UNIVERSITY OF SOUTHERN QUEENSLAND

STATISTICAL COMPARISON OF ADDITIVE REGRESSION TREE METHODS ON ECOLOGICAL GRASSLAND DATA

A thesis submitted for the award of
Honours

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CERTIFICATION OF DISSERTATION

I certify that the ideas, experimental work, results, analyses, software and conclusions reported in this dissertation are entirely my own effort, except where otherwise acknowledged. I also certify that the work is original and has not been previously submitted for any other award, except where otherwise acknowledged.

Emily Plant Date

Endorsement

Signature of Supervisors Date

Abstract

Boosted regression tree (BRT) and Bayesian additive regression tree (BART) models are both additive tree models that are theoretically well defined. However, BART is a relatively new technique to the field of ecology, while BRTs are widely used. By exploring the differences, range of obtainable results and relative limitations of both methods, this project aims to fill a gap in ecologists' collective knowledge to facilitate the use of both methods by ecologists in the future as well as determine if BART has some benefits over the widely used BRT method.

100 BRT and 729 BART models were fit on each of two grasslands datasets. One data set contained data from a period of drought, and the other dataset represents the recovery phase from the drought.

These two grassland datasets had 13 hydroclimatic and land use predictor variables. The response variable for both datasets was Enhanced Vegetation Index (EVI) trend, which is interpreted as a measure of grassland degradation and recovery. The settable parameters of both methods (BRT and BART) were varied to compare the performance of each method.

The models for each method were evaluated using three prediction error statistics; root mean square error (RMSE), mean absolute error (MAE), and the coefficient of determination (R^2). The best models across the two methods were assessed by inspecting the relative importance of predictor variables and two-way interactions, and the prediction error statistics. All analysis was conducted in R using the `dismo` package to fit boosted regression trees, and the `bartMachine` package to fit Bayesian regression trees.

BRT and BART models exhibited similar variable and interaction importance selection abilities, but the BART method generated models with similar or more favourable prediction error statistics than the BRT method (BART explained an additional 10.17% of variation than BRT on the drought dataset, and an additional 11.92% on the wetting dataset), indicating that BARTs may be more effective at modelling ecological data. BARTs also had other benefits including shorter run times, more reasonable defaults in its software implementation, and greater functionality of said software implementation, beyond model building and prediction functions.

There are some limitations to this study. Most notably, all models were only fit to two datasets from the one ecology scenario (grassland decline and recovery). Additionally, these datasets contained no missing data (and further, missing data was not simulated), so the relative abilities of BRT and BART to fit models and predict from missing data were not investigated. Therefore, future work in this area should include studies comparing BRT and BART models on multiple datasets (some of which should contain missing data) from a diverse range of ecological scenarios.

Acknowledgements

I have received much assistance and support in the course of writing this thesis. Firstly, I would like to thank my supervisors, Dr. Rachel King and Dr. Jarrod Kath, for their continuous encouragement, patient guidance, and insightful feedback. I also wish to extend my thanks to my partner and family for their unwavering support.

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Chapter 1

Introduction

Regression is a type of analysis which is used to predict values of one (dependant) variable using the values of one or more (predictor) variables. There are numerous types of regression analyses such as linear, generalized linear models, ordinary least squares, stepwise, ridge, and polynomial. Regression methods have different assumptions and uses, but they share similar challenges. Specifically, when fitting a regression model, the goal is to produce a model that fits the data well and produces good predictions with small errors, while not overfitting to the specific sample the model is being trained on.

1.1 Statistics Methods

1.1.1 Treed Regression

Decision tree models are one type of regression analysis and have two sub-types; regression trees which produce numeric predictions and classification trees which produce categorical predictions (Breiman et al., 1984). These tree models partition the predictor space into groups of homogeneous responses to these predictors by employing a series of binary splits. Binary splits are repeatedly applied to the output of the previous split until some pre-determined stopping condition is met (Elith et al., 2008).

Decision trees consist of internal nodes (including one root node) and terminal nodes (also known as leaves). Each internal node in the tree indexes a splitting rule which contains a splitting predictor variable and a splitting value. An observation is moved to the left child node if the splitting rule is satisfied for that observation and is assigned to the right child node if the rule is not satisfied (Breiman et al., 1984; Kapelner and Bleich, 2016).

Once the stopping condition is met, the final nodes are not split. These are the terminal nodes. Terminal nodes do not have associated splitting rules, instead they index a region in predictor space where the responses to predictor variables are approximately homogeneous (Breiman et al., 1984; Elith et al., 2008). Figure 1.1 displays an example of a single tree.

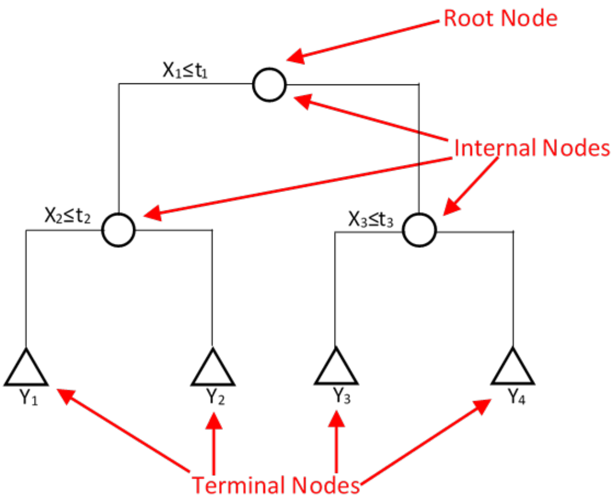

Figure 1.1: An example of a single decision tree, with four predicted responses; Y_1, Y_2, Y_3 and Y_4, three splitting predictor variables; X_1, X_2 and X_3 and three splitting values; t_1, t_2 and t_3. The root node, internal nodes, and terminal nodes are labelled.

Due to the hierarchical structure of trees, regression trees automatically model interactions as the response in any non-root node depends on responses to nodes higher in the tree (Elith et al., 2008). A terminal node represents a main effect only when it depends on only one variable. This occurs when the terminal node is a child of the root node, or when every internal node leading to the terminal node has the same splitting variable. Alternatively, an interaction effect occurs when the response depends on multiple variables, which is usually the case (Chipman et al., 2010).

Regression trees have many advantages. These single tree models can be easily visually represented in an intuitive way through the use of tree diagrams. Additionally, little data preparation is required because predictor variables can be of any type (numeric, binary, or categorical) and no assumptions are made about the distribution of the data or errors (as regression trees are a nonparametric method). They are also unaffected by differing scales of measurement among predictors, insensitive to outliers, and can not only accommodate missing data but also model any structure present in the missing data using surrogate splits (Breiman et al., 1984; Elith et al., 2008).

Decision trees also have some disadvantages which introduce uncertainty in their interpretation and limit their useability. They are not as accurate as other methods such as generalized linear models (GLM) and generalized additive models (GAM) and they also struggle to model smooth functions (Elith et al., 2008). Additionally, the structure of a single tree depends on the particular sample of data, and thus minor alterations in the training data can result in very different tree structures. This is mainly due to the hierarchical nature of trees; an erroneous split near the root of the tree is propagated to all the splits below (Elith et al., 2008). Additive tree methods such as bagging or boosting (Section 1.1.2) reduce this instability of single trees (Elith et al., 2008; Hastie et al., 2001).

1.1.2 Boosted Regression Trees

Boosted Regression Tree (BRT) models are a type of regression tree, specifically, they are additive regression trees. Additive regression trees are models where multiple individual simple trees are the terms of the model and BRTs use a process called boosting to fit these multiple simple trees and combine them into the one additive model (Elith et al., 2008).

Boosting is a sequential procedure, meaning that each simple tree is iteratively fitted and added to the model one after the other. This sequential nature is essential as each new tree added to the model is fit on the residuals of the (up to that point) current model. The processes of fitting these simple trees is also stochastic, which improves the predictive performance of the overall model. This stochasticity is introduced into the model via a parameter known as the "bag fraction" which controls the degree to which the model is stochastic. Figure 1.2 provides a graphical representation of the model fitting process (Elith et al., 2008).

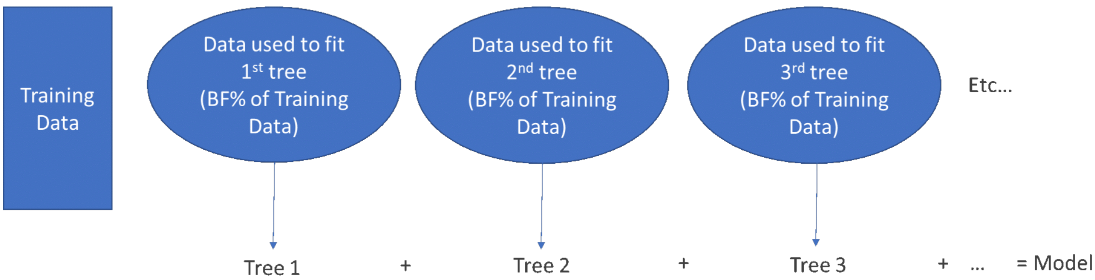

Figure 1.2: Illustration of the boosting process. Single trees are sequentially fit on the current model's residuals on some proportion of the training data known as the bag fraction (BF).

The bag fraction is the proportion of observations sampled from the complete training set, without replacement, that a single simple tree is fitted to (once the tree is fitted, then the data is replaced, and another sample is taken without replacement). Each time a new simple tree is added to the model, it is fitted on a subset of the training data and its structure is chosen (given the selected tree complexity) to explain the maximum amount of variation in the data that is not yet explained by the trees previously added to the model. This has the effect of creating many trees that could contain very different splitting variables and values from each other. Additionally, it also means that as the model acquires more trees, they are increasingly focused on the observations that are harder to accurately predict (Elith et al., 2008).

There is a similar process to boosting called bagging. While boosting is a sequential, iterative procedure which adds new trees to the model to explain the maximum amount of variation in the data that is not yet explained by the trees previously added to the model, bagging is a procedure that can be parallelised as fitting one simple tree does not depend on previous trees added to the model. This can be quite confusing as the proportion of observations to be sampled at each iteration of fitting a BRT model is called a bag fraction, however, BRTs employ boosting, not bagging (Breiman, 1996; Quantdare, 2017). Figure 1.3 provides a graphical illustration of the difference between the processes of fitting a single tree, multiple trees via bagging, and multiple trees via boosting.

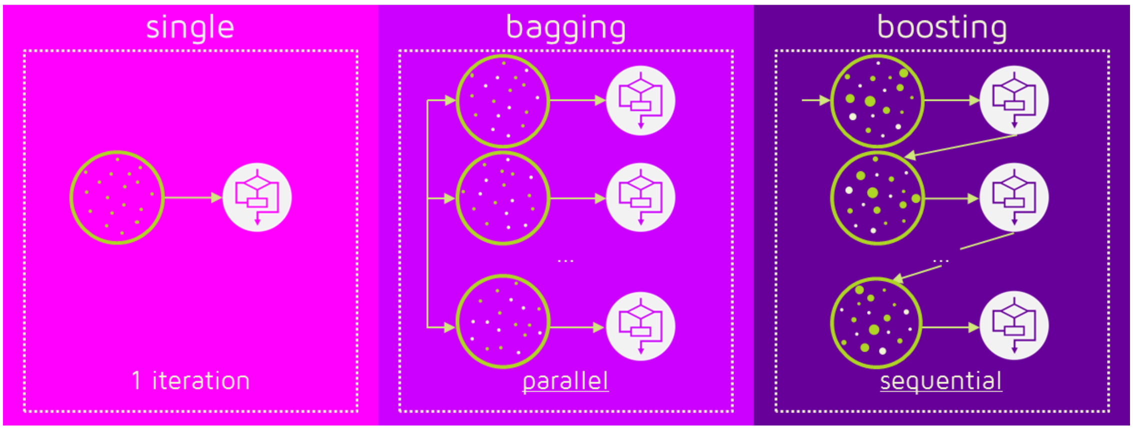

Figure 1.3: Bagging and Boosting both fit multple trees when building a model. When boosting, each new tree added to the model relys on all previous trees as when a tree is added to the model, it is fit to the residuals of the current (up to this point) model. However, when bagging, each tree is independant of one another. Figure from Quantdare (2017).

When creating a BRT model, values for three parameters must be provided, two of which provide regularisation (which is where some form of boundary or limitation is introduced to limit the model from overfitting the data) (Elith et al., 2008):

- The aforementioned bag fraction which controls the proportion of the training data to be sampled each time a tree is added to the model. Note that a different sample is generated for each tree and more importance is given to the observations that are not yet explained by the previous trees in the model.
- The tree complexity which determines the maximum depth of the simple trees and, by extension, whether or not interactions are fitted (and the

maximum order of interactions permitted to be modelled). Introduces regularisation to the model.

- The learning rate (or shrinkage) which regulates the contribution of each tree to the final model. Introduces regularisation to the model.

Together, the tree complexity and learning rate determine the number of trees in the final model. Decreasing or slowing the learning rate increases the number of trees required. Decreasing the tree complexity also increases the number of trees required. Therefore, as the tree complexity is decreased, the learning rate must be increased (and visa versa) to ensure sufficient trees are to be fitted in the model. Furthermore, regularisation in BRT models is achieved through jointly optimising the number of trees, learning rate, and tree complexity (Elith et al., 2008).

Optimal values for all three of these parameters can be estimated using k-fold cross validation (CV). CV is a re-sampling procedure used to test the effectiveness of (usually machine learning) models by withholding some proportion of the data when fitting the model to later test it. It can also be used to estimate optimal values of parameters in BRT models (Hastie et al., 2001). The general process for using cross validation to optimise a parameter of BRTs is outlined in Figure 1.4 (Elith et al., 2008).

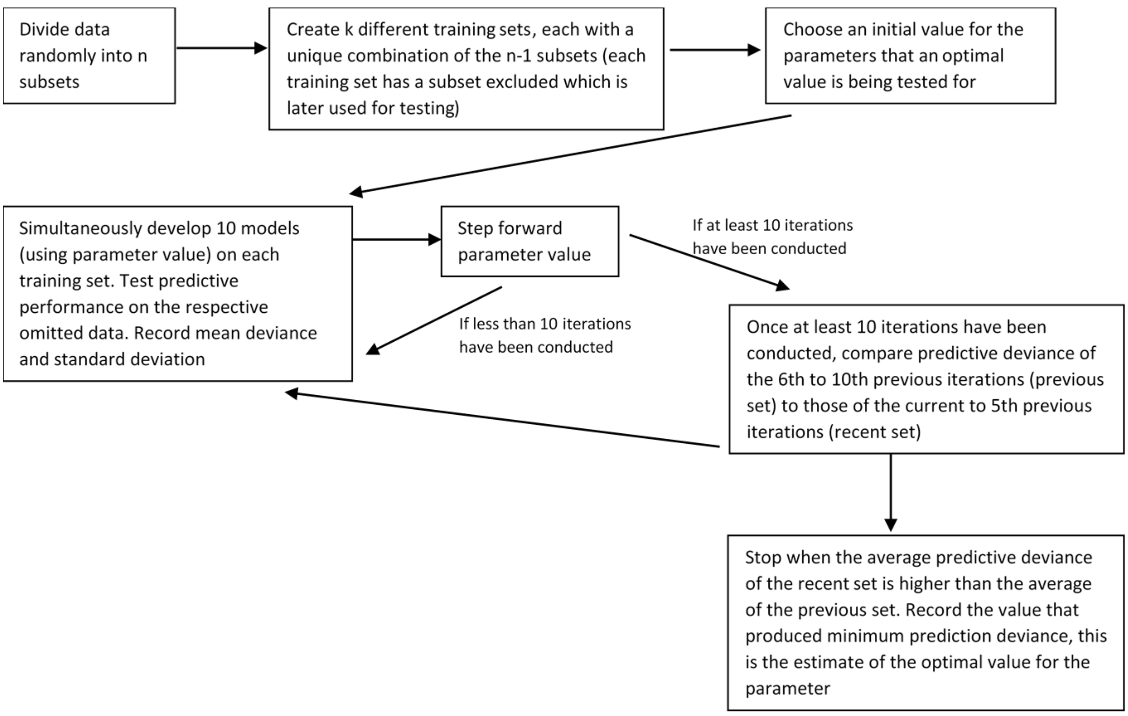

Figure 1.4: Flow diagram illustrating the process using of k-fold cross validation to estimate optimal values of the boosted regression tree parameters.

BRT models are often comprised of hundreds or thousands of individual trees so they can be quite complex. However, they are simple to generate predictions from and can be summarised, evaluated, and interpreted similarly to single regression trees. Additionally, standard errors can be estimated with bootstrap procedures (Elith et al., 2008).

BRTs can also be used to assess predictor importance. The relative influence or contribution of a predictor variable to the model can be calculated based on the number of times the variable has been used as a splitting variable, weighted by the squared improvement to the model as a result of each split and averaged over all the trees in the model (Elith et al., 2008).

1.1.2.1 *Assumptions of Boosted Regression Trees*

Boosted regression trees are a non-parametric method and therefore do not have any distribution assumptions. As such, they are very flexible and so it is acceptable to fit BRT models on many types of data including skewed, multimodal, and categorical (Biodiversity and Climate Change Virtual Laboratory, 2019).

1.1.2.2 *History of Boosted Regression Trees*

Boosted regression trees were originally derived from machine learning but have since been developed into an advanced form of regression (Elith et al., 2008). Freund (1995) introduced the AdaBoost algorithm (aka Discrete AdaBoost algorithm) which fit multiple weak classifying trees, on a sequential basis, to observations in a data set with evolving weights. The AdaBoost algorithm was later further refined by Freund and Schapire (1997) (Ridgeway, 1999).

Drucker (1997) first proposed and tested methods for extending boosting to regression problems (earlier works had only dealt with classification trees) and Breiman (1999) also explored the use of boosting in regression problems by involving optimisation of a regression loss function (Ridgeway, 1999). Additionally, Elith et al. (2008) wrote a paper which provided a working guide to boosted regression trees, primarily targeted towards ecologists.

1.1.3 *Bayesian Additive Regression Trees*

Bayesian Additive Regression Trees (BART) is an additive regression tree method that differs to Boosted Regression Trees as it relies on a Bayesian probability model and does not employ boosting Kapelner and Bleich (2016).

Bayesian techniques incorporate prior knowledge into statistical models in order to preserve and refine uncertainty by adjusting prior beliefs when considering new data. This contrasts with frequentist methods which simply model the probability of the data given a null hypothesis, whereas Bayesian methods allow the modelling of the probability of a hypothesis given the data (van de Schoot et al., 2013).

Bayesian techniques include three main components; a prior probability distribution, likelihood, and posterior probability distribution. The prior probability distribution (hereafter "prior(s)"), allows for knowledge based on previous experience about parameters to be included in a model. Priors can be either informative (strong) or uninformative (weak). The likelihood is derived from the data and is the probability of the observations in the dataset occurring assuming that the prior is accurate. The posterior probability distribution (hereafter "posterior(s)") is the refined predictions calculated from combining the prior and the likelihood (van de Schoot et al., 2013).

Because it is a Bayesian model, BART is comprised of several priors. Specifically, it includes priors for the tree structure, leaf parameters given the tree structure, and the error variance (which is independent of the other two priors). These priors exist in order to provide regularisation, which weakens the individual tree effects and thus prevents any single tree from dominating the model as a whole (Kapelner and Bleich, 2016). This ensures that that each of the trees explains a small and different part of the variation in the data (Chipman et al., 2010). In BART regularisation is introduced through the use of priors, while in BRT regularisation is introduced through the learning rate and tree complexity.

The tree structure prior is used to ensure shallow tree structures, so that the complexity of any single tree is limited. The shallower (less complex) the individual trees, the more model regularisation. Node depth is defined as the distance of the node to the root (so the depth of the root itself is zero). Nodes of depth d have prior probability $\alpha(1 + d)^{-\beta}$ of splitting and therefore of being non-terminal (Kapelner and Bleich, 2016). Alpha (α) is the "base" probability of splitting a terminal node (and therefore growing a tree), smaller values of α will tend to yield smaller trees. Beta (β) controls the rate at which the likelihood of a terminal splitting decreases as the terminal node's depth increases. Larger values of β make deeper nodes less likely to split, resulting in a lesser spread in the number of terminal nodes in the prior distribution (Chipman et al., 2010). For the tree structure prior, two hyper parameters (α and β) need to be chosen (Kapelner and Bleich, 2016). Figure 1.5 presents four possible tree structure priors generated using different combinations of α and β.

Figure 1.5 graphically illustrates how α and β affect the prior distribution of the number of terminal nodes. The top left and right plots display distributions that have been generated from the same β value ($\beta = 0.5$), but differing α values ($\alpha = 0.5$ and $\alpha = 0.95$ respectively). α controls the base probability of splitting a node, and this is reflected in the top two graphs; the top left has the smaller α value and therefore the distribution is shallower than that of the top right plot (which has the larger α value).

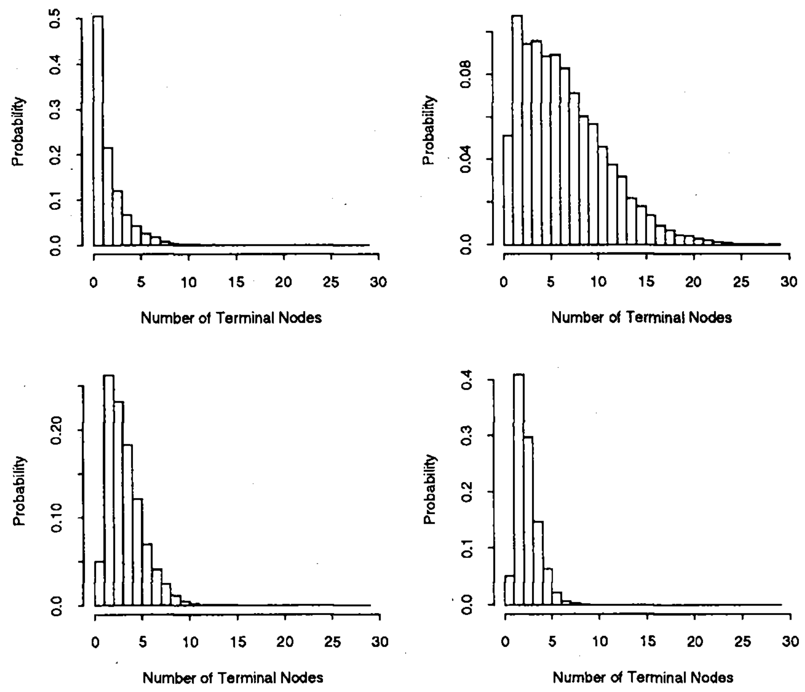

Figure 1.5: Prior Distribution on Number of Terminal Nodes. Top Left: $\alpha = 0.5$, $\beta = 0.5$, with prior mean 2.1; Top Right: $\alpha = 0.95$, $\beta = 0.5$, with prior mean 7; Bottom Left: $\alpha = 0.95$, $\beta = 1$, with prior mean 3.7; Bottom Right: $\alpha = 0.95$, $\beta = 1.5$, with prior mean 2.9. Figure from Chipman, George and McCulloch (1998).

