0.0008	6.9E-07	0.00059	0.00115	
75	10.0231	10.0226	0.0005	2.8E-07	0.00038	0.00074	-0.0017
	10.0210		-0.0016	2.5E-06	0.00111	0.00217	
	10.0236		0.0010	1.1E-06	0.00073	0.00143	

Face Left Observations at 30m range							
	TOPCON	ES-105N					
	Prism =	29.8211					Average
Angle	Dist	Mean (ẋ)	x-ẋ	(x-ẋ)²	Std Dev	S at 95%	Diff to prism
0	29.8214	29.8214	0.0000	0.0E+00	0.00000	0.00000	0.0003
	29.8214		0.0000	0.0E+00	0.00000	0.00000	
	29.8214		0.0000	0.0E+00	0.00000	0.00000	
25	29.8217	29.8217	0.0000	0.0E+00	0.00000	0.00000	0.0006
	29.8217		0.0000	0.0E+00	0.00000	0.00000	
	29.8217		0.0000	0.0E+00	0.00000	0.00000	
30	29.8210	29.8214	-0.0004	1.6E-07	0.00028	0.00055	0.0003
	29.8216		0.0002	4.0E-08	0.00014	0.00028	
	29.8216		0.0002	4.0E-08	0.00014	0.00028	
35	29.8204	29.8209	-0.0005	2.2E-07	0.00033	0.00065	-0.0002
	29.8208		-0.0001	4.4E-09	0.00005	0.00009	
	29.8214		0.0005	2.8E-07	0.00038	0.00074	
40	29.8216	29.8215	0.0001	1.8E-08	0.00009	0.00018	0.0004
	29.8214		-0.0001	4.4E-09	0.00005	0.00009	
	29.8214		-0.0001	4.4E-09	0.00005	0.00009	
45	29.8224	29.8220	0.0004	1.6E-07	0.00028	0.00055	0.0009
	29.8219		-0.0001	1.0E-08	0.00007	0.00014	
	29.8217		-0.0003	9.0E-08	0.00021	0.00042	
50	29.8228	29.8220	0.0008	6.4E-07	0.00057	0.00111	0.0009
	29.8217		-0.0003	9.0E-08	0.00021	0.00042	
	29.8215		-0.0005	2.5E-07	0.00035	0.00069	
55	29.8229	29.8219	0.0010	1.0E-06	0.00071	0.00139	0.0008
	29.8215		-0.0004	1.6E-07	0.00028	0.00055	
	29.8213		-0.0006	3.6E-07	0.00042	0.00083	
60	29.8227	29.8228	-0.0001	1.0E-08	0.00007	0.00014	0.0017
	29.8227		-0.0001	1.0E-08	0.00007	0.00014	
	29.8230		0.0002	4.0E-08	0.00014	0.00028	
65	29.8235	29.8229	0.0006	3.2E-07	0.00040	0.00079	0.0018
	29.8229		0.0000	1.1E-09	0.00002	0.00005	
	29.8224		-0.0005	2.8E-07	0.00038	0.00074	
70	29.8241	29.8238	0.0003	1.1E-07	0.00024	0.00046	0.0027
	29.8244		0.0006	4.0E-07	0.00045	0.00088	
	29.8228		-0.0010	9.3E-07	0.00068	0.00134	
75	29.8249	29.8239	0.0010	1.1E-06	0.00073	0.00143	0.0028
	29.8228		-0.0011	1.1E-06	0.00075	0.00148	
	29.8239		0.0000	1.1E-09	0.00002	0.00005	

Face Left Observations at 30m range							
	TOPCON	ES-105N					
	Prism =	29.8211					Average
Angle	**Dist**	**Mean (ẋ)**	**x-ẋ**	**(x-ẋ)²**	**Std Dev**	**S at 95%**	**Diff to prism**
0	29.8214	29.8214	0.0000	0.0E+00	0.00000	0.00000	0.0003
	29.8214		0.0000	0.0E+00	0.00000	0.00000	
	29.8214		0.0000	0.0E+00	0.00000	0.00000	
25	29.8214	29.8214	0.0000	0.0E+00	0.00000	0.00000	0.0003
	29.8214		0.0000	0.0E+00	0.00000	0.00000	
	29.8214		0.0000	0.0E+00	0.00000	0.00000	
30	29.8216	29.8205	0.0011	1.1E-06	0.00075	0.00148	-0.0006
	29.8211		0.0006	3.2E-07	0.00040	0.00079	
	29.8189		-0.0016	2.7E-06	0.00115	0.00226	
35	29.8208	29.8208	0.0000	0.0E+00	0.00000	0.00000	-0.0003
	29.8208		0.0000	0.0E+00	0.00000	0.00000	
	29.8208		0.0000	0.0E+00	0.00000	0.00000	
40	29.8200	29.8200	0.0000	0.0E+00	0.00000	0.00000	-0.0011
	29.8200		0.0000	0.0E+00	0.00000	0.00000	
	29.8200		0.0000	0.0E+00	0.00000	0.00000	
45	29.8207	29.8207	0.0000	0.0E+00	0.00000	0.00000	-0.0004
	29.8207		0.0000	0.0E+00	0.00000	0.00000	
	29.8207		0.0000	0.0E+00	0.00000	0.00000	
50	29.8205	29.8205	0.0000	0.0E+00	0.00000	0.00000	-0.0006
	29.8205		0.0000	0.0E+00	0.00000	0.00000	
	29.8205		0.0000	0.0E+00	0.00000	0.00000	
55	29.8199	29.8199	0.0000	0.0E+00	0.00000	0.00000	-0.0012
	29.8199		0.0000	0.0E+00	0.00000	0.00000	
	29.8199		0.0000	0.0E+00	0.00000	0.00000	
60	29.8195	29.8194	0.0001	1.0E-08	0.00007	0.00014	-0.0017
	29.8192		-0.0002	4.0E-08	0.00014	0.00028	
	29.8195		0.0001	1.0E-08	0.00007	0.00014	
65	29.8197	29.8191	0.0006	3.6E-07	0.00042	0.00083	-0.0020
	29.8188		-0.0003	9.0E-08	0.00021	0.00042	
	29.8188		-0.0003	9.0E-08	0.00021	0.00042	
70	29.8177	29.8175	0.0002	4.0E-08	0.00014	0.00028	-0.0036
	29.8171		-0.0004	1.6E-07	0.00028	0.00055	
	29.8177		0.0002	4.0E-08	0.00014	0.00028	
75	29.8165	29.8190	-0.0025	6.1E-06	0.00174	0.00342	-0.0021
	29.8194		0.0004	1.9E-07	0.00031	0.00060	
	29.8210		0.0020	4.1E-06	0.00144	0.00282	

Face Left Observations at 60m range							
	TOPCON	ES-105N					
	Prism =	59.4274					Average
Angle	**Dist**	**Mean (ẋ)**	**x-ẋ**	**(x-ẋ)²**	**Std Dev**	**S at 95%**	**Diff to prism**
	59.4274		0.0000	0.0E+00	0.00000	0.00000	
0	59.4274	59.4274	0.0000	0.0E+00	0.00000	0.00000	0.0000
	59.4274		0.0000	0.0E+00	0.00000	0.00000	
	59.4268		-0.0003	1.1E-07	0.00024	0.00046	
25	59.4271	59.4271	0.0000	1.1E-09	0.00002	0.00005	-0.0003
	59.4275		0.0004	1.3E-07	0.00026	0.00051	
	59.4276		-0.0002	4.0E-08	0.00014	0.00028	
30	59.4280	59.4278	0.0002	4.0E-08	0.00014	0.00028	0.0004
	59.4278		0.0000	0.0E+00	0.00000	0.00000	
	59.4278		0.0000	0.0E+00	0.00000	0.00000	
35	59.4278	59.4278	0.0000	0.0E+00	0.00000	0.00000	0.0004
	59.4278		0.0000	0.0E+00	0.00000	0.00000	
	59.4279		0.0001	1.0E-08	0.00007	0.00014	
40	59.4280	59.4278	0.0002	4.0E-08	0.00014	0.00028	0.0004
	59.4275		-0.0003	9.0E-08	0.00021	0.00042	
	59.4287		-0.0001	1.0E-08	0.00007	0.00014	
45	59.4287	59.4288	-0.0001	1.0E-08	0.00007	0.00014	0.0014
	59.4290		0.0002	4.0E-08	0.00014	0.00028	
	59.4287		0.0001	1.0E-08	0.00007	0.00014	
50	59.4285	59.4286	-0.0001	1.0E-08	0.00007	0.00014	0.0012
	59.4286		0.0000	0.0E+00	0.00000	0.00000	
	59.4283		-0.0006	4.0E-07	0.00045	0.00088	
55	59.4297	59.4289	0.0008	5.9E-07	0.00054	0.00106	0.0015
	59.4288		-0.0001	1.8E-08	0.00009	0.00018	
	59.4287		-0.0012	1.4E-06	0.00084	0.00165	
60	59.4311	59.4299	0.0012	1.5E-06	0.00085	0.00167	0.0025
	59.4299		0.0000	1.8E-10	0.00001	0.00002	
	59.4281		-0.0018	3.1E-06	0.00125	0.00245	
65	59.4289	59.4299	-0.0010	9.3E-07	0.00068	0.00134	0.0025
	59.4326		0.0027	7.5E-06	0.00193	0.00379	
	59.4308		0.0002	4.0E-08	0.00014	0.00028	
70	59.4304	59.4306	-0.0002	4.0E-08	0.00014	0.00028	0.0032
	59.4306		0.0000	0.0E+00	0.00000	0.00000	
	59.4350		0.0042	1.8E-05	0.00297	0.00582	
75	59.4267	59.4308	-0.0041	1.7E-05	0.00290	0.00568	0.0034
	59.4307		-0.0001	1.0E-08	0.00007	0.00014	