The bottom left and right plots display distributions that have been generated from the same α value ($\alpha = 0.95$), but differing β values ($\beta = 1$ and $\beta = 1.5$ respectively). β controls the rate at which the likelihood of a terminal node splitting decreases as the terminal node's depth increases, this is illustrated in the bottom two graphs; the bottom left has the smaller of β value and displays a greater spread in the number of terminal nodes in the posterior distribution than the bottom right plot (which has the larger β value).

The leaf prior controls the leaf parameter, which is the fitted value assigned to any observation that falls within a particular terminal node. In other words, it is the treed model's estimate of the response in the particular partition of predictor space that terminal node represents. The aim of this prior is to shrink the leaf parameter towards the centre of the response distribution, introducing additional model regularisation (Kapelner and Bleich, 2016).

The prior on each leaf parameter is given by $\mu_\ell \overset{iid}{\sim} N\left(\mu_\mu/m, \sigma_\mu^2\right)$ where:

- The expectation (μ_μ) is the range centre (Kapelner and Bleich, 2016).
- The variance hyperparameter (σ_μ^2) is chosen so that the range centre plus or minus some number (k) of variances cover 95% of the responses in the training set. Therefore, with m trees, σ_μ is chosen so that $m\mu_\mu + k\sqrt{m}\sigma_\mu = y_{\min}$

and $m\mu_\mu - k\sqrt{m}\sigma_\mu = y_{\max}$. As k is increased, σ_μ^2 decreases which results in greater model regularisation (Kapelner and Bleich, 2016).

For the leaf prior, one hyperparameter (k) must be chosen. The number of trees (m) must also be defined. As k is increased, more shinkage is applied to the leaf prior, resulting in a more conservative fit (Kapelner and Bleich, 2016).

The error variance prior controls the error variance and limits the probability mass assigned to small values of σ^2 to prevent overfitting. It is chosen to be $\sigma^2 \sim \text{InvGamma}\left(\frac{v}{2}, \frac{v\lambda}{2}\right)$. λ is calculated from the training data so that there is a "q" percent a priori chance that the model will improve upon the root mean square error (RMSE) from an ordinary least square regression. A larger value of q results in more model regularisation (Kapelner and Bleich, 2016). v specifies the shape of the inverse gamma distribution (Chipman et al., 2010).

In total, there are five adjustable hyperparameters; α, β, k, v, and q. The number of trees, m, must also be chosen. As with boosted regression trees, the estimates of optimal values for the hyperparameters can be obtained through use of cross validation methods (Kapelner and Bleich, 2016).

The entire posterior probability distribution cannot be computed (except in trivial models) because of the large number of possible trees, so a Metropolis-within-Gibbs sampler is employed to explore the posterior. This algorithm simulates a Markov chain sequence of trees which tends to gravitate towards regions of higher posterior probability and is therefore used to search for higher posterior probability trees (Chipman et al., 1998; Kapelner and Bleich, 2016).

The Metropolis-within-Gibbs sampler with an initial tree, T_o and iteratively transitions the tree from T_i to T_{i+1} by proposing a change to T_i's structure which is then either accepted or rejected. Depending on the specific implementation of BART employed, there are three or four types of changes that could be proposed (Chipman et al., 1998; Kapelner and Bleich, 2016):

- Grow: Randomly pick a terminal node, split it into two new ones by assigning a new splitting variable and value.
- Prune: Randomly pick a parent of two terminal nodes and turn it into a terminal node by collapsing the nodes below it.
- Change: Randomly pick an internal node, and reassign it a new splitting variable and value.
- Swap (optional type of change): Randomly pick a parent-child pair that are both internal nodes, and swap their splitting variables and values (unless the other child has the identical splitting variable and value, in which case swap the splitting variables and values of the parent with that of both children).

BART can also be used for classification problems with categorical response variables. In this case, the error variance prior is not necessary, and the model assumes $\sigma^2 = 1$. The prior on the tree structure remains the same but the leaf prior requires some minor modifications (Kapelner and Bleich, 2016).

BART can also be used as a tool for variable selection. It is less effective on models that have a large number of trees as the redundancy introduced by many trees incorporates many irrelevant predictors into the model along with the relevant ones. When the redundancy is reduced (by decreasing the number of trees in the model) the predictors must compete with each other to improve the fit of the overall model and thus, fewer irrelevant predictors are incorporated in the model and used as splitting variables (Chipman et al., 2010).

1.1.3.1 Assumptions of Bayesian Additive Regression Trees

Bayesian additive regression tree models assume that that the error structure of a model is normal, homoscedastic, and mean centred. Additionally, the likelihood of responses in the terminal nodes are assumed to be a priori normal, and for classification problems (when the response is a categorical variable) a probit model is assumed. BART also requires convergence of the Metropolis-within-Gibbs sampler (Kapelner and Bleich, 2016).

1.1.3.2 History of Bayesian Additive Regression Trees

Breiman et al. (1984) first popularised classification and regression tree models within the statistical community. Then Chipman et al. (1998) proposed a Bayesian approach for finding classification and regression tree (CART) models. Although model averaging could be used with this method, it is not an ensemble method, rather a single tree model. Chipman et al. (2010) then extended this previous work to be an ensemble method and called the resulting model Bayesian Additive Regression Trees.

1.1.4 Other Tree-Based Regression

There are several tree-based regression techniques other than BRT and BART. One such technique is Bayesian treed models which, like BART, rely on a Bayesian probability model. However, this method fits parametric sub-models to the terminal nodes, rather than a simple mean or proportion. It is not an ensemble tree method but could be extended to include it (Chipman et al., 2002).

Another tree-based regression method is random forests. It is an ensemble tree method which introduces randomness by utilising bagging in addition to fitting each tree using a random subset of the predictor variables. In other words, a different random selection of predictor variables and observations from the full data set are used to fit each tree. This process is similar to bagging, but instead of just selecting a subset of cases/observations (as in bagging) a subset of cases/observations and predictor variables are selected (Breiman, 2001).

Dynamic trees are yet another type of tree-based regression. This type of model is a sequential, ensemble method (like BRT), but like BART, relies on a Bayesian probability model. The dynamic trees model is designed to adapt with the accumulation of new data and changes with additional data in such a way that a new observation leads to only a small change in the posterior (Taddy et al., 2011).

Figure 1.6 presents a "tree" of methods, to summarise the discussed models' relationships to each other, and indicates the type of response variables the models can be used with (continuous data, categorical data, or either). Models positioned below another model type, are subtypes of that model (e.g. logistic regression is a type of generalised linear model).

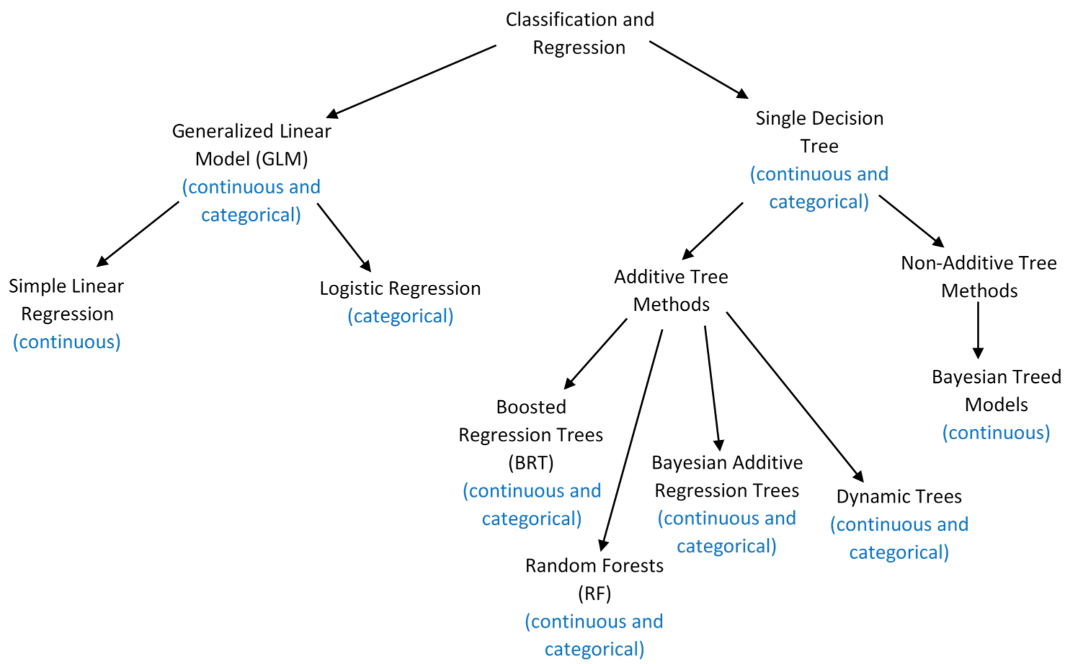

Figure 1.6: Tree of methods summarising the relationships between the discussed model types. Type of reponse variables that can be used with each model type is inticated in blue.

1.1.5 Inference and Prediction from Additive Regression Tree Models

Statistical models have the ability to be utilised for prediction (where the goal is to predict outcomes on new data points) and inference (where the goal is to learn about the data generation process and test hypotheses). Different types of modelling methods tend to be better suited to one of these two goals, although elements of both can be present in the one model type. When a method is used with the main goal of prediction, interpretability usually suffers, and when a method is used with inference in mind, the predictive power of the model typically is compromised. Generally, machine learning methods are more suited to predictive modelling as they are often treated as black boxes, greatly inhibiting their interpretability and thus usefulness for inference. In contrast, Bayesian methods are often used when inference is the main goal, due to their ability to incorporate assumptions or prior information about the data generating process (Döring, 2018; James et al., 2017).

Single decision trees have high interpretability, but due to the model limitations (Section 1.1.1), have low predictive performance. However, the additive and machine learning natures of BRT and BART inverts this relationship. Like other machine learning methods, they can be considered black box methods due to their low interpretability but have high predictive power. Despite their heavy association with prediction, both these methods can also be used with inference in mind. Both BRT and BART can be used for variable selection, to determine which variables are contributing most to the response (a valuable aspect of inference (James et al., 2017)). Additionally, BART is useful for inference as it relies on a Bayesian model which allows it to incorporate assumptions and prior knowledge of the data generating process.

1.1.6 Training and Test Sets

When building models, especially machine learning based methods, it is common practice to employ the use of training and test sets. This involves splitting a complete data set into two parts (one to train the model and one to test it). The model is then fit on the training set, and performance of the model is then evaluated using the test set. The purpose of this is to gain an unbiassed evaluation of the model fit, using data that was not used to build the model (Brownlee, 2017; James et al., 2017).

1.2 Ecological Context

Climate change is expected to increase the frequency and intensity of drought globally, so it is important to study the degradation and recovery of various environments during drought. Grasslands make an ideal candidate to study under drought because of their high sensitivity to variation in climate conditions. Kath et al. (2019) explored grassland responses a long-term drought using boosted regression trees.

The grassland data set that Kath et al. (2019) analysed was chosen to be used in this honours project for several reasons. Firstly, `dismo`, the R package that will be used here to fit boosted regression tree models, was created with ecologists in mind (Elith et al., 2008), so it was prudent to use ecological data here. Additionally, Kath et al. (2019) used the `gbm` package (Greenwell et al.,

2019) as well as several functions from the `dismo` package (Hijmans et al., 2017) to fit boosted regression tree models to the data and inspect them. Therefore, comparisons can be drawn between Kath et al's BRT results with the BRT and BART models fitted here.

Kath et al.'s grassland dataset was collected from an approximately 30,000 km^2 area in the Darling Downs, Australia (Figure 1.7). It contains a mix of extensively grazed native grasslands and dryland and irrigated cropping. It is also one of Australia's most agriculturally productive areas (Australian Bureau of Statistics, 2016; Kath et al., 2019).

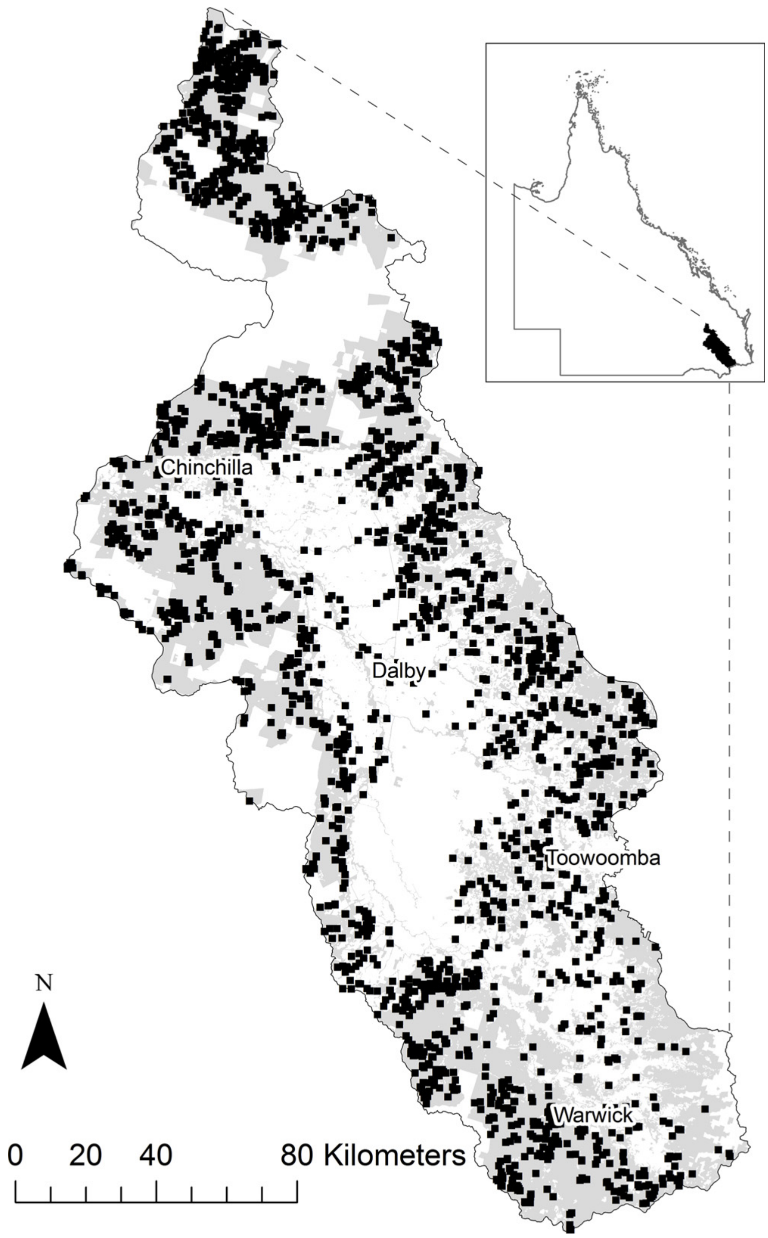

Figure 1.7: The study area of the Kath, Le Brocque, Reardon-Smith and Apan (2019) grassland dataset in southern Queensland, Australia. Grey areas are grazing lands from which enhanced vegetation index time series data from 2549 sites (black squares) were extracted.

The dataset contains 13 predictor variables (specifically eight hydroclimatic and five land-use variables) to model the response variable, enhanced vegetation index (EVI) trends in grasslands during drought and wet periods. The EVI measure represents vegetation primary productivity and was remotely sensed through satellite imagery (Kath et al., 2019).

1.2.1 *Remotely Sensed Data*

Satellite remote sensing (SRS) methods have been increasing in popularity amongst ecologists in recent years. These methods have many advantages compared to the more conventional alternatives such as unmanned aerial vehicles (UAVs) and in situ monitoring (Pasetto et al., 2018).

In situ methods are time, cost and labour intensive, and UAVs have low spectral resolution, limited flight endurance, and are often restricted by laws limiting the use of drones. In comparison, SRS has high spectral resolution, almost regular repetition of measurements (subject to atmospheric conditions), has been freely available for the past 2-3 decades, and there is an ever-expanding list of ready-to-use SRS products available (such as Landsat and MODIS). Additionally, SRS products are certified through quality assurance tests and standards (Pasetto et al., 2018).

1.2.2 *Creating EVI Response Variable*

EVI values in this dataset were gathered from the MODIS terra sensor. At each site in the study area (of which there were 2549), EVI values were derived as a time series. For each site, these values were then aggregated to mean monthly values in order to be consistent with the monthly format of the hydroclimatic predictor variables. Kath et al. (2019) then calculated trends over time in the EVI using Mann-Kendall tests. A negative trend in the EVI was taken as a measure of grassland decline and a positive change as a measure of grassland recovery.

1.2.3 *Splitting the Dataset into Drought and Recovery Phases*

Kath et al. (2019) used rainfall anomalies to split the complete dataset and classify multiple year-long drought (May 2002–May 2007) and wetting (May 2006–May 2011) phases. Boosted Regression Trees were then used to determine drivers of degradation and recovery in the drought and wetting phases respectively.

An overlap between the end of the drought phase and commencement of the wetting phase was included in order to ensure that any trend identified in the wetting phase was relative to drought condition – so the wetting phase data began towards the end of the drought.

1.2.4 *Currently Used Methods in Ecology*

Numerous statistical methods are currently used in ecology. Such techniques include; generalised additive models (GAMs), generalised linear models (GLMs), principal component regression (PCR), penalised regression methods such as ridge regression and least absolute shrinkage and selection operator (LASSO) and dimension reduction methods such as sliced-inverse regression (SIR) and principal Hessian directions (PHD). More recently various machine learning techniques such as boosted regression trees (BRT), random forests (RF) and multivariate adaptive regression splines (MARS) have begun to be utilised in ecological contexts (Dormann et al., 2013).

1.3 Research Questions

This project aims to answer three main research questions:

1. Do BRT and BART differ in identifying important variables and pair-wise interactions?
2. Is either model type (BRT or BART) more effective at modelling ecological data (in terms of reduced prediction error and ease of use)?
3. Are the defaults of the BRT and BART software implementations reasonable?

1.4 Importance of Research

Although both the BRT and BART methods are theoretically well defined within statistical literature, BRTs are widely used in the field of ecology while the BART method is a relatively new technique to the field. Specifically, a preliminary search of the literature has found no evidence of the use of Bayesian trees in ecology, while at the time of writing, Elith et al. (2008) "A working guide to boosted regression trees" (which outlines the use of BRTs in ecology) has been cited over 2000 times. As such, the differences, range of obtainable results and relative limitations of BARTs has had limited or no exploration in an ecological context. This project aims to contribute to filling this gap in ecologists' collective knowledge to facilitate the use of both methods, by ecologists in the future as well as determine if BART has some benefits over the widely used BRT method.

Chapter 2

Methods

2.1 Reference Analysis Models

In order to illustrate the benefits of BRT and BART, some more common and simple model types were first fit to the data as a form of reference analysis. The methods considered here are; linear regression, single regression tree, and random forest models.

The linear models were fit in base-R (R Core Team, 2018) using the `lm` function, however the single regression tree and the random forest models required R packages. The single regression tree analysis was conducted using the "`rpart`" package (Therneau and Atkinson, 2019) and graphs were created with the package "`rpart.plot`" (Milborrow, 2019). The random forest models were fit using the package "`randomForest`" (Liaw and Wiener, 2002).

All of these models were fit using the defaults, with the exception of one parameter in the random forest call. This parameter, "`importance`", was set to "`TRUE`" (its default is "`FALSE`"). This allowed the importance of predictors to be assessed.

2.2 Boosted Regression Tree Models

2.2.1 Software

All boosted regression tree analysis was conducted in R (R Core Team, 2018), using the R package `gbm` (Greenwell et al., 2019) and additional custom functions written by Elith et al. (2008), contained in the R package `dismo` (Hijmans et al., 2017). All plots that were not created in base R, were constructed in the R package `ggplot2` (Wickham, 2016).

The `gbm` package, or "Generalized Boosted Regression Models" package was originally developed by Greg Ridgeway (2013), but has since been written by Brandon Greenwell, Bradley Boehmke, and Jay Cunningham (2019). It contains functions for both fitting, evaluating and interpreting boosted regression tree models (Greenwell et al., 2019).

The `dismo` package, or "Species Distribution Modelling" package, was written by Robert J. Hijmans, Steven Phillips, John Leathwick, and Jane Elith (2017) for modelling species distributions using BRT models and other methods such as the "MaxEnt" (maximum entropy) model (Hijmans et al., 2017).

dismo is an extension of gbm and includes a function called gbm.step which fits a BRT model to one (or more) response variables and utilises cross-validation to determine the optimum number of trees to include in the model. This function is an extension of the gbm function "gbm" which also fits BRT models (Elith and Leathwick, 2017; Hijmans et al., 2017).

All dismo functions are designed to return exactly what the gbm functions return, but with extra components. Specifically, the gbm.step function includes 14 additional components, including gbm.call (contains details of the original gbm.step call), fitted (contains fitted values from the final tree), and fitted.vars (contains the variance of the fitted values) (Elith and Leathwick, 2017; Hijmans et al., 2017).

Aside from extending already existing gbm functions, dismo also includes some original functions designed to improve interpretation of the BRT models produced in either gbm or dismo. Such functions include gbm.plot which plots the fitted functions from a BRT model and gbm.perspec which plots pairwise interactions between variables (Hijmans et al., 2017).