Face Right Observations at 60m range							
	TOPCON	ES-105N					
	Prism =	59.4274					Average
Angle	**Dist**	**Mean (ẋ)**	**x-ẋ**	**(x-ẋ)²**	**Std Dev**	**S at 95%**	**Diff to prism**
0	59.4275	59.4275	0.0000	0.0E+00	0.00000	0.00000	0.0001
	59.4275		0.0000	0.0E+00	0.00000	0.00000	
	59.4275		0.0000	0.0E+00	0.00000	0.00000	
25	59.4274	59.4272	0.0002	4.0E-08	0.00014	0.00028	-0.0002
	59.4271		-0.0001	1.0E-08	0.00007	0.00014	
	59.4271		-0.0001	1.0E-08	0.00007	0.00014	
30	59.4277	59.4276	0.0001	1.0E-08	0.00007	0.00014	0.0002
	59.4273		-0.0003	9.0E-08	0.00021	0.00042	
	59.4278		0.0002	4.0E-08	0.00014	0.00028	
35	59.4276	59.4276	0.0000	0.0E+00	0.00000	0.00000	0.0002
	59.4279		0.0003	9.0E-08	0.00021	0.00042	
	59.4273		-0.0003	9.0E-08	0.00021	0.00042	
40	59.4270	59.4273	-0.0003	9.0E-08	0.00021	0.00042	-0.0001
	59.4272		-0.0001	1.0E-08	0.00007	0.00014	
	59.4277		0.0004	1.6E-07	0.00028	0.00055	
45	59.4272	59.4273	-0.0001	1.0E-08	0.00007	0.00014	-0.0001
	59.4270		-0.0003	9.0E-08	0.00021	0.00042	
	59.4277		0.0004	1.6E-07	0.00028	0.00055	
50	59.4255	59.4260	-0.0005	2.5E-07	0.00035	0.00069	-0.0014
	59.4263		0.0003	9.0E-08	0.00021	0.00042	
	59.4262		0.0002	4.0E-08	0.00014	0.00028	
55	59.4277	59.4270	0.0007	4.9E-07	0.00049	0.00097	-0.0004
	59.4265		-0.0005	2.5E-07	0.00035	0.00069	
	59.4268		-0.0002	4.0E-08	0.00014	0.00028	
60	59.4255	59.4255	0.0000	0.0E+00	0.00000	0.00000	-0.0019
	59.4255		0.0000	0.0E+00	0.00000	0.00000	
	59.4255		0.0000	0.0E+00	0.00000	0.00000	
65	59.4190	59.4241	-0.0051	2.6E-05	0.00358	0.00702	-0.0033
	59.4270		0.0029	8.6E-06	0.00207	0.00407	
	59.4262		0.0021	4.6E-06	0.00151	0.00296	
70	59.4241	59.4241	0.0000	0.0E+00	0.00000	0.00000	-0.0033
	59.4239		-0.0002	4.0E-08	0.00014	0.00028	
	59.4243		0.0002	4.0E-08	0.00014	0.00028	
75	59.4192	59.4228	-0.0036	1.3E-05	0.00255	0.00499	-0.0046
	59.4226		-0.0002	4.0E-08	0.00014	0.00028	
	59.4266		0.0038	1.4E-05	0.00269	0.00527	