Given the additional functionality available in dismo, this package was used instead of gbm to fit the BRT models in this project. Additionally, Kath et al. (2019), also used dismo functions when fitting their models, allowing a direct comparison between their models and the models fitted here.

It should be noted that there exists a new version of gbm, gbm3 (Metcalfe, 2016). However, this package was not completely developed at the time of writing, and thus was not employed here.

Table 2.1 illustrates several adjustable parameters when fitting a BRT model in both gbm (using gbm.fit) and dismo (using gbm.step). There are several other adjustable parameters (such as the maximum number of trees the model-building process will fit before stopping) but these are the most important as they are required parameters used to fit the model.

2.2.2 *Process*

For each dataset (drought and wetting; each with a sample size of $n = 2549$), 100 different models (using all combinations of the parameter values in Table 2.2) were fitted to a 70% training set ($n = 1784$). The number of trees included in the models were recorded. These models were fit using dismo's gbm.step function, which uses k-fold cross validation (See Section 1.1.2 for a brief discussion) to determine the optimal number of trees given the other parameter values when fitting a model. All models were then tested on 30% testing sets ($n = 765$) by predicting EVI on the test set observations and then calculating the RMSE (root mean square

Table 2.1: Adjustable parameters with their descriptions, code parameter name, default values, and possible values for `gbm.fit` in the `gbm` package and `gbm.step` in the `dismo` package (Greenwell et al., 2019; Hijmans et al., 2017).

Adjustable Parameter	Description	gbm (gbm.fit)		dismo (gbm.step)		Possible Values
		Code	Default Value	Code	Default Value	
Distribution	Character string specifying the name of the distribution to use for the error structure.	`distribution`	If not specified, `gbm.fit` will try to guess.	`family`	`bernoulli`	`bernoulli`, `poisson`, `laplace` or `gaussian` (dismo) AND As above plus: `tdist`, `huberized`, `adaboost`, `coxph`, `quantile`, `pairwise` (gbm)
Tree Complexity (tc)	Sets the complexity of individual trees - the maximum depth of variable interactions.	`interaction.depth`	1	`tree.complexity`	1	$tc \in (0, \infty)$ (must be integer value)
Learning Rate (lr)	The shrinkage parameter applied to each tree. Sets the weight applied to individual trees.	`shrinkage`	0.001	`learning.rate`	0.01	$lr \in (0, \infty)$
Bag Fraction (bf)	Sets the proportion of observations used in selecting variables - the fraction of the training set observations randomly selected to train the next tree in the model-building process.	`bag.fraction`	0.5	`bag.fraction`	0.75	$bf \in (0, 1)$
Number of Trees (nt)	The total number of trees to fit in the BRT model. This is equivalent to the number of iterations.	`n.trees`	100	`gbm.step` will use cross validation to determine optimum number of trees.		$nt \in (0, \infty)$ (must be integer value)

error), MAE (mean absolute error) and R^2 (coefficient of determination) using the observed and predicted EVI values of the test set data.

Kath et al. (2019) fitted boosted regression tree models to each dataset with a bag fraction of 0.75, learning rate of 0.01, and tree complexity of 5 (Kath, J 2019, pers. comm., 21 October). This parameter combination is one of the 100 combinations considered here (Table 2.2).

Table 2.2: All parameter values that were considered in this study, with a total of 100 combinations. Default values for the `gbm.step` function in the `dismo` package are in red.

Parameter	Values Tested
Bag Fraction (*bf*)	0.3, 0.6, 0.75, 0.9
Learning Rate (*lr*)	0.001, 0.005, 0.01, 0.05, 0.1
Tree Complexity (*tc*)	1, 3, 5, 9, 13

Values for the bag fraction lower than 0.3 were not considered because that would mean that each tree would be trained on less than 30% of the data, which would drastically increase the likelihood that the models would be trained on non-representative data. Learning rate values were chosen to test the full range of feasible choices on this specific dataset. The smallest learning rate (0.001) is computationally slow and the `gbm.step` function would most often reach the default limit of the number of trees it could fit (10000), but the tree limit was then increased until the model-building process reached the optimum number of trees. The largest learning rate (0.1) allowed the trees to learn so fast that gbm occasionally refused to run the model because the model-building process was moving too fast down the gradient of the loss function. The values for tree complexity were also chosen to cover a broad range. The absolute minimum value for tree complexity is 1, denoting a decision tree stump, so this is the smallest value tested. Four more values for tree complexity, up to *tc* of 13 were also tested (so in total, five tree complexities were tested). A *tc* of 13 is quite large and indicates that each tree has the potential to fit 13-way interactions, meaning that it is possible that each tree is fitting an interaction with all 13 variables (however this is not certain as predictor variables can be used as splitting variables more than once in a tree).

Elith et al. (2008) indicate that higher tree complexity paired with smaller learning rates will provide the best model results. However, this is restricted by computing time. The longest models (with tree complexities of 1 and learning rates of 0.001) to fit took between 2 and 2.5 minutes to run.

Graphs were produced in R using `ggplot2` to display how the number of trees, and error statistics were affected by varying the tree complexity, learning rate, and bag fraction. Graphs to display variable importance, and calculations to determine inclusion proportions were produced using the `dismo` package.

2.3 Bayesian Additive Regression Tree Models

2.3.1 Software

All Bayesian additive regression tree analysis was conducted in R (R Core Team, 2018), using the R package `bartMachine` (Kapelner and Bleich, 2016). This `bartMachine` package, or "Bayesian Additive Regression Trees" package was written by Adam Kapelner and Justin Bleich, and is maintained by Adam Kapelner. It is an implementation of Bayesian Additive Regression Trees and contains some expanded features for data analysis and visualization and has several useful functions for testing the assumptions of BART models. Note that this implementation of BART does not include the "Swap" proposal as mentioned in Section 1.1.3 (Kapelner and Bleich, 2016).

There exists another BART implementation package for R. This package, called `BayesTree` (Chipman and McCulloch, 2016), was written by the original authors of the BART algorithm but has many drawbacks that make it unsuitable for this project. Firstly, unlike `bartMachine`, `BayesTree` cannot save models across R sessions, making it unsuitable for the comparison of multiple (several hundred) models, as is this case in this study. Secondly, `BayesTree` has no external predict function. It can predict on new data, but the test data must be included as an argument to the model building function. Because of this, `BayesTree` cannot predict on new data without refitting the model, making it cumbersome to use. `bartMachine` avoids this issue by including an external predict function. Additionally, `BayesTree` is significantly slower than `bartMachine` due it it's lack of parallelization. In contrast, `bartMachine`'s model building process is parallelized, as are several features including prediction and variable selection (Kapelner and Bleich, 2016).

`bartMachine` has several other useful features not found in `BayesTree`, but were not strictly relevant to this project. These features include; plotting functions for posterior credible and prediction intervals, plotting functions for visually inspecting convergence of the Markov chain Monte Carlo, ability to incorporate prior information on covariates, and ability to handle missing data in both the model building and prediction processes (Kapelner and Bleich, 2016).

Table 2.3 illustrates several adjustable parameters when fitting a BART model in `bartMachine` (using the function `bartMachine`). There are several other adjustable parameters (such as the number of MCMC samples to be discarded as "burn-in") but these are the most important as they are required parameters used to fit the model.

Table 2.3: Adjustable priors and hyperparamters with their descriptions, code parameter name, default values, and possible values for the `bartMachine` function in the `bartMachine` package (Chipman et al., 1998, 2010; Kapelner and Bleich, 2016).

Adjustable Prior	Adjustable Prior Description	Adjustable Hyperparameter	Adjustable Hyperparameter Description	`bartMachine` (`bartMachine`) Code	Default Value	Possible Values
Tree Prior	Has ability to enforce shallow tree structures, resulting in model regularisation. Nodes at depth d are nonterminal with prior probability $\alpha\,(1+d)^{-\beta}$	α	Controls the depth of trees. Shallower values tend to produce shallower trees.	`alpha`	0.95	$\alpha \in (0, \infty)$
		β	Smaller values tend to result in greater spread in the number of terminal nodes in the prior distribution (Figure 1.5).	`beta`	2	$\beta \in [0, \infty)$
Leaf Prior	This prior has the ability to shrink the tree parameters towards zero, limiting the effect of the individual tree components by keeping them small. The prior on each of the leaf parameters is given as: $\mu_\ell \overset{iid}{\sim} N\left(\mu_\mu/m, \sigma_\mu^2\right)$	k	Defines the variance of the leaf prior which is chosen so that the range centre plus or minus k variances cover 95% of the provided response values in the training set. Larger values result in greater shrinkage.	`k`	2	$k \in (0, \infty)$
		m	Number of trees to be included in the BART model.	`num_trees`	50	$m \in (0, \infty)$ (must be integer value)
Error Variance Prior	Controls the error variance. The prior is chosen to be $\sigma^2 \sim \text{InvGamma}\left(\frac{\upsilon}{2}, \frac{\upsilon\lambda}{2}\right)$	υ	Specifies the shape of the gamma distribution.	`nu`	3	$\upsilon \in (0, \infty)$
		q	λ is chosen from the data so that there is a q a priori chance that the BART model will improve upon the RMSE from an ordinary least squares regression. q limits the probability mass placed on small values of σ^2 to prevent overfitting. So, the higher values of q, the larger the values of the sampled σ^2, resulting in more model regularisation.	`q`	0.9	$q \in (0, 1)$

It should be noted that while Table 2.3 displays the ranges of each hyperparameter, bartMachine is coded in such a way that it will allow values of α that are negative as well as values greater than 1. Additionally, negative values are also allowed for β and k, and non-integer values are accepted for m, the number of trees.

2.3.2 Process

For each dataset (drought and wetting; each with a sample size of N=2549), 729 different models (using all combinations of the hyperparameter values in Table 2.4) were fitted to a 70% training set (n = 1784). These models were fit using bartMachine's bartMachine function, which simply fits a BART model using the given hyperparameter values. All models were then tested on a 30% testing set (n = 765) by predicting EVI on the test set observations and then calculating RMSE, MAE, and R^2 using the observed and predicted EVI values of the test set data.

Table 2.4: All parameter values that were considered in this study, with a total of 729 combinations. Default values for the bartMachine function in the bartMachine package are in red.

Parameter	Values Tested
α	0.5, 0.95, 2
β	0.5, 0.95, 2
m	50, 100, 200
k	0.25, 1, 2
v	3, 7, 10
q	0.8, 0.9, 0.99

The hyperparameter values were chosen to display a range of possible values for each hyperparameter. Because there are six different hyperparameter to be set, time constraints dictated that only three values of each hyperparameter could be tested (as opposed to the BRT analysis where hyperparameters had four or five tested values). This gave a total of 729 models, much more than the 100 BRT models that were fitted. This was achievable because the BART models were much faster to fit (around 20 to 40 seconds compared to the 2 to 2.5 minutes required for the largest BRT models).

To ensure reasonable values of each hyperparameter was chosen to be tested, multiple models were initially fitted on both data sets using the bartMachineCV function which builds a BART model by cross-validating over a grid of hyperparameter choices, and fits a model with what it determines as the optimal hyperparameter values from the options passed to it. The hyper parameter values that these models determined to be optimal (which were generally the same for both the drought and wetting datasets), were then included in the range of hyper parameter values tested Kapelner and Bleich (2016).

The final consideration when selecting which hyperparameter values should be tested, was the defaults of the package. All defaults (marked in red in Table 2.4) were tested.

Graphs were produced in R using `ggplot2` to display how the error statistics were affected by varying the hyperparameters. Graphs to display variable importance, and calculations to determine inclusion proportions were produced using the `bartMachine` package.

Note that the packages used to fit both model types (BRT and BART), have more functionality for researches to employ than is discussed here (e.g. plotting fitted functions and assessing normality assumptions). The research presented here is focused on determining the optimal parameters and best fitting models.

2.4 Comparison of Models

2.4.1 *Statistics Employed*

To find the "best" models, prediction error was assessed using three statistics; root mean square error (RMSE), mean absolute error (MAE), and of coefficient of determination (R^2). To calculate the MAE, the magnitudes of the errors are summed to obtain the total error, which is then divided by n. This is shown numerically in Equation 2.1. Calculating RMSE is only slightly more complex, the errors are squared and then summed to obtain the total squared error, which is then divided by n, and finally, the square root is taken (shown numerically in Equation 2.2) (Willmott and Matsuura, 2005). R^2 is calculated using Equation 2.3.

$$\mathrm{MAE} = n^{-1} \sum_{i=1}^{n} |\mathrm{P}_i - \mathrm{O}_i| \tag{2.1}$$

$$\mathrm{RMSE} = \left[n^{-1} \sum_{i=1}^{n} |\mathrm{P}_i - \mathrm{O}_i|^2 \right]^{\frac{1}{2}} \tag{2.2}$$

$$\mathrm{R}^2 = \frac{\sum_{i=1}^{n} \left(\mathrm{P}_i - \overline{\mathrm{O}}_i\right)^2}{\sum_{i=1}^{n} \left(\mathrm{O}_i - \overline{\mathrm{O}}_i\right)^2} \tag{2.3}$$

Where, Predictions: $\mathrm{P}_i; i = 1, 2, \cdots, n$ and Observations: $\mathrm{O}_i; i = 1, 2, \cdots, n$

Both RMSE and MAE are utilised here because although RMSE is a commonly used measurement of error, it is known to often be an inappropriate indicator of "average" error due to the fact that each error contributes to the total in proportion to its square (as opposed to its magnitude), meaning that large errors have a greater influence on the total square error than the smaller ones. This has the effect of RMSE becoming increasing larger than MAE calculated on the same data as the distribution of error magnitudes becomes more variable (Willmott and Matsuura, 2005). R^2 was also used to provide a measure of prediction error that was dissimilar in calculation to RMSE and MAE. The `dismo` and `bartMachine` packages do not natively provide these statistics, they were instead calculated in R using the formulae above.

2.4.2 Variable and Interaction Importance

Once the models with the least prediction error were identified, the importance of each predictor variable and interactions among predictors were investigated, to see if this relationship was consistent between both individual models and model type (BRT and BART). This was done because part of the functionality of treed methods is to assess predictor and interaction importance in the data generation process. So, it is of interest if BRT and BART identified the same predictor variables and interactions as influential.

2.5 Ecology Methods

No changes were made to the original dataset from Kath et al. (2019). Table 2.5 gives all the variables (and their units) that were used to form these datasets, as well as the abbreviations used to denote them. Kath et al. (2019) describes in detail how each variable was collected or calculated.

Table 2.5: All variable names, units, and abbreviations in the dataset from Kath et al. (2019).

Variable	Abbreviation
Potential evapotranspiration (PET) anomaly (atmospheric demand) (mm)	`evap_anom`
Mid soil moisture layer anomaly (10-100cm) (mm)	`midsoil_anom`
Top soil moisture layer anomaly (0-10cm) (mm)	`topsoil_anom`
Deep soil moisture layer anomaly (100- 500cm) (mm)	`deepsoil_anom`
Mean cattle density	`Mean_cattle_density`
Top Soil moisture layer trend (mm month^{-1})	`s0_trend`
Deep Soil moisture layer trend (mm month^{-1})	`sd_trend`
Mid Soil moisture layer trend (mm month^{-1})	`ss_trend`
Potential evapotranspiration (PET) trend (atmospheric demand) (mm month^{-1})	`e0_trend`
Ratio of C3 to C4 grasses	`c3_c4ratio`
Woody vegetation cover (proportion)	`AHGF_FPC`
Dryland agriculture (proportion)	`Dryag_prop`
Irrigated agriculture (proportion)	`Irrag_prop`
Enhanced vegetation index (EVI) Trend	`EVItrend`

2.6 Inclusion of predictors

Correlations between the variables in the drought and wetting datasets are shown in Table 2.6. Both datasets contain both positive and negative correlations between the variables, however most correlations are less than | 0.7 |, which is the generally accepted level above which collinearity (when two predictor variables are highly correlated with each other) can affect regression models and their interpretation. The only exceptions are in the drought data set, where topsoil moisture layer trend was correlated with both midsoil moisture layer anomaly (r = 0.75) and topsoil moisture layer anomaly (r = 0.78) (highlighted in yellow in Table 2.6).

When two variables have a correlation indicating collinearity, one of the variables is often removed from the dataset. In this case, top soil moisture layer trend would be removed, rather than removing the two variables it is highly correlated with (mid soil moisture layer anomaly and topsoil moisture layer anomaly). However, the top soil moisture layer trend variable was not removed in this study for two reasons. Firstly, leaving it in allows for direct comparison between the models for both datasets, which would not be possible if the drought data was missing a variable that was included in the wetting data. Secondly, Kath et al. (2019) also retained the topsoil moisture layer trend variable (also to allow direct comparison between the datasets), so this will allow comparison of our models with theirs.

Table 2.6: Matrix of correlations between variables. Correlations between variables in the drought dataset are in the lower half (red), and correlations between variables in the wetting dataset are in the upper half (blue text). Correlations that exceeded | 0.7 | are highlighted in yellow.

	PET anomaly	Mid soil moisture layer anomaly	Top soil moisture layer anomaly	Deep soil moisture layer anomaly	Mean cattle density	Top Soil moisture layer trend	Deep Soil moisture layer trend	Mid Soil moisture layer trend	PET trend	Ratio of C3 to C4 grasses	Woody vegetation cover	Dryland agriculture	Irrigated agriculture
PET anomaly		0.07	-0.14	0.21	-0.02	0.34	-0.15	-0.12	-0.15	0.27	-0.08	0.35	0.20
Mid soil moisture layer anomaly	-0.06		0.52	0.36	-0.13	0.19	-0.11	-0.04	-0.38	0.03	-0.23	0.25	-0.06
Top soil moisture layer anomaly	0.34	0.62		0.26	-0.20	0.30	-0.46	-0.23	-0.39	-0.34	-0.22	0.11	-0.12
Deep soil moisture layer anomaly	-0.40	-0.33	-0.57		-0.11	0.44	-0.37	-0.25	-0.19	0.14	-0.19	0.25	0.01
Mean cattle density	0.20	0.13	0.09	-0.02		-0.13	0.03	-0.04	0.06	-0.09	0.03	-0.03	0.07
Top Soil moisture layer trend	0.11	0.75	0.78	-0.56	0.05		-0.55	-0.02	0.01	0.00	0.20	-0.57	-0.15
Deep Soil moisture layer trend	-0.44	-0.37	-0.59	0.63	-0.23	-0.55		0.34	0.42	0.29	0.25	-0.35	-0.09
Mid Soil moisture layer trend	0.11	0.10	-0.05	-0.18	0.03	-0.02	0.12		0.07	0.39	0.17	-0.16	-0.07
PET trend	0.31	0.01	0.04	-0.09	0.24	0.01	-0.28	-0.06		0.17	0.09	-0.32	0.06
Ratio of C3 to C4 grasses	0.25	-0.34	0.33	-0.30	-0.09	0.00	-0.08	-0.08	-0.16		-0.10	0.15	0.02
Woody vegetation cover	0.07	0.23	0.15	-0.21	0.03	0.20	-0.32	-0.21	0.21	-0.10		-0.41	-0.19
Dryland agriculture	0.01	-0.49	0.40	0.32	-0.03	-0.57	0.33	0.13	-0.05	0.15	-0.41		0.12
Irrigated agriculture	0.08	-0.11	-0.04	0.11	0.07	-0.15	0.02	0.07	0.08	0.02	-0.19	0.12	

Chapter 3

Results and Discussion

To evaluate model performance, three error statistics (RMSE, MAE, and R^2) and variable and interaction importance were considered. This necessitates that the results be presented in context and thus, the results and discussion sections have been merged rather than presented as two separate sections. After the results for each model type have been discussed, comparisons between them are given, followed by discussions of the limitations of the model types, limitations of this study, significance of the project from both statistical and ecological standpoints, and possible future work.

3.1 Summary Statistics of Data Sets

Table 3.1 shows the five-number summary as well as the mean and standard deviation for each of the variables in both ecological datasets. Four of the variables (mean cattle density, woody vegetation cover, proportion of dryland agriculture and proportion of irrigated agriculture) contain the same values across both datasets as data for those variables was measured once for each site (Figure 3.1).

Of all the variables, mean cattle density has the greatest absolute mean and standard deviation in both datasets and the dependant variable, EVI trend, has the smallest absolute mean and standard deviation in both datasets. This however does not indicate anything substantial as both variables (mean cattle density and EVI trend) have units that are different from those of all other variables.

Each of the four variables that are consistent across wet and dry datasets are shown in Figure 3.1. Mean cattle density, and the proportions of woody vegetation cover, dryland agriculture, and irrigated agriculture are extremely positively skewed and have outliers. This is to be expected for the mean cattle density as it is bound on one end at zero but unbounded at the other, allowing for a few more extreme positive observations which, when combined with a high frequency of low cattle density, creates a positively skewed distribution. The proportions of woody vegetation cover, dryland agriculture, and irrigated agriculture are also constrained between zero and one. This does not necessarily lead to a positive skew, but these three land use variables have low proportions at most sites, with a few high proportion outliers which, in this case, creates a positive skew.

Table 3.1: Five number summary, mean and standard deviation for each variable in each dataset (wetting and drought).

Variable	Data Set	Minimum	Q1	Median	Q3	Maxiumum	Mean	Standard Deviation
Potential evapotranspiration (PET) anomaly (atmospheric demand) (mm)	Drought	0.0258	0.0306	0.0318	0.0331	0.037	0.0318	0.002
	Wetting	-0.0412	-0.0357	-0.0339	-0.0325	-0.0287	-0.0341	0.0023
Mid soil moisture layer anomaly (10-100cm) (mm)	Drought	-0.475	-0.305	-0.2332	-0.1879	-0.0776	-0.2485	0.0845
	Wetting	0.0002	0.0779	0.1112	0.1626	0.3066	0.1221	0.059
Top soil moisture layer anomaly (0-10cm) (mm)	Drought	-0.2242	-0.1477	-0.1182	-0.1047	-0.0598	-0.1258	0.0318
	Wetting	0.0354	0.0729	0.094	0.1162	0.184	0.0962	0.0283
Deep soil moisture layer anomaly (100- 500cm) (mm)	Drought	-0.2162	-0.0441	-0.0128	0.0026	0.0105	-0.0285	0.0425
	Wetting	-0.148	-0.0381	-0.0226	-0.0107	0.1102	-0.0243	0.0219
Mean cattle density	Drought	0.0055	7.9003	13.7614	25.9185	524.422	25.9158	43.904
	Wetting	0.0055	7.9003	13.7614	25.9185	524.422	25.9158	43.904
Top Soil moisture layer trend (mm month^{-1})	Drought	-0.1049	-0.0392	-0.0219	-0.0056	0.0073	-0.0251	0.0223
	Wetting	0.0174	0.0398	0.0558	0.094	0.2308	0.0718	0.0425
Deep Soil moisture layer trend (mm month^{-1})	Drought	-2.3822	-0.6193	-0.2918	-0.1381	0.1706	-0.4123	0.351
	Wetting	-0.1165	0.2127	0.6863	1.4192	7.7164	0.9427	0.9908
Mid Soil moisture layer trend (mm month^{-1})	Drought	-2.1425	-0.5265	-0.38	-0.2889	0.0707	-0.4232	0.2166
	Wetting	-0.076	0.3839	0.5083	0.6916	4.7674	0.5803	0.3537
Potential evapotranspiration (PET) trend (atmospheric demand) (mm month^{-1})	Drought	-0.0308	-0.0211	-0.0185	-0.0155	-0.0075	-0.0182	0.004
	Wetting	-0.0362	-0.027	-0.0246	-0.0223	-0.015	-0.0248	0.0035
Ratio of C3 to C4 grasses	Drought	0.2043	0.2323	0.2696	0.3488	0.7925	0.2969	0.0788
	Wetting	0.2054	0.2336	0.2718	0.3509	0.7983	0.2986	0.0795
Woody vegetation cover (proportion)	Drought	0.0002	0.04	0.0736	0.1333	0.5047	0.0974	0.0773
	Wetting	0.0002	0.04	0.0736	0.1333	0.5047	0.0974	0.0773
Dryland agriculture (proportion)	Drought	0	0	0.034	0.2363	0.9256	0.1576	0.2264
	Wetting	0	0	0.034	0.2363	0.9256	0.1576	0.2264
Irrigated agriculture (proportion)	Drought	0	0	0	0	0.9239	0.02	0.072
	Wetting	0	0	0	0	0.9239	0.02	0.072
Enhanced vegetation index (EVI) Trend	Drought	-0.0047	-0.0012	-0.0007	-0.0004	0.0012	-0.0008	0.0006
	Wetting	-0.001	0.001	0.0015	0.0019	0.0052	0.0015	0.0007

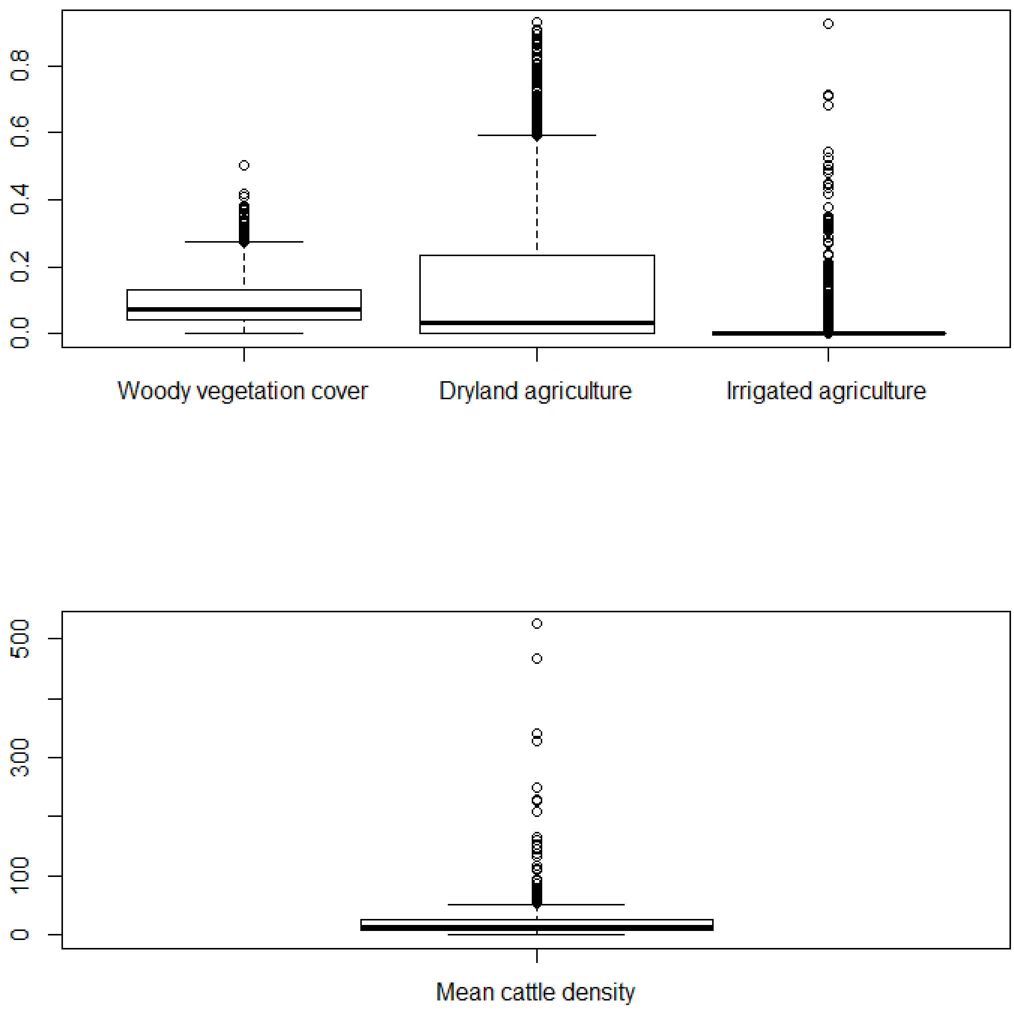

Figure 3.1: Boxplots of the four variables that are consistent across the wetting and drought datasets. Top: Woody vegetation cover, Dryland agriculture, and Irrigated agriculture (proportion of land used). Bottom: Mean cattle density.

3.1.1 *Drought Dataset*

The other 10 variables differ between the drought and wetting datasets. Figure 3.2 shows boxplots of these variables for the drought dataset in a single plot and Figure 3.3 contains multiple boxplots to better display the variables' distributions.

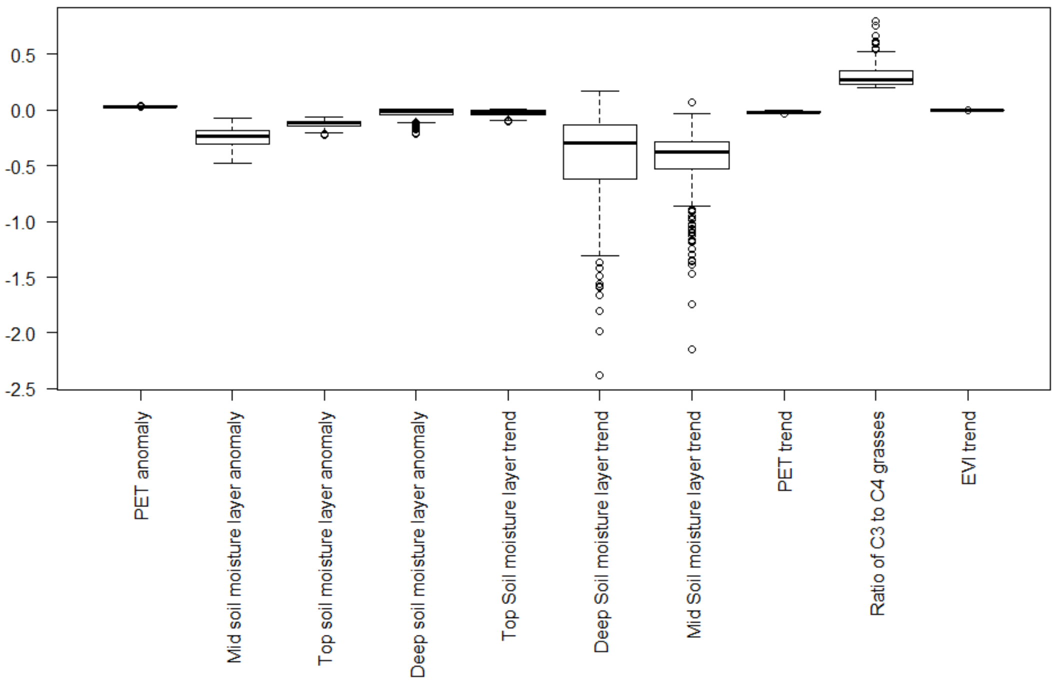

Figure 3.2: Boxplots of variables in the drought dataset. EVI trend and Ratio of C3 to C4 grasses are unitless, all anomaly variables are measured in mm, and all trend variables are measured in mm month^{-1}.

PET anomaly (mm), PET trend (mm month^{-1}), mid soil moisture layer anomaly (mm), and top soil moisture anomaly (mm) all appear to have approximately symmetric distributions (Figure 3.3). EVI trend, deep soil moisture layer trend (mm month^{-1}), mid soil moisture layer trend (mm month^{-1}), deep soil moisture layer anomaly (mm), and top soil moisture layer trend (mm month^{-1}) appear to have negatively skewed distributions. The ratio of C3 to C4 grasses has an extremely positively skewed distribution.

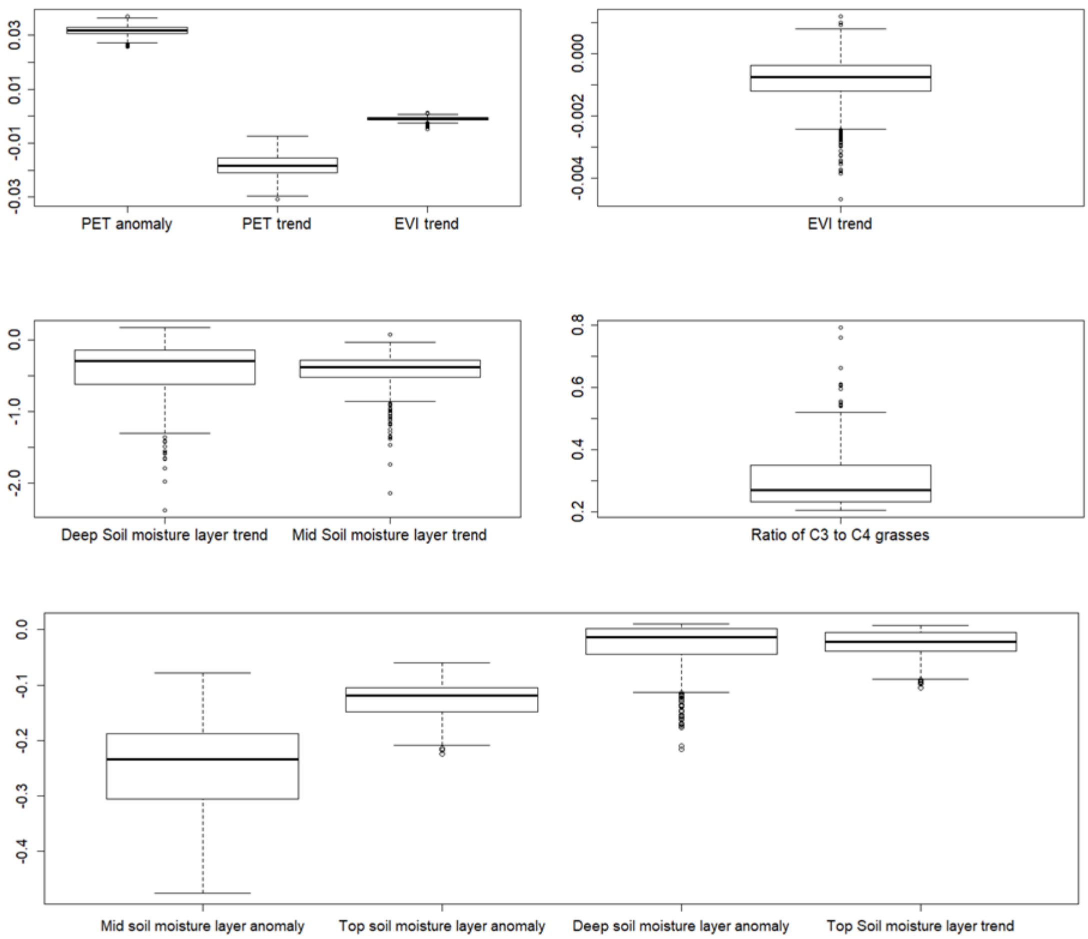

Figure 3.3: Boxplots of variables in the drought dataset arranged on separate y axes for enhanced readability. EVI trend and Ratio of C3 to C4 grasses are unitless, all anomaly variables are measured in mm, and all trend variables are measured in mm month^{-1}.

3.1.2 *Wetting Dataset*

Figure 3.4 shows boxplots of the 10 variables that differ between the two datasets variables for the wetting dataset in the one plot and Figure 3.5 contains multiple boxplots to better display the variables' distributions.

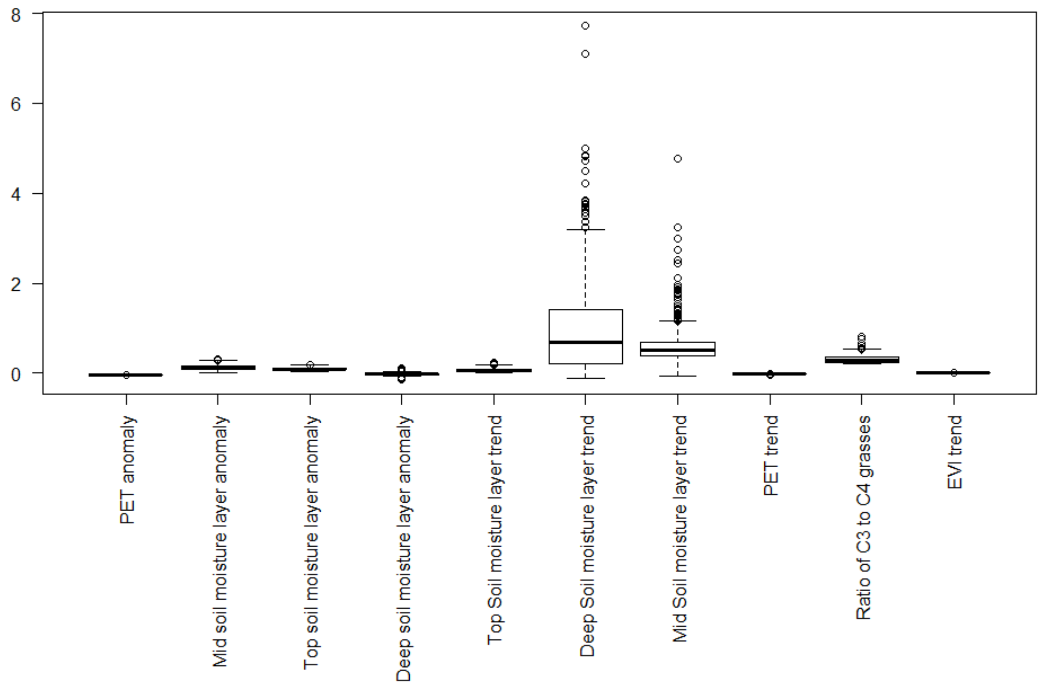

Figure 3.4: Boxplots of variables in the wetting dataset. EVI trend and Ratio of C3 to C4 grasses are unitless, all anomaly variables are measured in mm, and all trend variables are measured in mm month^{-1}.

Interestingly, many of the distributions of the variables in the wetting dataset have switched direction of skewness compared to in the drought dataset (Figure 3.3 and Figure 3.5). In the drought dataset, EVI trend, deep soil moisture layer trend, mid soil moisture layer trend, deep soil moisture layer anomaly, and top soil moisture layer trend all have negatively skewed distributions. But in the wetting dataset, they have positively skewed distributions (except for perhaps deep soil moisture layer anomaly, which appears more symmetric).

EVI trend, the dependant variable in this study, has the smallest range of all the variables, with a range of-0.0047 to 0.0012 in the drought dataset and -0.001 to 0.0052 in the wetting dataset (Table 3.1). These ranges indicate that there was both grassland decline (negative EVI trend values) and recovery (positive EVI trend values) in both the drought and wetting datasets. However, the majority of the values in the drought dataset were negative, and positive in the wetting dataset. Additionally, the outliers in the drought dataset generally consisted of very negative values while the outliers in the wetting dataset were mostly very positive values.

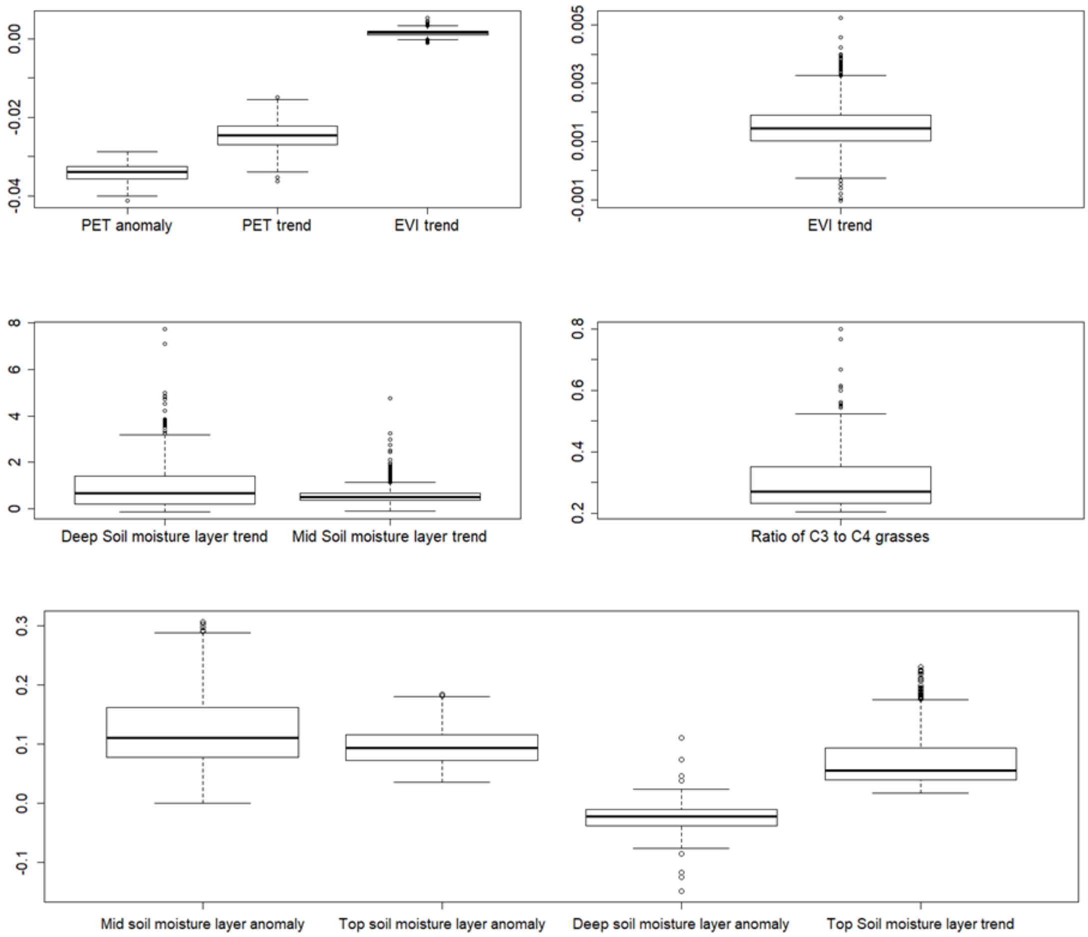

Figure 3.5: Boxplots of variables in the wetting dataset arranged on separate y axes for enhanced readability. EVI trend and Ratio of C3 to C4 grasses are unitless, all anomaly variables are measured in mm, and all trend variables are measured in mm month^{-1}.

3.2 Model Type Results

3.2.1 Reference Analysis Results

As part of the reference analyses, linear regression, single tree, and random forest models were fitted to the wetting and drought datasets as examples of initial baseline methods. Although these methods were not needed to address the research aims of this project, they help to illustrate the improvements in model fit that can be gained by employing more complex methods and the ways in which model performance can be evaluated. Figure 3.6 presents six graphs, each of which plots the predicted EVI trend against the observed EVI trend for a dataset regressed using one of the three methods employed here. It is evident that none of these methods' predictions were particularly accurate, with the random forest models, performing slightly better than the linear regression and single tree models.

Figure 3.7 graphically displays error statistics to quantify this prediction error. Three statistics were used, root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2). See Appendix Table A.1 for table of containing the numeric values of the error statistics displayed in Figure 3.7.

The RMSE and MAE statistics are both on the scale of the response variable, in this case EVI trend. These statistics indicate that the single tree method produced slightly less prediction error than the linear regression, and that the random forest had far more prediction accuracy than either of the linear regression or single tree. The R^2 statistic can be interpreted as the fraction of variance in the response variable (EVI trend) that can be explained by the model. This statistic indicates that the single tree modelled the data better than the linear regression, but that the random forest explained more variation in EVI trend than either of the other two methods. These reference analysis results highlight the improvements in model fit that can be gained as more complex methods are employed.