Appendix D

Summary of REDM Error

REDM Observation Errors

	10m												
	Horizontal Angle of Incidence							Combined Angle of Incidence					
0	-0.0004	-0.0002	-0.0006	-0.0002	-0.0004	-0.0006		-0.0002	0.0000	0.0000	0.0001	0.0000	0.0002
25	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002		0.0005	-0.0006	-0.0006	-0.0002	0.0000	0.0002
30	-0.0002	-0.0002	-0.0002	-0.0004	-0.0002	-0.0002		0.0005	-0.0005	-0.0005	-0.0001	-0.0001	0.0002
35	-0.0002	-0.0002	-0.0002	-0.0006	-0.0002	-0.0002		0.0004	-0.0005	-0.0004	0.0004	0.0004	0.0004
40	0.0002	-0.0004	-0.0004	0.0002	-0.0002	-0.0002		0.0004	-0.0012	-0.0011	0.0004	0.0009	0.0011
45	-0.0002	-0.0002	-0.0002	0.0000	0.0006	-0.0002		0.0003	-0.0003	-0.0002	0.0008	0.0003	0.0002
50	-0.0002	-0.0004	-0.0002	0.0000	0.0006	0.0004		-0.0018	-0.0018	-0.0002	0.0011	0.0000	0.0011
55	-0.0002	-0.0004	-0.0002	0.0002	0.0006	0.0008		-0.0016	-0.0013	-0.0016	0.0004	0.0008	0.0012
60	0.0000	-0.0002	0.0002	-0.0004	0.0002	0.0008		-0.0003	-0.0023	-0.0023	0.0017	0.0011	0.0011
65	-0.0002	0.0000	0.0008	0.0000	0.0004	0.0000		-0.0009	-0.0011	-0.0009	0.0011	0.0019	0.0011
70	-0.0002	-0.0006	0.0008	0.0000	0.0006	0.0015		-0.0012	-0.0027	-0.0007	0.0011	0.0005	0.0011
75	-0.0006	-0.0004	0.0004	0.0002	0.0018	0.0018		-0.0012	-0.0033	-0.0007	0.0011	0.0004	0.0015

	30m												
	Horizontal Angle of Incidence							Combined Angle of Incidence					
0	0.0000	-0.0004	0.0002	-0.0016	0.0000	-0.0006		0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
25	0.0004	0.0002	0.0002	0.0004	0.0002	0.0004		0.0003	0.0003	0.0003	0.0006	0.0006	0.0006
30	-0.0006	0.0004	-0.0002	0.0002	0.0004	0.0006		0.0005	0.0000	-0.0022	-0.0001	0.0005	0.0005
35	-0.0002	0.0004	-0.0006	0.0004	0.0004	0.0000		-0.0003	-0.0003	-0.0003	-0.0007	-0.0003	0.0003
40	-0.0002	0.0000	-0.0006	0.0006	0.0004	0.0002		-0.0011	-0.0011	-0.0011	0.0005	0.0003	0.0003
45	-0.0006	-0.0006	-0.0006	0.0006	0.0004	0.0004		-0.0004	-0.0004	-0.0004	0.0013	0.0008	0.0006
50	-0.0006	-0.0006	-0.0008	0.0012	0.0012	0.0004		-0.0006	-0.0006	-0.0006	0.0017	0.0006	0.0004
55	0.0002	-0.0006	-0.0008	0.0014	0.0014	0.0012		-0.0012	-0.0012	-0.0012	0.0018	0.0004	0.0002
60	-0.0006	-0.0002	-0.0010	0.0018	0.0006	0.0014		-0.0016	-0.0019	-0.0016	0.0016	0.0016	0.0019
65	-0.0006	-0.0014	-0.0010	0.0022	0.0014	0.0024		-0.0014	-0.0023	-0.0023	0.0024	0.0018	0.0013
70	-0.0022	-0.0004	-0.0018	0.0020	0.0018	0.0026		-0.0034	-0.0040	-0.0034	0.0030	0.0033	0.0017
75	-0.0030	-0.0025	-0.0015	0.0022	0.0018	0.0026		-0.0046	-0.0017	-0.0001	0.0038	0.0017	0.0028