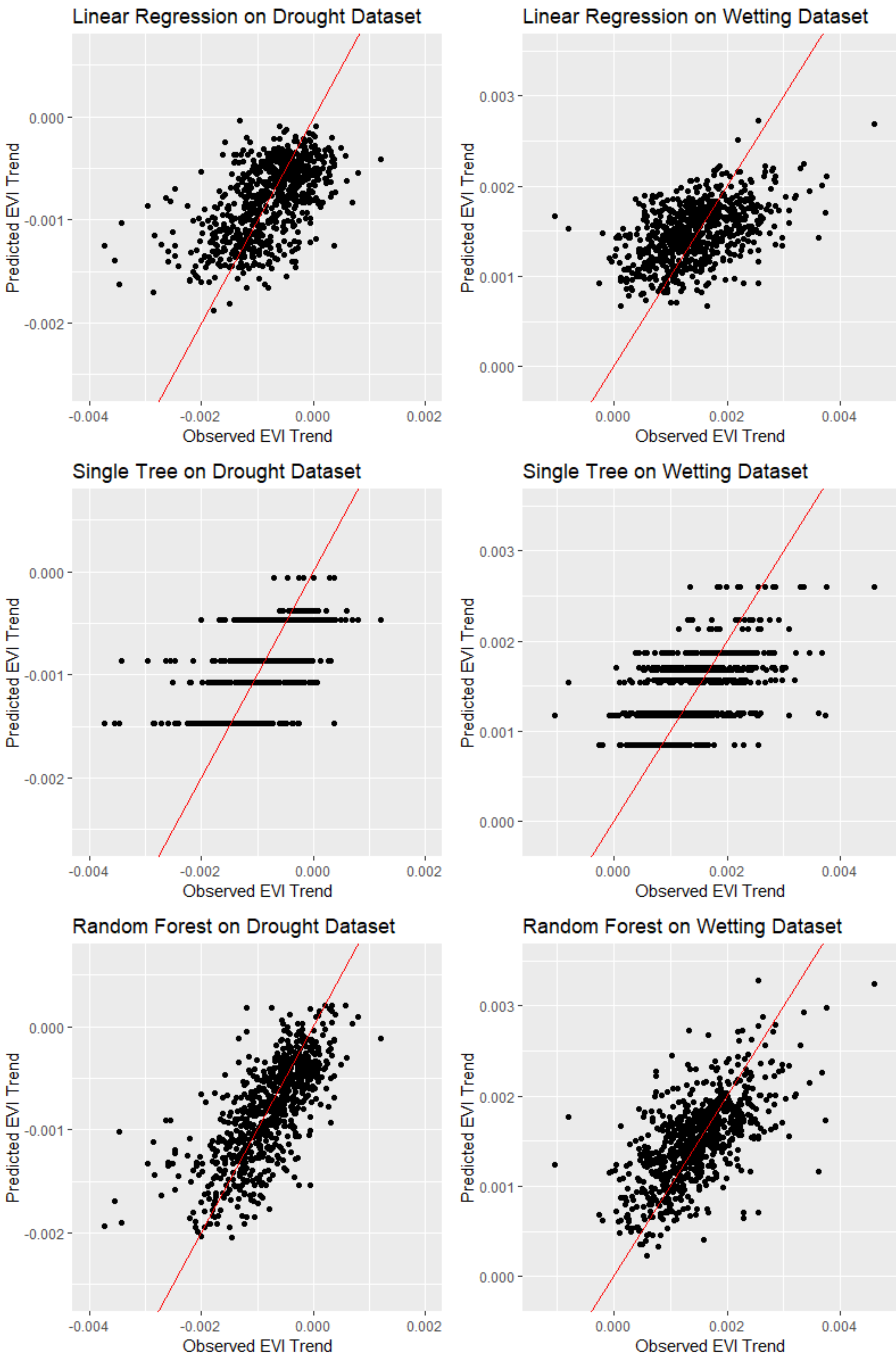

Figure 3.6: Predicted EVI trend against observed EVI trend for three model types (linear regression, single tree, and random forest) on both the drought (top graphs) and the wetting datasets (bottom graphs). The reference line where the observed EVI trend equals the predicted EVI trend is shown in red.

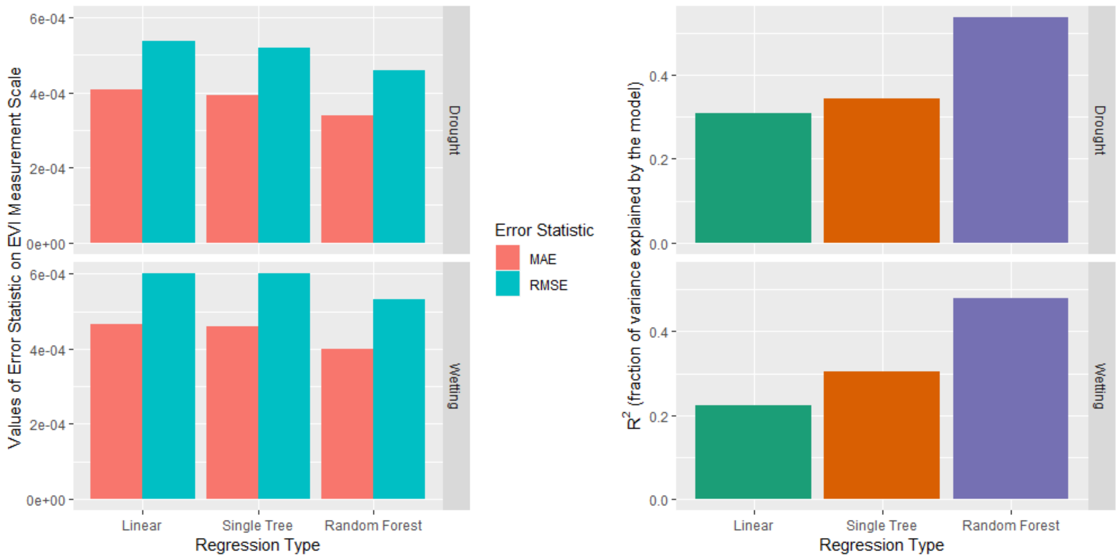

Figure 3.7: Graphical representation of the error statistics (RMSE, MAE, and R^2) for each model in Figure 3.6. Left: RMSE and MAE. Right: R^2. Graphs are faceted on dataset. See Appendix Table A.1 for table with numerical values of these statistics.

3.2.1.1 *Variable Importance*

When interpreting both single trees and additive treed methods (such as random forest), it is important to investigate variable importance. Additionally, linear regression can also be used for variable importance inference by examining the standardised coefficients (the higher the absolute value of the beta coefficient, the greater the importance of the variable) (Gelman and Hill, 2006). Standardised coefficients for the linear regression models fit on the drought and wetting datasets are presented in Table 3.2. Tree graphs for the drought and wetting single tree models were produced (Figure 3.8). Variable importance can be inferred from these trees based on the variables that informed the branching at each node. It is worth noting that the single tree model required a deeper tree to model the wetting dataset than it did for the drought dataset. Determing variable importance based on the random forest models is slightly more complex. Due to the number of individual trees in an additive tree model (such as a random forest model), it would be impractical to inspect every tree in the two random forest models to determine variable importance. Instead, variable importance measures are calculated. Figure 3.9 displays the random forest models' rankings of predictor variables on two importance measures (`%IncMSE` and `IncNodePurity`, see Liaw and Wiener (2002) for a description of how these measures are calculated) for the drought and wetting dataset respectively.

Table 3.2: Table of standardised coefficients for the linear regression models fit on the drought and wetting data. The higher the absolute value of the coefficent, the greater the importance of the variable.

Variable	Drought Data	Wetting Data
Potential evapotranspiration (PET) anomaly	0.0204	-0.2617
Mid soil moisture layer anomaly	0.1934	0.0552
Top soil moisture layer anomaly	0.1265	0.0451
Deep soil moisture layer anomaly	0.0609	-0.0085
Mean cattle density	0.0024	-0.0769
Top Soil moisture layer trend	0.0380	0.0930
Deep Soil moisture layer trend	-0.0795	-0.1570
Mid Soil moisture layer trend	0.2195	0.2062
Potential evapotranspiration (PET) trend	-0.1197	-0.0228
Ratio of C3 to C4 grasses	-0.1126	0.1838
Woody vegetation cover	0.1546	-0.1294
Dryland agriculture	-0.1678	0.0907
Irrigated agriculture	-0.0068	-0.0185

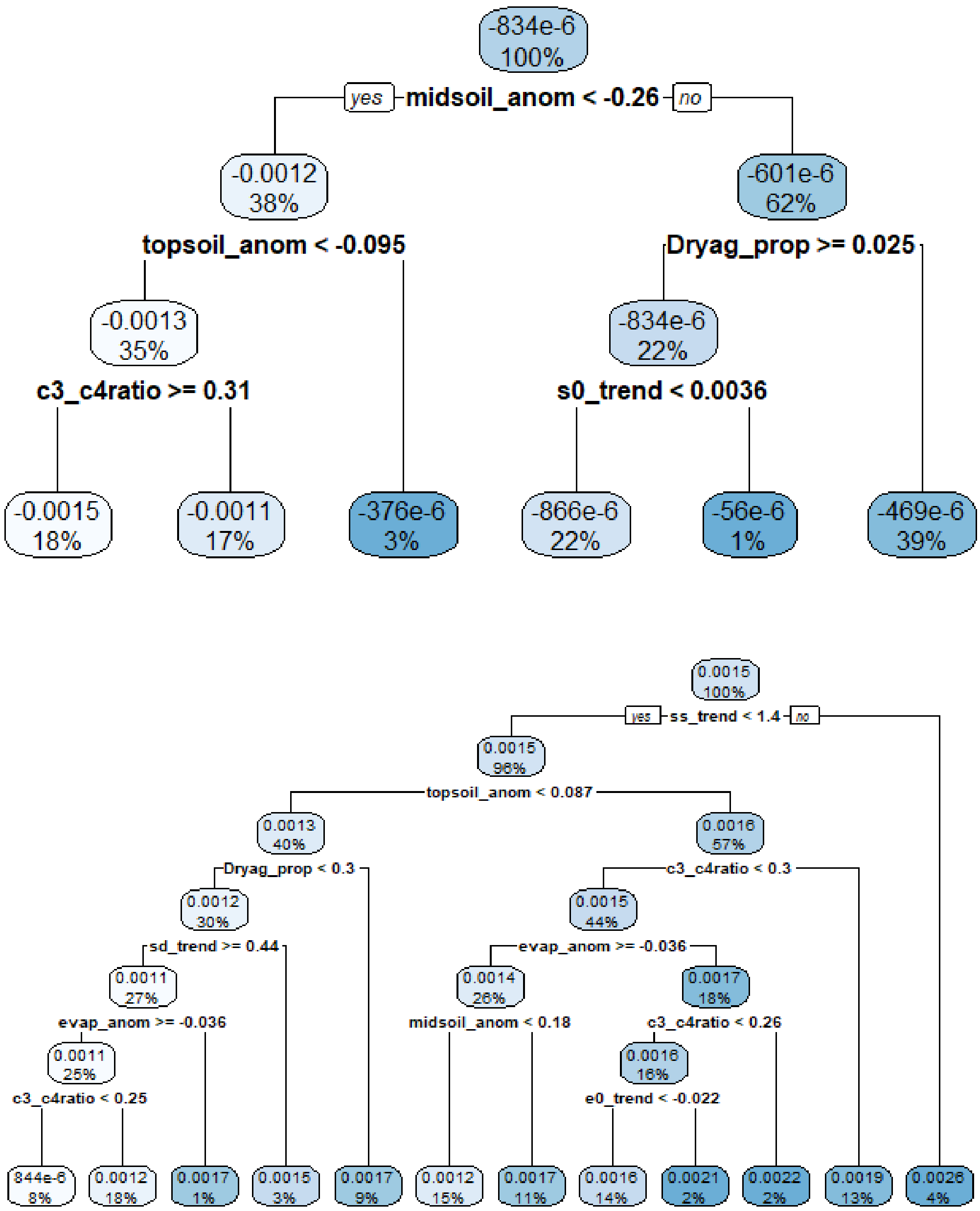

Figure 3.8: Tree graphs for the single tree models. Top: single tree on drought dataset. Bottom: single tree on wetting dataset. See Table 2.5 for abbreviated name meanings.

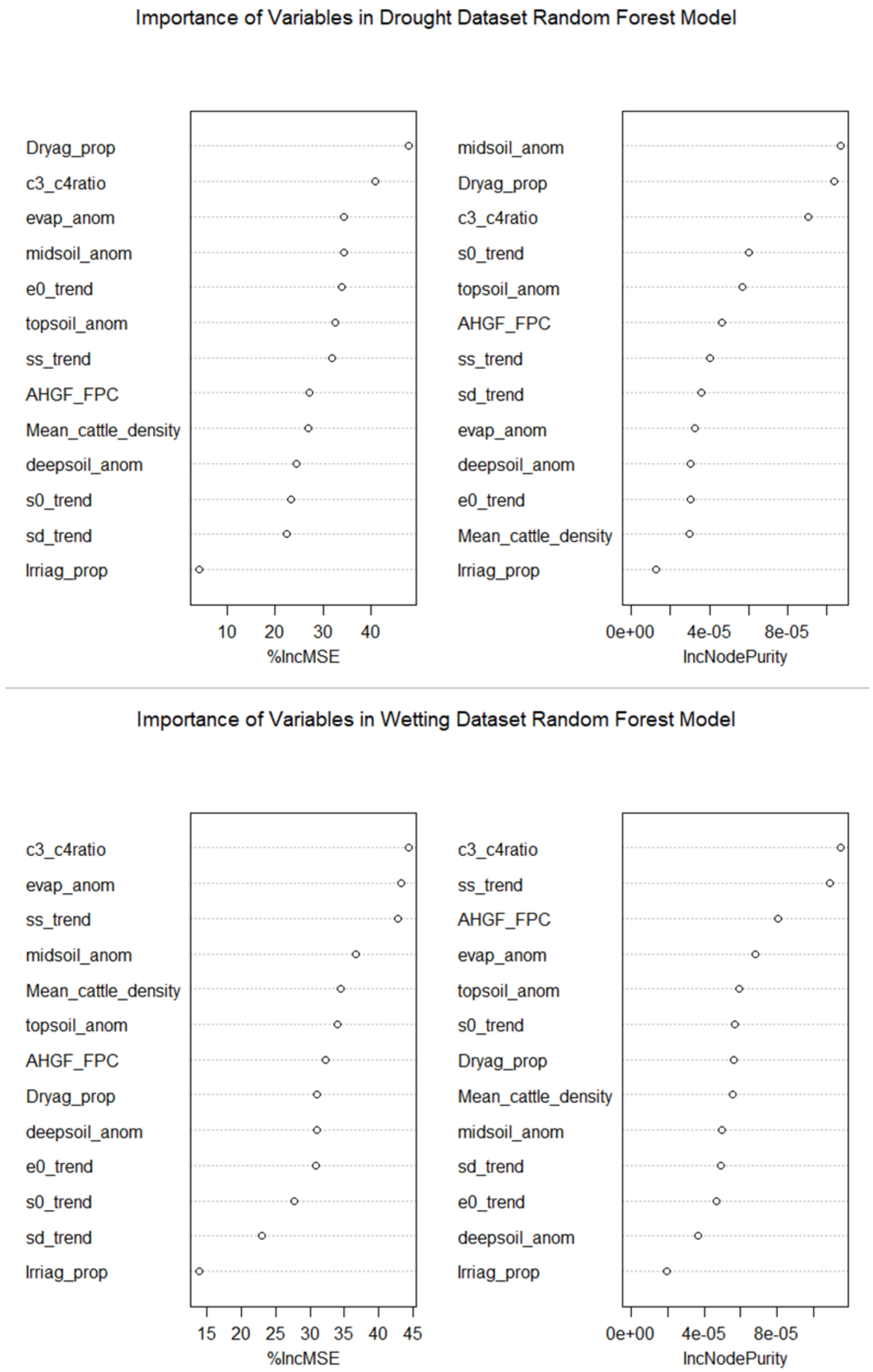

Figure 3.9: Rankings of predictor variables identified by the random forest models on two importance measures (See Liaw and Wiener (2002) for a description of how these measures are calculated). Top Left: Variable importance of the model built on the drought dataset, calculated using the `%IncMSE` method. Top Right: Variable importance of the model built on the drought dataset, calculated using the `IncNodePurity` method. Bottom Left: Variable importance of the model built on the wetting dataset, calculated using the `%IncMSE` method. Bottom Right: Variable importance of the model built on the wetting dataset, calculated using the `IncNodePurity` method. See Table 2.5 for abbreviated name meanings.

The single decision trees and the random forest models do show some commonality in variable importance. For the drought dataset, the single tree model (Figure 3.8) identified mid soil moisture layer anomaly (`midsoil_anom`), top soil moisture layer anomaly (`topsoil_anom`), proportion of dryland agriculture (`Dryag_prop`), ratio of C3 to C4 grasses (`c3_c4ratio`), and top soil moisture layer trend (`s0_trend`) as important variables (the variables that informed splits at each node). The random forest method (Figure 3.9) also identified these variables as important, but it indicated that PET anomaly (`evap_anom`) and PET trend (`e0_trend`) were important as well. The linear regression (Table 3.2) was in slightly less agreement. It, like the single tree and random forest models, identified mid soil moisture layer anomaly (`midsoil_anom`), proportion of dryland agriculture (`Dryag_prop`), and top soil moisture layer anomaly (`topsoil_anom`) as important, but it also identified mid soil moisture layer trend (`ss_trend`) and the proportion of woody vegetation (`AHGF_FPC`) as important predictors of EVI trend in the drought dataset.

The wetting models also had some variable importance commonality. The single decision tree model (Figure 3.8) identified the ratio of C3 to C4 grasses (`c3_c4ratio`) as the most important variable (as it appears three times in the tree), and PET anomaly (`evap_anom`) as the second most important variable (appears twice in the tree). Other variables identified by the decision tree as important are; mid soil moisture layer trend (`ss_trend`), top soil moisture layer anomaly (`topsoil_anom`), proportion of dryland agriculture (`Dryag_prop`), and mid soil moisture layer anomaly (`midsoil_anom`). The random forest method (Figure 3.9) also identified these variables as important (and one of the metrics indicated that the ratio of C3 to C4 grasses (`c3_c4ratio`) and PET anomaly (`evap_anom`) are the first and second most important variables respectively). However, each method also identified several predictor variables as important that the other did not. The decision tree also indicated that deep soil moisture layer trend (`sd_trend`) and PET trend (`e0_trend`) were important and the random forest indicated that the proportion of woody vegetation (`AHGF_FPC`), top soil moisture layer trend (`s0_trend`), and mean cattle density (`Mean_cattle_density`) were important. The linear regression (Table 3.2) indicated many of the same variables as important that the single tree and random forest models did. Specifically, it identified PET anomaly (`evap_anom`), mid soil moisture layer trend (`ss_trend`), the ratio of C3 to C4 grasses (`c3_c4ratio`), deep soil moisture layer trend (`sd_trend`), and the proportion of woody vegetation (`AHGF_FPC`) as important predictors of EVI trend in the wetting dataset.

When choosing the most appropriate model to use and interpret, both the variable importance and the error statistics must be considered. Here the random forest model would be preferred as it produced the best predictions (low RMSE and MAE, and high R^2), and also identified many of the same important variables that the linear regression and single tree did, indicating that these variables are likely to indeed be important (as all three model types mostly agree). The linear method would be a poor model choice as it produced the worst prediction error statistics, however, the single tree method would also be a poor choice it has similarly poor error statistics to the linear regression and also produced the grouped predictions that single decision trees suffer from (due to their method of repeating binary splits to group the observations).

3.2.2 *Boosted Regression Trees*

The number of trees comprising each boosted regression tree model is presented graphically in Figure 3.10. Figure 3.11 presents the three prediction error statistics (RMSE, MAE, and R^2) for each model. See Appendix Table A.2 and Appendix Table A.3 for values (rather than plots) of the number of trees and error statistics.

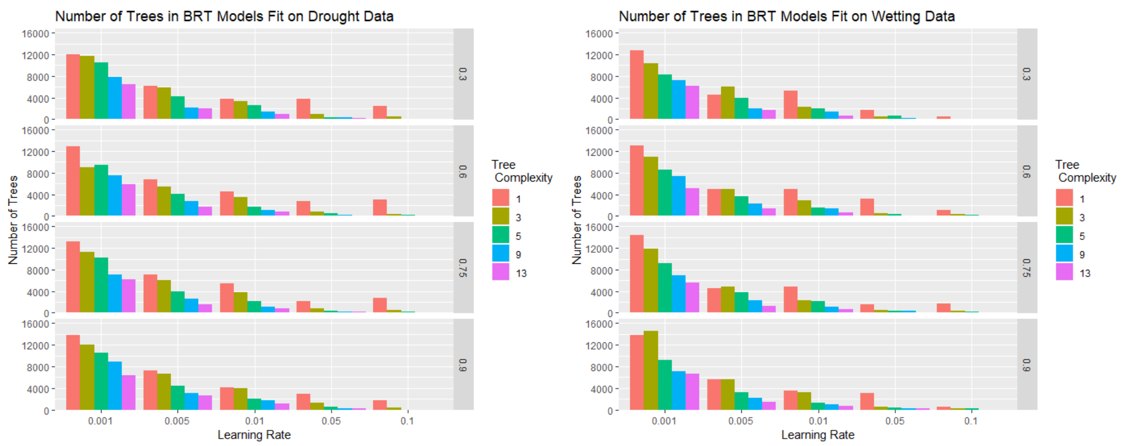

Figure 3.10: Number of trees each BRT model fit with combinations of the parameters Learning Rate (x axis), Tree Complexity (coloured), and Bag Fraction (faceted). Left: drought dataset models. Right: wetting dataset models. See Appendix Table A.4 and Appendix Table A.5 for lists of missing models.

Figure 3.11 illustrates an interesting result where the models fit on the wetting dataset had greater RMSE and MAE values, and lower R^2 values than the models fit on the drought dataset. This indicates that observations in the wetting dataset are more difficult to predict than their counterparts in the drought dataset.

In general, a tree complexity of 1 (simplest tree) performs badly on both datasets in terms of prediction error (Figure 3.11) and also requires more trees in the model which increases computing time (Figure 3.10). The prediction error and number of trees steadily decreases, and the proportion of variance in the data explained by the model increases as the tree complexity increases. The highest two tree complexities tested (9 and 13) generally produced models with less trees,

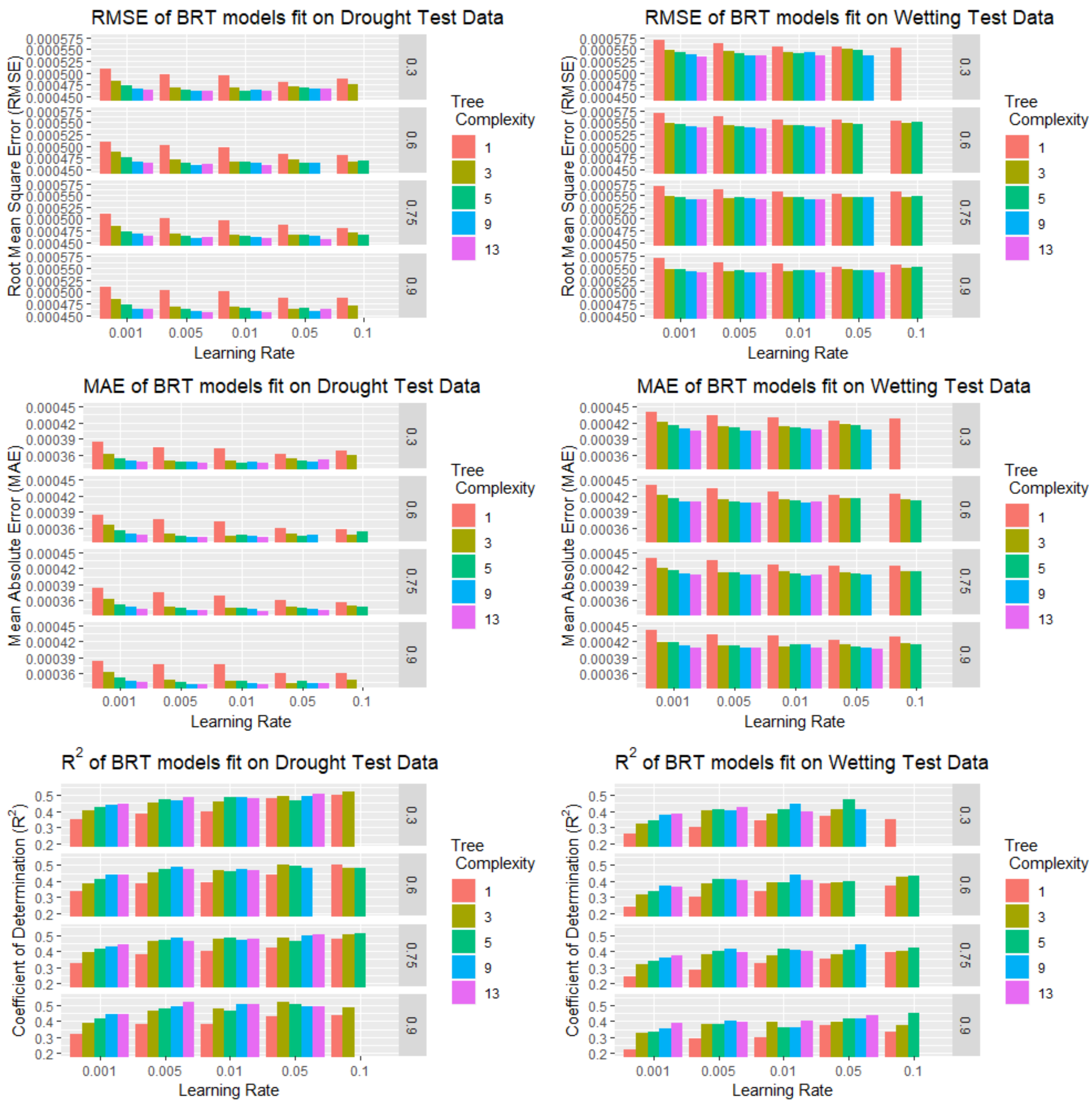

Figure 3.11: Error statistics (RMSE, MAE, and R^2) for each BRT model fit with combinations of the parameters Learning Rate (x axis), Tree Complexity (coloured), and Bag Fraction (faceted). Left Plots: drought dataset models. Right Plots: wetting dataset models. See Appendix Table A.4 and Appendix Table A.5 for lists of missing models.

lower RMSE and MAE statistics and higher percentages of variation explained by the models (R^2).

The value of the learning rate also has a similar effect on the number of trees (Figure 3.10) and prediction error (Figure 3.11). That is, as the learning rate increases, the prediction error and number of trees decreases and the variance in the data explained by the model increases. However, too high value of learning rate (around 0.1) seems to increase the prediction error. Therefore, it seems the best learning rate for these datasets is around 0.01 or 0.05, although 0.005 also appears to do well. It is also important to note that combinations of high learning rate and tree complexity results in the software being unable to fit the model, as the individual steps down the loss function become too large (Appendix Table A.4 and Appendix Table A.5). Overall, it appears that for these two datasets the best models have a high tree complexity (9 to 13), and a moderate learning rate (0.01 to 0.05).

Interestingly, the value of the bag fraction does not appear to impact the number of trees (Figure 3.10) or prediction error (Figure 3.11). This is counter to the theory which states that that the stochasticity introduced by the bag fraction would improve predictive performance (Elith et al., 2008).

3.2.2.1 *Variable Importance*

The six best performing models (in terms of RMSE, MAE and R2) are presented in Table 3.3. The variable importance of the best performing model for each dataset, using each of the tree prediction error statistics (highlighted red in Table 3.3) were calculated and presented in Figure 3.12.

Table 3.3: Best performing BRT models calculated on each dataset, with each of the three (RMSE, MAE, and R^2) error statistics. Best models for each combination of dataset and error statistic are highlighted in red.

	Drought Data				Wetting Data			
	BF	LR	TC	Error Statistic	BF	LR	TC	Error Statistic
RMSE	0.9	0.01	13	4.560×10^{-04}	0.3	0.001	13	5.353×10^{-04}
	0.9	0.005	13	4.568×10^{-04}	0.3	0.005	13	5.363×10^{-04}
	0.75	0.05	13	4.572×10^{-04}	0.3	0.01	13	5.375×10^{-04}
	0.9	0.01	9	4.576×10^{-04}	0.6	0.005	13	5.377×10^{-04}
	0.9	0.005	9	4.580×10^{-04}	0.3	0.005	9	5.377×10^{-04}
	0.75	0.01	13	4.585×10^{-04}	0.3	0.05	9	5.378×10^{-04}
MAE	0.9	0.005	13	3.398×10^{-04}	0.3	0.005	13	4.051×10^{-04}
	0.9	0.01	13	3.399×10^{-04}	0.3	0.001	13	4.056×10^{-04}
	0.9	0.005	9	3.400×10^{-04}	0.3	0.005	9	4.057×10^{-04}
	0.75	0.01	13	3.408×10^{-04}	0.6	0.005	13	4.060×10^{-04}
	0.9	0.01	9	3.411×10^{-04}	0.6	0.005	9	4.062×10^{-04}
	0.75	0.005	9	3.415×10^{-04}	0.6	0.01	9	4.066×10^{-04}
R2	0.3	0.1	3	0.5213	0.3	0.05	5	0.4732
	0.9	0.005	13	0.5166	0.9	0.1	5	0.4489
	0.9	0.05	3	0.5161	0.3	0.01	9	0.4464
	0.75	0.1	5	0.5141	0.75	0.05	9	0.4436
	0.3	0.05	13	0.5078	0.6	0.01	9	0.4371
	0.9	0.01	9	0.5071	0.9	0.05	13	0.4357

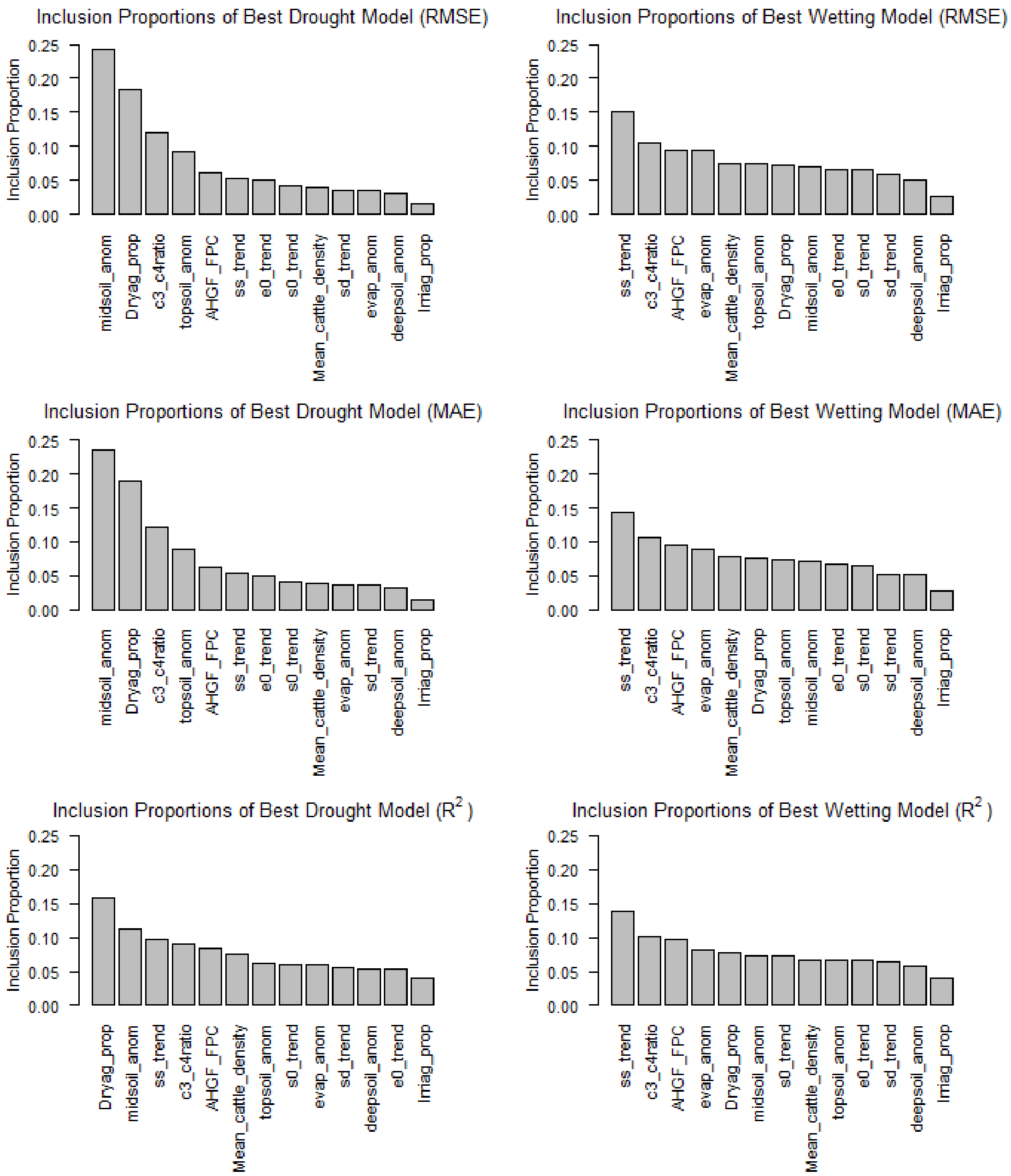

Figure 3.12: Variable Importance (measured with inclusion proportions – essentially the proportion of times that a variable has been used in a splitting rule) of each variable in the BRT models selected as "best" by three error statistics (RMSE, MAE, R^2). Left: models applied to drought dataset. Right: Models applied to wetting dataset. Inclusion proportions calculated using the `dismo` package (Hijmans et al., 2017).

Each plot in Figure 3.12 shows that all of the six models clearly indicate a single most important variable, and a single least important variable. The models fit on the drought dataset have a steeper drop off in importance between the variables than their wetting dataset counterparts, but all models display little difference in variable inclusion between the mid-ranked variables.

The drought dataset models selected by RMSE and MAE are in almost total agreement in terms of the variable importance rankings (Figure 3.12). The only difference in rankings between the two models is that deep soil moisture layer trend (`sd_trend`) and PET anomaly (`evap_anom`) swapped rankings. The most important variables identified by the best R^2 model differ somewhat from those identified by the best RMSE and MAE models, however most differences are in order of importance. For example, across the three drought models; the proportion of dryland agriculture (`Dryag_prop`) and mid soil moisture layer anomaly (`midsoil_anom`) have been identified as the top two most important variables, the ratio of C3 to C4 grasses (`c3_c4ratio`) and proportion of woody vegetation cover (`AHGF_FPC`) are also somewhat important, and proportion of irrigated agriculture (`Irrag_prop`) is consistently the least important variable.

Similarly, the wetting dataset models selected as "best" by the RMSE and MAE statistics were also in very close agreement in terms of variable importance, with the proportion of dryland agriculture (`Dryag_prop`) and top soil moisture layer anomaly (`topsoil_anom`) flipped in ranking of importance (Figure 3.12). The best R^2 model differs more from those two models, but is still rather similar. All models fit on the wetting dataset have identified the same top four most important variables (in the same order) and have also all selected the proportion of irrigated agriculture (`Irrag_prop`) to be least important.

In general, the drought dataset models indicate that the proportion of dryland agriculture (`Dryag_prop`), mid soil moisture layer anomaly (`midsoil_anom`), and ratio of C3 to C4 grasses (`c3_c4ratio`) are the most important variables influencing EVI trend in the drought, and that the proportion of irrigated agriculture (`Irrag_prop`), and deep soil moisture layer anomaly (`deepsoil_anom`) are the least important. The wetting dataset models indicate that the mid soil moisture layer trend (`ss_trend`), ratio of C3 to C4 grasses (`c3_c4ratio`), and proportion of wood vegetation cover (`AHGF_FPC`) are the most important variables influencing EVI trend during the wet phase, and that the proportion of irrigated agriculture (`Irrag_prop`), deep soil moisture layer anomaly (`deepsoil_anom`), and deep soil moisture layer trend (`sd_trend`) are the least important.

3.2.2.2 Interaction Importance

To assess the models' identifications of interaction importance, the top eight interactions identified by the models selected as "best" by RMSE, MAE, and

R^2 for each dataset (highlighted red in Table 3.3) are presented in Table 3.4. It is important to note that all interactions presented here are two-way interactions, as the software (dismo) was not capable of calculating the importance of three-way or higher interactions (Hijmans et al., 2017).

In terms of the drought dataset, the models selected by RMSE and MAE have almost identical interaction importance, with just two interactions swapping places (the interaction between mid soil moisture layer trend (ss_trend) and mid soil moisture layer anomaly (midsoil_anom) swapped with the interaction between the ratio of C3 to C4 grasses (c3_c4ratio) and top soil moisture layer anomaly (topsoil_anom)). However, the model selected as best by using the R^2 statistic has identified many interactions as important that the other two models did not. In fact, this model only shares two interactions in common with the interactions identified by the RMSE and MAE "best" models (interaction between mid soil moisture layer trend (ss_trend) and mid soil moisture layer anomaly (midsoil_anom), and between and top soil moisture layer anomaly (topsoil_anom) and mid soil moisture layer anomaly (midsoil_anom)).

The wetting dataset models selected by the RMSE and MAE statistics are in slightly less agreement in terms of interaction importance than their drought data counterparts; they only share six (out of the top 8) interactions in common. However, the R^2 model has identified five of the six interactions that the RMSE and MAE models also identified, which is much more consistent than the drought model selected by the R^2 statistic.

The models selected by the RMSE and MAE statistics tend to agree on variable and interaction importance measures much more closely than the models selected by the R^2 statistic. This is perhaps not surprising due to the similar nature of the calculations used to produce the RMSE and MAE statistics (see Equation 2.1 and Equation 2.2).

Table 3.4: Top eight interactions identified by the BRT models selected as "best" by RMSE, MAE, and R^2 for each dataset.

		Variable 1	Variable 2	Relative Strength of Interaction
Drought Data Models	Best RMSE Model (TC=13, LR=0.01, BF=0.9)	topsoil_anom	midsoil_anom	1.86×10^{-06}
		Dryag_prop	midsoil_anom	1.39×10^{-06}
		ss_trend	midsoil_anom	1.07×10^{-06}
		c3_c4ratio	topsoil_anom	9.06×10^{-07}
		Dryag_prop	c3_c4ratio	8.10×10^{-07}
		AHGF_FPC	s0_trend	7.30×10^{-07}
		AHGF_FPC	ss_trend	7.18×10^{-07}
		Irriag_prop	e0_trend	5.95×10^{-07}
	Best MAE Model (TC=13, LR=0.005, BF=9)	topsoil_anom	midsoil_anom	1.18×10^{-06}
		Dryag_prop	midsoil_anom	1.13×10^{-06}
		c3_c4ratio	topsoil_anom	1.03×10^{-06}
		ss_trend	midsoil_anom	1.02×10^{-06}
		Dryag_prop	c3_c4ratio	8.58×10^{-07}
		AHGF_FPC	s0_trend	6.87×10^{-07}
		AHGF_FPC	midsoil_anom	6.29×10^{-07}
		Irriag_prop	e0_trend	5.35×10^{-07}
	Best R^2 Model (TC=3, LR=0.1, BF=3)	sd_trend	midsoil_anom	3.38×10^{-06}
		c3_c4ratio	evap_anom	2.30×10^{-06}
		ss_trend	midsoil_anom	1.99×10^{-06}
		Dryag_prop	s0_trend	1.84×10^{-06}
		deepsoil_anom	evap_anom	1.80×10^{-06}
		topsoil_anom	midsoil_anom	1.32×10^{-06}
		Dryag_prop	AHGF_FPC	1.20×10^{-06}
		Dryag_prop	ss_trend	1.15×10^{-06}
Wetting Data Models	Best RMSE Model (TC=13, LR=0.001, BF=3)	evap_anom	ss_trend	2.26×10^{-06}
		evap_anom	c3_c4ratio	1.70×10^{-06}
		AHGF_FPC	ss_trend	8.27×10^{-07}
		topsoil_anom	c3_c4ratio	8.18×10^{-07}
		evap_anom	AHGF_FPC	7.06×10^{-07}
		e0_trend	c3_c4ratio	6.14×10^{-07}
		AHGF_FPC	c3_c4ratio	5.66×10^{-07}
		evap_anom	Dryag_prop	5.42×10^{-07}
	Best MAE Model (TC=13, LR=0.005, BF=3)	evap_anom	ss_trend	1.90×10^{-06}
		ss_trend	e0_trend	1.25×10^{-06}
		evap_anom	c3_c4ratio	1.15×10^{-06}
		AHGF_FPC	ss_trend	9.02×10^{-07}
		topsoil_anom	c3_c4ratio	8.58×10^{-07}
		e0_trend	c3_c4ratio	7.42×10^{-07}
		evap_anom	AHGF_FPC	7.21×10^{-07}
		evap_anom	s0_trend	6.65×10^{-07}
	Best R^2 Model (TC=5, LR=0.05, BF=3)	topsoil_anom	c3_c4ratio	2.56×10^{-06}
		evap_anom	AHGF_FPC	2.41×10^{-06}
		evap_anom	c3_c4ratio	1.92×10^{-06}
		ss_trend	c3_c4ratio	1.90×10^{-06}
		deepsoil_anom	midsoil_anom	1.71×10^{-06}
		AHGF_FPC	c3_c4ratio	1.64×10^{-06}
		evap_anom	ss_trend	1.43×10^{-06}
		ss_trend	s0_trend	1.43×10^{-06}

3.2.3 *Bayesian Additive Regression Trees*

Figure 3.13 and Figure 3.14 present the three prediction error statistics (RMSE, MAE, and R^2) for each of the Bayesian regression tree models. These statistics are averaged (with ±1 standard error) over all 729 models fit for each data set. Due to the high number of hyperparameters, two graphs for each error statistic on each dataset were produced. See Appendix Table A.6 and Appendix Table A.7 for values of error statistics.

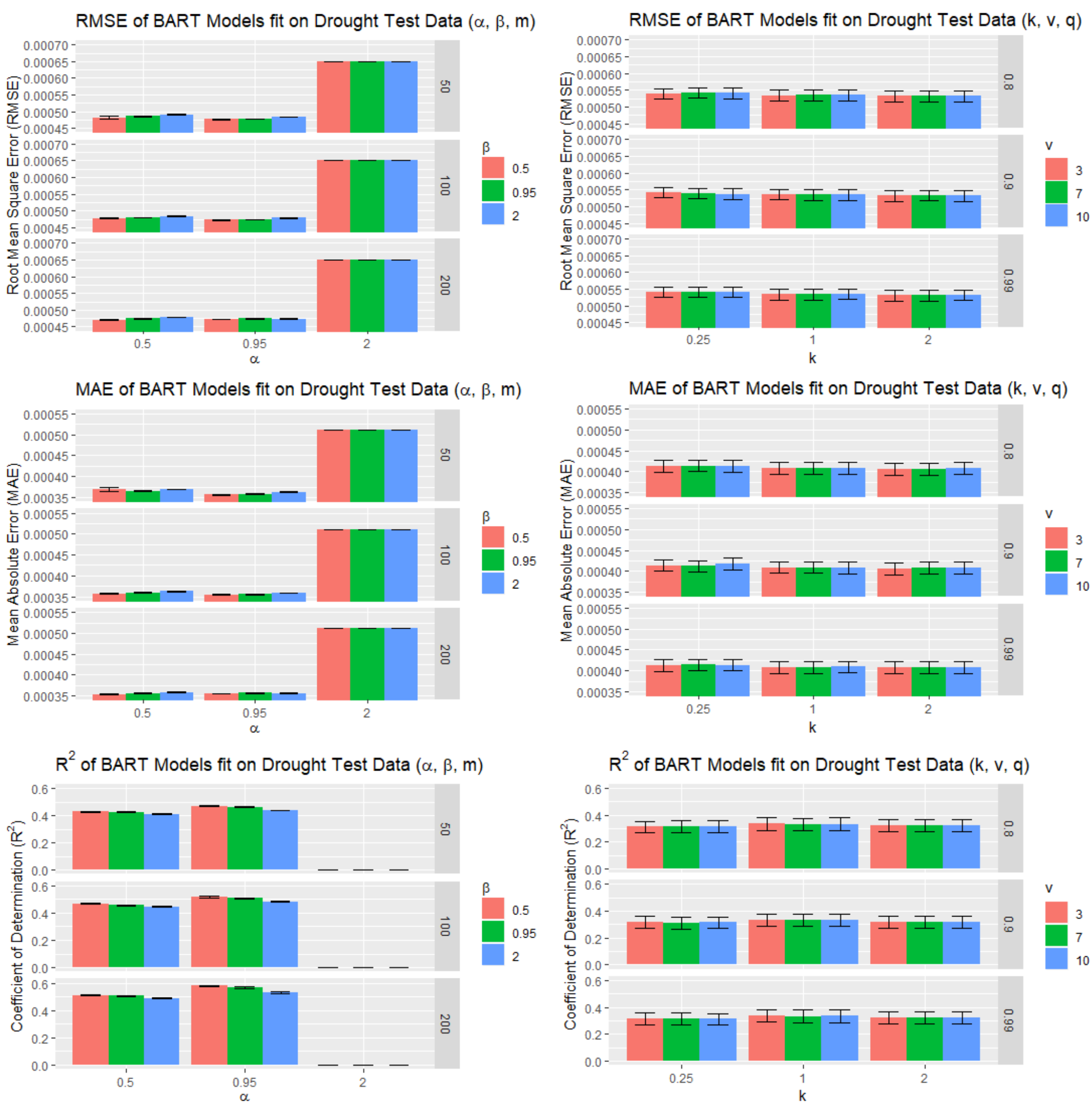

Figure 3.13: Error statistics (RMSE, MAE, and R^2) for each BART model fit on the drought dataset. Left Plots: Average error statistics for models with combinations of α (x axis), β (coloured), and m (faceted). Right Plots: Average error statistics for models with combinations of k (x axis), v (coloured), and q (faceted). Error bars denote ±1 standard error.