	60m												
	Horizontal Angle of Incidence							Combined Angle of Incidence					
0	0.0000	-0.0008	-0.0006	-0.0001	0.0002	-0.0006		0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
25	0.0002	-0.0002	0.0002	0.0000	0.0002	-0.0006		0.0000	-0.0003	-0.0003	-0.0006	-0.0003	0.0001
30	0.0004	0.0000	-0.0004	0.0006	0.0004	-0.0006		0.0003	-0.0001	0.0004	0.0002	0.0006	0.0004
35	0.0004	0.0004	0.0000	0.0008	0.0008	-0.0010		0.0002	0.0005	-0.0001	0.0004	0.0004	0.0004
40	0.0004	-0.0006	0.0004	0.0014	0.0010	-0.0010		-0.0004	-0.0002	0.0003	0.0005	0.0006	0.0001
45	0.0008	-0.0006	0.0004	0.0014	0.0014	-0.0002		-0.0002	-0.0004	0.0003	0.0013	0.0013	0.0016
50	0.0022	-0.0006	0.0014	0.0026	0.0014	-0.0004		-0.0019	-0.0011	-0.0012	0.0013	0.0011	0.0012
55	-0.0002	-0.0008	0.0014	0.0046	0.0018	0.0004		0.0003	-0.0009	-0.0006	0.0009	0.0023	0.0014
60	0.0002	-0.0020	0.0018	0.0036	0.0035	0.0036		-0.0019	-0.0019	-0.0019	0.0013	0.0037	0.0025
65	-0.0024	-0.0088	-0.0022	-0.0014	-0.0004	0.0048		-0.0084	-0.0004	-0.0012	0.0007	0.0015	0.0052
70	0.0042	-0.0076	-0.0044	0.0044	-0.0008	-0.0028		-0.0033	-0.0035	-0.0031	0.0034	0.0030	0.0032
75	0.0062	-0.0064	-0.0056	0.0056	0.0001	0.0000		-0.0082	-0.0048	-0.0008	0.0076	-0.0007	0.0033

*red cells denote observations outside manufacturer’s specifications (no confidence interval applied)