Figure 3.13 and Figure 3.14 show that models fit on the wetting dataset had greater RMSE and MAE values, and lower R^2 values than the models fit on the

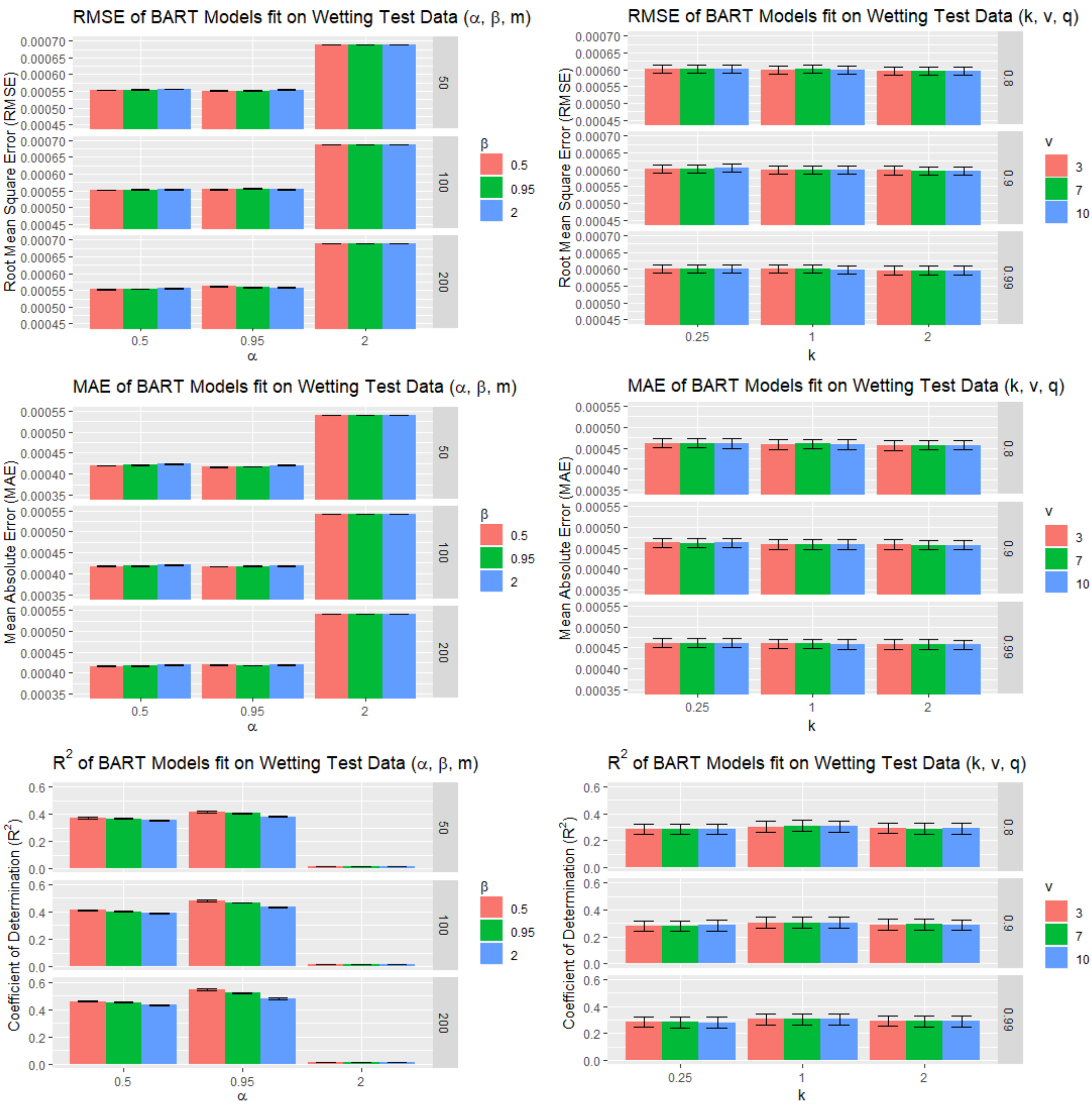

Figure 3.14: Error statistics (RMSE, MAE, and R^2) for each BART model fit on the wetting dataset. Left Plots: Average error statistics for models with combinations of α (x axis), β (coloured), and m (faceted). Right Plots: Average error statistics for models with combinations of k (x axis), v (coloured), and q (faceted). Error bars denote ±1 standard error.

drought dataset. This is similar to the results of the BRT models (Figure 3.11) which also showed that the wetting models had worse error statistics. This indicates that observations in the wetting dataset are more difficult to predict than their counterparts in the drought dataset.

Across all hyperparameter combinations, and both datasets, the models with an α value of 2 constantly performed terribly, explaining close to 0% of the variation in the data. As the α hyperparameter controls the "base" tree depth and forces regularisation, it is likely that the models where α=2 did not have enough regularisation due to trees that were too deep. It also appears that for both datasets, there is little difference between model performance between models with the differing values of k (hyperparameter for the leaf prior), v and q (hyperparameters for the error variance prior) tested here. The values of α and β (hyperparameters for the tree prior), and m (number of trees) seem to be driving the variation in model performance seen here.

Overall, it appears that for these two datasets, the best models have a lower α value (0.5 to 0.95) and a lower β value (0.5 to 0.95) which would result in shallow trees with a moderate to large spread in the number of terminal nodes in the prior distribution. Despite the number of trees (m) influencing model performance, it is difficult to assess the appropriateness of any tested value of m as it is unclear if a lesser or greater number of trees is preferable for these datasets (Table 3.5). Additionally, the values of k, v, and q tested here did not appear to affect model performance.

3.2.3.1 *Variable Importance*

The six best performing models (in terms of RMSE, MAE and R^2) are presented in Table 3.5. The variable importance of the six best performing model for each dataset, using each of the tree prediction error statistics (highlighted red in Table 3.5) were calculated and presented in Figure 3.15.

Table 3.5: Best performing BART models calculated on each dataset, with each of the three (RMSE, MAE, and R^2) error statistics. Best models for each combination of dataset and error statistic are highlighted in red.

	Drought Data							Wetting Data						
	α	β	m	k	v	q	Error Statistic	α	β	m	k	v	q	Error Statistic
RMSE	0.5	0.5	50	0.25	10	0.9	3.713×10^{-04}	0.95	0.5	50	2	3	0.8	5.386×10^{-04}
	0.95	0.5	200	2	7	0.8	4.615×10^{-04}	0.95	0.95	50	2	10	0.9	5.407×10^{-04}
	0.95	0.95	200	2	10	0.9	4.625×10^{-04}	0.95	0.5	50	2	3	0.99	5.416×10^{-04}
	0.95	0.5	100	1	10	0.8	4.630×10^{-04}	0.95	0.5	50	2	7	0.99	5.425×10^{-04}
	0.95	0.5	100	1	10	0.99	4.643×10^{-04}	0.95	0.5	50	2	7	0.9	5.427×10^{-04}
	0.95	0.95	200	2	10	0.99	4.649×10^{-04}	0.95	0.95	50	2	3	0.8	5.428×10^{-04}
MAE	0.95	0.5	200	2	7	0.8	3.457×10^{-04}	0.95	0.5	50	2	7	0.9	4.066×10^{-04}
	0.95	0.5	50	2	3	0.9	3.464×10^{-04}	0.95	0.5	50	2	3	0.99	4.080×10^{-04}
	0.5	0.5	200	0.25	10	0.99	3.468×10^{-04}	0.95	0.95	200	2	10	0.8	4.083×10^{-04}
	0.95	0.5	100	1	10	0.8	3.473×10^{-04}	0.95	0.5	50	1	10	0.8	4.084×10^{-04}
	0.95	0.5	200	2	10	0.9	3.477×10^{-04}	0.95	0.5	200	2	10	0.9	4.091×10^{-04}
	0.5	0.5	100	2	7	0.99	3.477×10^{-04}	0.95	2	200	2	7	0.9	4.092×10^{-04}
R^2	0.95	2	200	0.25	3	0.9	0.6230	0.95	0.5	200	1	10	0.9	0.5924
	0.95	0.5	200	1	10	0.99	0.6216	0.95	0.5	200	1	3	0.99	0.5906
	0.95	0.5	200	1	3	0.9	0.6203	0.95	0.5	200	1	7	0.9	0.5898
	0.95	0.5	200	1	7	0.9	0.6195	0.95	0.5	200	1	10	0.99	0.5837
	0.95	0.5	200	1	3	0.8	0.6180	0.95	0.5	200	1	7	0.99	0.5829
	0.95	0.95	200	0.25	7	0.99	0.6165	0.95	0.5	200	1	3	0.9	0.5818

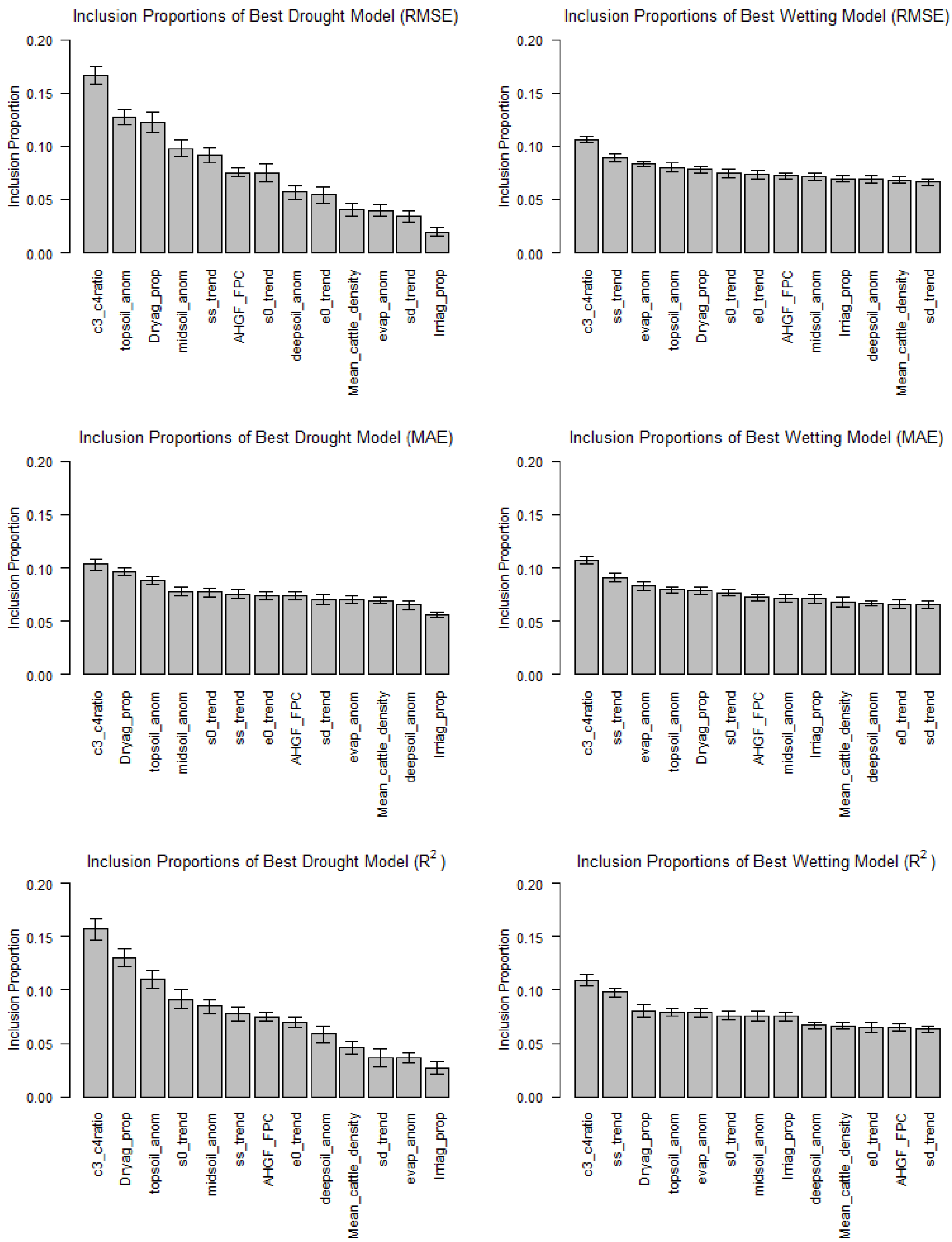

Figure 3.15: Variable Importance (measured with inclusion proportions – essentially the proportion of times that a variable has been used in a splitting rule) of each variable in the BART models selected as "best" by three error statistics (RMSE, MAE, R^2). Left: models applied to drought dataset. Right: Models applied to wetting dataset. Inclusion proportions calculated using the `bartMachine` `package` (Kapelner and Bleich, 2016). Error bars denote ±1 standard error.

Note that the plots in Figure 3.15 have error bars while equivalent BRT plots in Figure 3.12 do not. This is because each model method requires a different technique to calculate the inclusion proportions. BART is a Bayesian method meaning that the output consists of multiple posteriors, and thus the inclusion proportions must be averaged across all these posteriors, requiring standard error bars on the plots (Bleich et al., 2013).

Figure 3.15 shows that the three models on the drought dataset indicate a most important variable, and a least important variable. The wetting dataset models tend to only have a clear most important variable, with several variables being similarly least important. The models fit on the drought dataset also have a steeper drop off in importance between the variables than their wetting dataset counterparts.

All three drought dataset models have similar variable rankings. They all identified the ratio of C3 to C4 grasses (`c3_c4ratio`) as the most important variable and identified the proportion of dryland agriculture (`Dryag_prop`) and top soil anomaly (`topsoil_anom`) as the next two most important variables. All the models also identified the proportion of irrigated agriculture (`Irrag_prop`) as the least important variable.

All three wetting dataset models also have similar variable rankings. They all identified the ratio of C3 to C4 grasses (`c3_c4ratio`) as the most important variable and identify mid soil moisture layer trend (`ss_trend`) as the second most important. The models selected by the RMSE and MAE statistics identified PET anomaly (`evap_anom`) as the third most important variable, while the best R^2 model places it towards the middle of the rankings as 5^{th}. Additionally, all the wetting dataset models identified deep soil moisture layer trend (`sd_trend`) as the least important variable.

In general, the drought dataset models indicate that the ratio of C3 to C4 grasses (`c3_c4ratio`), proportion of dryland agriculture (`Dryag_prop`), and top soil moisture layer anomaly (`topsoil_anom`) as the most important variables influencing EVI trend in the drought. The proportion of irrigated agriculture (`Irrag_prop`) is the least important variable. The wetting dataset models indicate that the ratio of C3 to C4 grasses (`c3_c4ratio`), mid soil moisture layer trend (`ss_trend`), and PET anomaly (`evap_anom`) are the most important variables influencing EVI trend in recovery, and that the proportion of irrigated agriculture (`Irrag_prop`) is the least important.

3.2.3.2 *Interaction Importance*

To assess the models' identifications of interaction importance, the top eight interactions identified by the models selected as "best" by RMSE, MAE, and R^2 for each dataset are presented in Table 3.6. It is important to note that similar to the BRT interaction importance results (Section 3.2.2.2), all interactions presented here are two-way interactions only, as the software (`bartMachine`) was not capable of calculating the importance of three-way or higher interactions (Kapelner and Bleich, 2016). Additionally, note that Table 3.6 contains a column for the standard deviation, whereas the BRT equivalent (Table 3.4) does not. This is due to the same reason that Figure 3.15 has error bars but the BRT equivalent (Figure 3.12) does not. That is, each model method requires a different technique to calculate the inclusion proportions. BART is a Bayesian method meaning that the output consists of multiple posteriors, and thus the relative interaction importance must be averaged across all these posteriors, requiring a standard deviation measure (Bleich et al., 2013).

In terms of the drought dataset, the models selected by RMSE and R^2 have almost identical interaction importance, identifying seven of the same interactions as the most important (Table 3.6). The model selected by MAE, however, only has identified four of these interactions as important.

The wetting dataset models selected by the MAE and R^2 statistics identified six of the same interactions, while the model identified by RMSE identified five of these (Table 3.6). It is also interesting to note that almost all of the top eight interactions identified by all three of the wetting models were comprised of either ratio of C3 to C4 grasses (`c3_c4ratio`) or mid soil moisture layer trend (`ss_trend`) as a component in the pairwise interaction.

Table 3.6: Top eight interactions identified by the BART models selected as "best" by RMSE, MAE, and R^2 for each dataset.

		Variable 1	Variable 2	Relative Strength of Interaction	Standard Deviation of Interaction
Drought Data Models	Best RMSE Model ($\alpha = 0.5, \beta = 0.5, m = 50, k = 0.25, \upsilon = 10, q = 0.9$)	s0_trend	topsoil_anom	575.2	286.30
		c3_c4ratio	topsoil_anom	423.53	280.83
		Dryag_prop	c3_c4ratio	409.9	200.92
		topsoil_anom	midsoil_anom	406.37	224.32
		c3_c4ratio	s0_trend	347.17	198.50
		Dryag_prop	s0_trend	332.73	217.63
		e0_trend	midsoil_anom	304.37	191.11
		s0_trend	evap_anom	294.83	238.84
	Best MAE Model ($\alpha = 0.95, \beta = 0.5, m = 200, k = 2, \upsilon = 7, q = 0.8$)	c3_c4ratio	topsoil_anom	3565.7	559.68
		Dryag_prop	c3_c4ratio	3516.9	660.73
		Dryag_prop	topsoil_anom	3293.67	611.31
		c3_c4ratio	midsoil_anom	3287.03	576.51
		Dryag_prop	s0_trend	3216.1	526.44
		topsoil_anom	midsoil_anom	3198.9	558.31
		Dryag_prop	ss_trend	3162.03	540.27
		c3_c4ratio	ss_trend	3122.1	686.30
	Best R^2 Model ($\alpha = 0.95, \beta = 2, m = 200, k = 0.25, \upsilon = 3, q = 0.9$)	s0_trend	topsoil_anom	512.77	247.46
		topsoil_anom	midsoil_anom	421.7	263.30
		c3_c4ratio	topsoil_anom	406.6	258.88
		Dryag_prop	s0_trend	374.07	160.48
		Dryag_prop	c3_c4ratio	357.33	245.95
		c3_c4ratio	s0_trend	274.43	185.85
		sd_trend	topsoil_anom	262.33	189.24
		e0_trend	midsoil_anom	240.3	159.07
Wetting Data Models	Best RMSE Model ($\alpha = 0.95, \beta = 0.5, m = 50, k = 2, \upsilon = 3, q = 0.8$)	evap_anom	c3_c4ratio	3546.3	595.55
		ss_trend	c3_c4ratio	3512.27	477.57
		topsoil_anom	c3_c4ratio	3345.9	649.97
		AHGF_FPC	c3_c4ratio	3148.17	576.77
		Dryag_prop	c3_c4ratio	3141.07	614.42
		s0_trend	c3_c4ratio	3140.07	430.70
		e0_trend	c3_c4ratio	3097.13	441.41
		ss_trend	s0_trend	3080.6	514.76
	Best MAE Model ($\alpha = 0.95, \beta = 0.5, m = 50, k = 2, \upsilon = 7, q = 0.9$)	ss_trend	c3_c4ratio	3564.03	511.95
		evap_anom	c3_c4ratio	3464.6	737.26
		topsoil_anom	c3_c4ratio	3453	494.58
		Dryag_prop	c3_c4ratio	3349.43	464.56
		AHGF_FPC	c3_c4ratio	3202.17	603.71
		topsoil_anom	ss_trend	3125.1	567.20
		evap_anom	ss_trend	3011.53	499.69
		Dryag_prop	ss_trend	3010.17	591.55
	Best R^2 Model ($\alpha = 0.95, \beta = 0.5, m = 200, k = 1, \upsilon = 10, q = 0.9$)	ss_trend	c3_c4ratio	2211.97	552.60
		evap_anom	c3_c4ratio	2204.67	468.41
		topsoil_anom	c3_c4ratio	2091.73	375.88
		Dryag_prop	c3_c4ratio	2039.57	413.10
		Irriag_prop	ss_trend	2027.93	463.32
		Irriag_prop	c3_c4ratio	2027.37	464.97
		topsoil_anom	ss_trend	2008.93	538.21
		AHGF_FPC	c3_c4ratio	1981.33	450.96

3.3 Comparison of Model Types

Figure 3.16 presents the error statistics of the best performing BRT and BART models (highlighted in red in Table 3.3 and Table 3.5). The models fit on the wetting dataset appear to be producing less accurate predictions than those fit on the wetting data due to the models fit on the wetting dataset having higher RMSE and MAE values, and lower R^2 values than their drought dataset counterparts. This discrepancy in prediction capability can also be seen in the graphs illustrating the predicted EVI trend against observed EVI trend (Appendix Figure A.1 and Appendix Figure A.2) as the models fit on the wetting dataset show increased scatter around the reference lines (shown in red). This indicates that EVI trend is more difficult to predict when the grassland is in recovery than when in drought.