REDM Observation Errors at 95% CI

	10m												
	Horizontal Angle of Incidence							**Combined Angle of Incidence**					
0	-0.0004	-0.0002	-0.0006	-0.0002	-0.0004	-0.0006		-0.0002	0.0000	0.0000	0.0001	0.0000	0.0002
25	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002		0.0005	-0.0006	-0.0006	-0.0002	0.0000	0.0002
30	-0.0002	-0.0002	-0.0002	-0.0004	-0.0002	-0.0002		0.0005	-0.0005	-0.0005	-0.0001	-0.0001	0.0002
35	-0.0002	-0.0002	-0.0002	-0.0006	-0.0002	-0.0002		0.0004	-0.0005	-0.0004	0.0004	0.0004	0.0004
40	0.0002	-0.0004	-0.0004	0.0002	-0.0002	-0.0002		0.0004	-0.0012	-0.0011	0.0004	0.0009	0.0011
45	-0.0002	-0.0002	-0.0002	0.0000	0.0006	-0.0002		0.0003	-0.0003	-0.0002	0.0008	0.0003	0.0002
50	-0.0002	-0.0004	-0.0002	0.0000	0.0006	0.0004		-0.0018	-0.0018	-0.0002	0.0011	0.0000	0.0011
55	-0.0002	-0.0004	-0.0002	0.0002	0.0006	0.0008		-0.0016	-0.0013	-0.0016	0.0004	0.0008	0.0012
60	0.0000	-0.0002	0.0002	-0.0004	0.0002	0.0008		-0.0003	-0.0023	-0.0023	0.0017	0.0011	0.0011
65	-0.0002	0.0000	0.0008	0.0000	0.0004	0.0000		-0.0009	-0.0011	-0.0009	0.0011	0.0019	0.0011
70	-0.0002	-0.0006	0.0008	0.0000	0.0006	0.0015		-0.0012	-0.0027	-0.0007	0.0011	0.0005	0.0011
75	-0.0006	-0.0004	0.0004	0.0002	0.0018	0.0018		-0.0012	-0.0033	-0.0007	0.0011	0.0004	0.0015
	30m												
	Horizontal Angle of Incidence							**Combined Angle of Incidence**					
0	0.0000	-0.0004	0.0002	-0.0016	0.0000	-0.0006		0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
25	0.0004	0.0002	0.0002	0.0004	0.0002	0.0004		0.0003	0.0003	0.0003	0.0006	0.0006	0.0006
30	-0.0006	0.0004	-0.0002	0.0002	0.0004	0.0006		0.0005	0.0000	-0.0022	-0.0001	0.0005	0.0005
35	-0.0002	0.0004	-0.0006	0.0004	0.0004	0.0000		-0.0003	-0.0003	-0.0003	-0.0007	-0.0003	0.0003
40	-0.0002	0.0000	-0.0006	0.0006	0.0004	0.0002		-0.0011	-0.0011	-0.0011	0.0005	0.0003	0.0003
45	-0.0006	-0.0006	-0.0006	0.0006	0.0004	0.0004		-0.0004	-0.0004	-0.0004	0.0013	0.0008	0.0006
50	-0.0006	-0.0006	-0.0008	0.0012	0.0012	0.0004		-0.0006	-0.0006	-0.0006	0.0017	0.0006	0.0004
55	0.0002	-0.0006	-0.0008	0.0014	0.0014	0.0012		-0.0012	-0.0012	-0.0012	0.0018	0.0004	0.0002
60	-0.0006	-0.0002	-0.0010	0.0018	0.0006	0.0014		-0.0016	-0.0019	-0.0016	0.0016	0.0016	0.0019
65	-0.0006	-0.0014	-0.0010	0.0022	0.0014	0.0024		-0.0014	-0.0023	-0.0023	0.0024	0.0018	0.0013
70	-0.0022	-0.0004	-0.0018	0.0020	0.0018	0.0026		-0.0034	-0.0040	-0.0034	0.0030	0.0033	0.0017
75	-0.0030	-0.0025	-0.0015	0.0022	0.0018	0.0026		-0.0046	-0.0017	-0.0001	0.0038	0.0017	0.0028
	60m												
	Horizontal Angle of Incidence							**Combined Angle of Incidence**					
0	0.0000	-0.0008	-0.0006	-0.0001	0.0002	-0.0006		0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
25	0.0002	-0.0002	0.0002	0.0000	0.0002	-0.0006		0.0000	-0.0003	-0.0003	-0.0006	-0.0003	0.0001
30	0.0004	0.0000	-0.0004	0.0006	0.0004	-0.0006		0.0003	-0.0001	0.0004	0.0002	0.0006	0.0004
35	0.0004	0.0004	0.0000	0.0008	0.0008	-0.0010		0.0002	0.0005	-0.0001	0.0004	0.0004	0.0004
40	0.0004	-0.0006	0.0004	0.0014	0.0010	-0.0010		-0.0004	-0.0002	0.0003	0.0005	0.0006	0.0001
45	0.0008	-0.0006	0.0004	0.0014	0.0014	-0.0002		-0.0002	-0.0004	0.0003	0.0013	0.0013	0.0016
50	0.0022	-0.0006	0.0014	0.0026	0.0014	-0.0004		-0.0019	-0.0011	-0.0012	0.0013	0.0011	0.0012
55	-0.0002	-0.0008	0.0014	0.0046	0.0018	0.0004		0.0003	-0.0009	-0.0006	0.0009	0.0023	0.0014
60	0.0002	-0.0020	0.0018	0.0036	0.0035	0.0036		-0.0019	-0.0019	-0.0019	0.0013	0.0037	0.0025
65	-0.0024	-0.0088	-0.0022	-0.0014	-0.0004	0.0048		-0.0084	-0.0004	-0.0012	0.0007	0.0015	0.0052
70	0.0042	-0.0076	-0.0044	0.0044	-0.0008	-0.0028		-0.0033	-0.0035	-0.0031	0.0034	0.0030	0.0032
75	0.0062	-0.0064	-0.0056	0.0056	0.0001	0.0000		-0.0082	-0.0048	-0.0008	0.0076	-0.0007	0.0033

*red cells denote observations outside the 95% CI

Appendix E

Risk Assessment Matrix

Risk Matrix

Probability	Catastrophic	Major	Moderate	Minor
Frequent	3	3	2	1
Occasional	3	2	1	1
Uncommon	3	2	1	1
Remote	3	2	1	1

3=	Highest Risk
2=	Intermediate Risk
1=	Lowest Risk

Probability	Catastrophic	Major	Moderate	Minor
Frequent			**Driving to site** - Ensure vehicle is on good working order and follow road rules	**Data Collection** - Carry spare batteries and mobile phone in case of emergency **Party Size** - Work in pairs at a minimum **PPE** - Wear steel capped boots, high vision clothing, hat and sunglasses
Occasional			**Slips, Trips and Falls** - Avoid carrying sets of legs over uneven ground, avoid obvious hazards such as steep slopes	**Working Conditions** - Stay hydrated, check weather forecast prior to commencing fieldwork and wear/apply sun protection
Uncommon				
Remote				