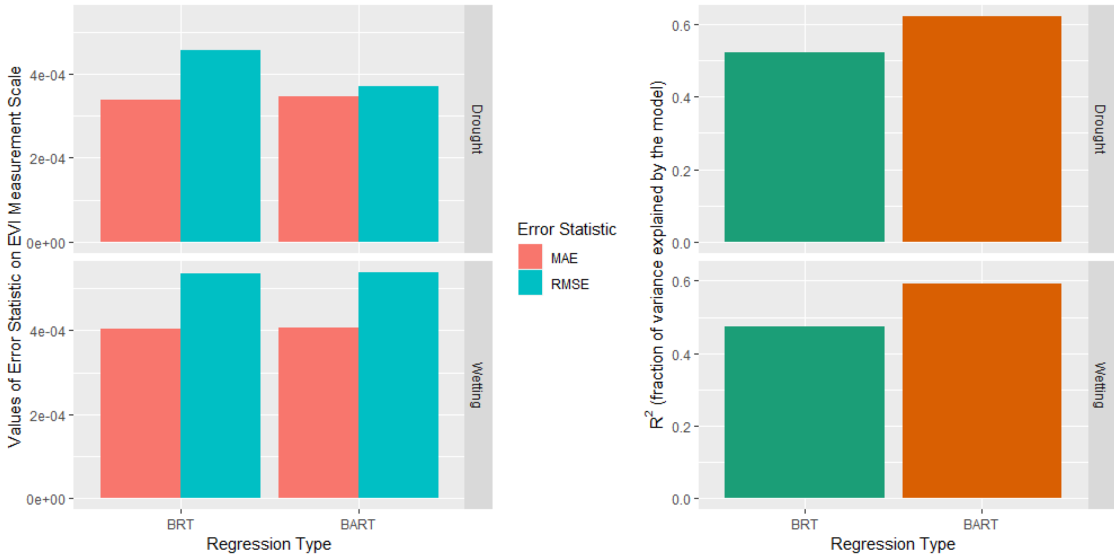

Figure 3.16: Graphical representation of the error statistics (RMSE, MAE, and R^2) for the best performing BRT and BART models calculated on each dataset. Left: RMSE and MAE. Right: R^2. Graphs are faceted on dataset. See Appendix Table A.8 for table with numerical values of these statistics.

3.3.1 Comparison of BRT and BART with Reference Analysis Methods

To briefly compare the reference methods (linear regression, single regression tree, and random forest models) with BRT and BART, the error statistics of the reference analysis methods, and the error statistics for the best performing BRT and BART models respectively are compared (Figure 3.7 and Figure 3.16). By comparing the values in these figures, the improvements in model fit that can be gained by employing the more complex methods of BRT and BART can be evaluated. As expected, the BRT and BART models consistently performed better than the linear regression and single tree models across both datasets and all three error statistics.

However, the random forest model results in error statistics that are similar to the error statistics of the best BRT models, and in general the best BART models performed slightly better than the BRT and random forest models (see Section 3.3.2 for a more in depth comparison between BRT and BART). This is somewhat to be expected as random forests, BRT, and BART are all additive treed methods, so it makes sense that they would perform somewhat similarly to each other in comparison to linear regression and single tree models.

It is also worth noting that the random forest model presented here was fit with the defaults of the `randomForest` package (Liaw and Wiener, 2002), so it is possible that an even better fit could be accomplished by using different values for the settable parameters (like what was done here for the BRT and BART methods). However, even if such a model was found, and performed better than the best BRT model, it is important to note that although random forests reduce variance more than single trees, they do not reduce bias like BRTs. This is because random forests employ bagging, not boosting. This means that each tree is built on a bootstrap sample that is distributed in a similar way as the full training set, and therefore the average bias of the model is the same as any one tree. BRT models, by contrast, utilise boosting, which sequentially models the residuals throughout the entire data space, including atypical observations that are not well explained by the initial trees. In this way, boosting (and BRTs) reduce both bias, through forward stagewise fitting, as well as variance, through model averaging (Elith et al., 2008). Therefore, although the random forest and BRT models fitted here perform similarly, with respect to error statistics, BRTs are the better choice as they reduce not only variance, but bias as well.

Variable selection is an important facet of regression tree methods, therefore it is of interest if the linear regression, single tree and random forest models produced similar rankings of variable importance as the BRT and BART models. In general, for both datasets, all five methods selected the same variables as most important and least important. Section 3.3.2 contains a more comprehensive comparison of variable importance selections between BRT and BART than the brief comparison with single trees and random forests presented here.

The five methods (linear regression, single tree, random forest, BRT, and BART) all identified the proportion of dryland agriculture (`Dryag_prop`) important in predicting EVI trend in the drought data, and all but the linear regression identified the ratio of C3 to C4 grasses (`c3_c4ratio`) as important. The linear regression, single tree, random forests and BRT models also identified mid soil moisture layer anomaly (`midsoil_anom`) as important, but BART did not. Additionally, the linear regression, single tree, random forest and BART models identified top soil moisture layer anomaly (`topsoil_anom`) as important, but BRT did not. All methods identified the proportion of irrigated agriculture (`Irrag_prop`) as the least important predictor of grassland decline.

For the wetting dataset, all methods identified the ratio of C3 to C4 grasses (`c3_c4ratio`) and mid soil moisture layer trend (`ss_trend`) as important in predicting EVI trend. Additionally, the linear regression, single tree, random forest and BART models also identified PET anomaly (`evap_anom`) as important, but BRT did not. All methods besides the linear regression and BART identified the proportion of irrigated agriculture (`Irrag_prop`) as the least important predictor, but the linear regression and BART models still indicated that it was not an important predictor.

As all five methods generally were in agreement about variable selection (see Table 3.2, Figure 3.8, Figure 3.9, Figure 3.12, and Figure 3.15 for variable importance measures for each method), it appears that in terms of variable selection, there is little to gain by using BRT or BART rather than a random forest. Additionally, it is likely worthwhile using an additive tree method (random forest, BRT, or BART) rather than a single tree as additive tree methods provide a numerical measure of variable importance that single trees do not.

3.3.2 *Comparison of BRT VS BART*

To compare BRTs and BARTs, the principal aim of this study, numerous aspects should be evaluated. Here, the prediction error statistics, variable and interaction importance, run time, difficulty in understanding how the model works and what the settable parameters do, and the extra functionality of the packages will be considered.

In this study three prediction error statistics were reported for every model; RMSE, MAE, and R^2. To assess the performance of the BRT and BART methods relative to each other, the prediction error statistics of the best models on each dataset (Table 3.3, Table 3.5, and Figure 3.16) will be compared.

In general, the BART models fitted on the drought dataset performed better than the BRT models. The BRT and BART models chosen as "best" by the RMSE statistic had RMSE values of 4.560×10^{-04} and 3.713×10^{-04} respectively. Lower RMSE values indicate lower prediction error, and thus the BART model has less prediction error than the BRT model. The BRT and BART models selected as "best" by the MAE statistic had MAE values of 3.398×10^{-04} and 3.457×10^{-04} respectively. Lower MAE values indicate lower prediction error, and thus the BRT model had less prediction error than the BART model. However, the values are much closer together than the RMSE values (a difference of 0.057×10^{-04} compared to 0.847×10^{-04}) indicating that the models had similar prediction errors, rather than one being clearly better than the other. The final error statistic considered is R^2 which indicates the proportion of variance in the data that is explained by the model. The BRT and BART models selected as "best" by the R^2 statistic had values of 0.5213 (52.13% of variance explained) and 0.6230 (62.30% of variance explained) respectively. The BART model explains the most variance in the data, an additional 10.17% compared to the BRT model. Therefore, the BART method appears to be more effective at modelling the drought data as it produced models with preferable error statistics.

Similarly, the BART models fitted on the wetting dataset typically performed similarly or better than the BRT models. The BRT and BART models chosen as "best" by the RMSE statistic performed similarly, with RMSE values of $5.353{\times}10^{-04}$ and $5.386{\times}10^{-04}$ respectively (difference of $0.033{\times}10^{-04}$). The BRT and BART models selected as "best" by the MAE statistic also performed similarly, with MAE values of $4.051{\times}10^{-04}$ and $4.006{\times}10^{-04}$ (difference of $0.045{\times}10^{-04}$). However, the BART model chosen as "best" by the R^2 statistic had a higher R^2 value (0.5924) than it's BRT counterpart (0.4732). Specifically, the BART model explains an additional 11.92% of the variation in the dataset compared to the BRT model. Therefore, the BART method appears to be more effective at modelling the wetting data as it produced models with similar or preferable error statistics with compared with the BRT method.

It is of interest if the BRT models identified the same variables and interactions as important as the BART models. To this end, the variable importance in Figure 3.12 and Figure 3.15 are compared, as are the top eight interactions of each model in Table 3.4 and Table 3.6. Typically, both model types (BRT and BART) agreed on the importance of the variables for each dataset. In the drought dataset, they both identified the ratio of C3 to C4 grasses (`c3_c4ratio`) and the proportion of dryland agriculture (`Dryag_prop`) as important, and the proportion of irrigated agriculture (`Irrag_prop`) as least important. However, the BRT models selected the mid soil moisture layer anomaly (`midsoil_anom`) as important, but the BART models did not, and the BART models identified top soil moisture layer anomaly (`topsoil_anom`) as important, but the BRT models did not.

The BRT and BART models fit on the wetting dataset also typically agree on variable importance. Both models identified the ratio of C3 to C4 grasses (`c3_c4ratio`) and mid soil moisture layer trend (`ss_trend`) as important. They also both identified the proportion of irrigated agriculture (`Irrag_prop`) as an unimportant variable. However, most of the BRT models identified woody vegetation cover (`AHGF_FPC`) as important, but the BART models did not, and most of the BART models identified PET anomaly (`evap_anom`) as important, but the BRT models did not.

All six models fit on the drought dataset (three BRT and three BART) indicated that the interaction between top soil moisture layer anomaly (`topsoil_anom`) and mid soil moisture layer anomaly (`midsoil_anom`) was important. This was the only interaction that all six models identified, but there are several more that most of these models selected. Namely, all three of the BART models, and two of the BRT models fit on the drought dataset identified the interactions between the ratio of C3 to C4 grasses (`c3_c4ratio`) and top soil moisture layer anomaly (`topsoil_anom`), as well as the interaction between the proportion of dryland agriculture (`Dryag_prop`) and the ratio of C3 to C4 grasses (`c3_c4ratio`) as important.

The six models fit on the wetting dataset all identified two specific interactions as important. Namely, the interactions between PET anomaly (`evap_anom`) and the ratio of C3 to C4 grasses (`c3_c4ratio`), as well as top soil moisture layer anomaly (`topsoil_anom`) and the ratio of C3 to C4 grasses (`c3_c4ratio`). Additionally, all three of the BRT models, and one of the BART models identified the interaction between PET anomaly (`evap_anom`) and mid soil moisture layer trend (`ss_trend`) as important, and one model of each method identified the interaction between mid soil moisture layer trend (`ss_trend`) and top soil moisture layer trend (`s0_trend`) as important.

Overall, there was some agreement across methods about which interactions were important for each dataset, but it was not as stark as the agreement for variable importance. This possibly is because the software can only assess importance of pairwise interactions; but the models are all fitting higher order interactions to reduce error as much as possible. This means that the selection of important interactions is not reflective of all of the interactions that the models are fitting and therefore the pairwise interactions that are selected as important may vary between models, even if the models are all fitting similar higher order interactions (Hijmans et al., 2017; Kapelner and Bleich, 2016).

In terms of running time, BART models are superior. The BART models considered here usually took between 20 and 40 seconds to fit. The BRT models varied greatly, with the longest taking around 2.5 minutes, and the shortest taking around 30 seconds. However, neither run times for BRT or BART are particularly long, so it is not of great concern.

Both BRTs and BARTs can be difficult to understand at first as they are not simple, straightforward methods. However, it is decidedly easier to understand how BRTs work and what the settable parameters do, compared to the BART method. Firstly, the Bayesian framework of BARTs makes the methods challenging to understand and for non-statisticians, possibly extremely daunting on first use, and users may therefore be likely to simply use the defaults of the software. Secondly, BRTs have only three settable parameters, compared to BART's six hyperparameters. Furthermore, the concepts of bag fractions, learning rates, and tree complexities are far more straightforward and easier to understand than BART's hyperparameters, especially as multiple hyperparameters are used to control just one prior (e.g. α and β both control the tree structure prior). However, the developers of both packages have published detailed tutorials and documentations to help researchers understand how these methods work and how to implement them (Elith and Leathwick, 2017; Elith et al., 2008; Hijmans et al., 2017; Kapelner and Bleich, 2016).

Both dismo (BRT implementation) and bartMachine (BART implementation) also have additional functionality that was not implemented in the course of this study. It is also worthwhile comparing this functionality to determine if one package offers more useful functions than the other.

The dismo package has three extra uses related to BRTs other than what was used in this study. Firstly, it allows for model simplification by testing dropping up to n variables (where the user chooses n). Secondly, it can plot the fitted functions of the predictor variables, although this should be used with caution as it can give misleading indications about the distributions of fitted values relative to each predictor, especially if there are strong interactions present in the data or if predictors are strongly correlated. Finally, it can plot the pairwise interactions to a 3D surface which is useful when investigating how the predictors interact (Elith and Leathwick, 2017; Elith et al., 2008; Hijmans et al., 2017).

The bartMachine package has six additional BART uses other than what was discussed in this study. Firstly, it can check assumptions, including the mean centredness of noise, and the convergence of the Gibbs sampler. Secondly, it can generate uncertainty estimates in the form of credible intervals and prediction intervals. Thirdly, bartMachine has the ability to test if any predictors in a user-chosen set matter in predicting the dependant variable. Fourth, it can examine partial dependence (how a predictor affects the dependant variable on average, when controlling for all other predictors). Fifth, it has several variable selection methods beyond simply calculating and graphing variable importance as done here. Finally, due to BART's Bayesian framework, bartMachine has the ability to incorporate informed prior information about the predictors into a BART model (Kapelner and Bleich, 2016). This can be incredibly useful and is one of the main benefits of Bayesian methods.

3.4 Limitations of the Methods

It is important to note that while both datasets contained variables with skewed distributions and outliers, these aspects of the data do not violate the assumptions of either BRT or BART as they are non-parametric methods. There are however some limitations to these methods. Namely, the dismo package (used to fit the

BRT models) and the `bartMachine` package (used to fit the BART models), can only calculate the importance of two-way interactions, even though the model itself may model three way, or even higher, interactions. This limits the user's ability to assess the appropriate tree complexity (beyond inspecting error statistics of multiple models with different tree complexities as done here), as well as the ability to assess the importance of the interactions of the variables and make inferences in context based on the interactions.

3.5 Limitations of the Study

This study has several limitations. Most importantly, all methods were applied to only one ecology scenario (grassland decline and recovery), with two datasets. It is possible that different types of ecology datasets would include data that would be better fit by BRT rather than BART. Additionally, the nature of the data might mean that it would be better modelled with different values of the parameters than what was most effective for the grassland data in this study. A further limitation is that both BRT and BART are capable of modelling data with missing values (Elith et al., 2008; Kapelner and Bleich, 2016), however the grassland datasets considered here had no missing data, and furthermore, missing data was not simulated (due to time limitations imposed on this study). This means that the impact of missing data on BRT and BART were not compared. It is possible that one method may handle missing data more effectively than the other, but this has not been assessed.

Another limitation of this study is that partial dependence of predictors were not investigated (both `dismo` and `bartMachine` have the ability to inspect partial dependence (Hijmans et al., 2017; Kapelner and Bleich, 2016)). The effects of a predictor (when controlling for all other predictors) can have important ecological and management implications. It is possible, for example, that although the BRT and BART models both identified the ratio of C3 to C4 grasses as an important predictor of grassland degradation, the partial effect size at any given ratio of the grasses could be different between model types. If this difference in partial effect size is large enough, it could produce different ecological and management implications for each model type. However, since the partial dependence was not investigated, it is unknown if BRT and BART produced differing partial effect sizes and would therefore produce differing ecological and management implications.

3.6 Significance of Study

3.6.1 Statistical Significance

As machine learning methods may in some cases be used as "black boxes" by the users who apply them, it is of great interest if the packages that implement BRT and BART can be used "out of the box" with their default parameter values (Table 2.1 and Table 2.3). The majority of the default parameters in the `dismo` and `bartMachine` packages were reasonable for the grassland datasets, however there were some notable exceptions.

Of the three parameters that need to be defined to run a BRT model, `dismo`'s `gbm.step` function offers two reasonable defaults, and one unrealistic default (note that `dismo` has several other functions for diverse use-cases which also differ in several ways, including default parameter settings) (Hijmans et al., 2017). Firstly, for the data considered here, the bag fraction did not seem to affect the number of trees in the model, nor any of the error statistics. Additionally, Elith et al. (2008) determined that bag fractions between 0.5 and 0.75 perform best for presence-absence data. Although the grasslands data is not presence-absence, due to the lack of a clear best bag fraction value, the default of 0.75 seems reasonable.

The default learning rate for `gbm.step` is 0.01. Again, this value is reasonable for the grasslands data as Figure 3.11 indicates that learning rate values of 0.05, 0.01, and 0.005 produced the smallest error statistics without the learning rate being so small that when combined with a moderate or high tree complexity the resulting model could not be fit. Additionally, the default learning rate of 0.01 is the middle option of the three sensible values identified for this data, and this lends credence to a learning rate of 0.01 being a reasonable default.

The default tree complexity for `gbm.step` is 1. Theoretically this tree complexity is unreasonable for most data as this tree complexity results in trees that have one split and two terminal nodes (known as a decision stump), meaning that no interactions are modelled. This would only be appropriate for data where the researchers would expect no interactions between variables in the data generating process. Figure 3.11 supports the assessment that a tree complexity of 1 is unreasonable as it shows that this tree complexity was consistently the worst performing tree complexity of all the values tested.

It should also be noted that Elith et al. (2008) recommends fitting models with at least 1000 trees as a heuristic. Both models that were fit using the default parameter values, determined that the optimal number of trees required to model the data were well above 1000 (5500 for the drought dataset and 4800 for the wetting dataset). Therefore, the defaults of `gbm.step` did not violate the number of trees heuristic when applied to these datasets.

Of the six hyperparameters that are required for a BART model, `bartMachine`'s `bartMachine` function contains five reasonable defaults and only one unreasonable default. Firstly, none of the values tested for k, ν, or q (which include the default values) appeared to alter the model performance (Figure 3.13 and Figure 3.14). Therefore, the default values of $k = 2$, $\nu = 3$, and $q = 0.9$ are sensible values for the two grassland datasets.

It is difficult to assess the appropriateness of the default $m = 50$ value as it is unclear if a lesser or greater number of trees is preferable for these datasets (Figure 3.13 and Figure 3.14). However, the default 50 trees is most likely the best choice as a smaller number of trees leads to better variable selection because it creates a bottleneck that forces the trees to only use predictor variables which most improve prediction of the response variable as splitting variables (Chipman et al., 2010)).

The default α value of 0.95 is a reasonable value for the grasslands data as Figure 3.13 and Figure 3.14 indicate that lower α values (between 0.5 to 0.95) resulted in better performing models.

The only hyperparameter that does not have an appropriate default value is β (the default is 2). Figure 3.13 and Figure 3.14 indicate that lower β (0.5 to 0.95) resulted in (albeit slightly) better performing models.

Overall, it appears that at least for the grasslands datasets, `bartMachine` (BART implementation) offers better "out of the box" performance than `dismo` (BRT implementation). This is because only one of its six hyperparameter's defaults is not appropriate, compared to `dismo`'s one of three inappropriate parameters. Additionally, the default value of $\beta = 2$ only resulted in slightly worse models, whereas the default tree complexity of 1 not only resulted in substantially worse models, but is also clearly theoretically inappropriate for most data (including the grasslands dataset used in this study).

3.6.2 Ecological Significance

In Section 3.2.2 it was determined that for BRT models, the two highest tree complexities tested produced the better performing models than the lower tree complexities, however, such high tree complexities usually do not facilitate ecological inference as high order interactions are complex and difficult to make practical inferences from (Kath et al., 2018). Furthermore, the software used in this study to fit BRT models (`dismo`) is unable to evaluate the importance of non-pairwise interactions, therefore it is currently impossible to even attempt to draw practical inferences from such high order interactions (Hijmans et al., 2017). Fitting models with high tree complexities only serves to produce more accurate predictions, not to make inferences about predictor interactions which could be used to make ecological management policy.

This is especially true for a tree complexity of 13 as, for the grasslands datasets, this means that potentially all 13 predictor variables are interacting. Therefore, although these high tree complexities produced the models with better error statistics, it would be extremely difficult to make any meaningful interaction inference from these models (even if the software allowed for the evaluation of high order interactions). Despite having more prediction error, models with lower tree complexity may facilitate better understanding of the data generating process.

Sections 3.2.2 and 3.2.3 discussed the variable and interactions that the BRT and BART models identified as important for each dataset. Therefore, the identified important variables and interactions for grassland drought and recovery can be compared to the findings of Kath et al. (2019).

Kath et al. (2019) identified the proportion of dryland agriculture, mid soil moisture later anomaly, ratio of C3 to C4 grasses and top soil moisture layer anomaly as important for grassland decline. These predictors are the same ones that the BRT and BART models selected as important. Kath et al. (2019) also identified the proportion of irrigated agriculture as the least important predictor of grassland decline, which is consistent with the BRT and BART models fitted here.

Kath et al. (2019) determined that important predictors of grassland recovery are mid soil moisture layer trend, the ratio of C3 to C4 grasses, PET anomaly, and the proportion of woody vegetation. They also identified the proportion of irrigated agriculture as the least important predictor of grassland recovery. Again, this is consistent with the BRT and BART models fitted here.

Kath et al. (2019) does not report on interaction importance, but the models fit on the drought dataset in this report indicate that the interaction between top soil moisture layer anomaly and mid soil moisture layer anomaly was the most important interaction of predictors grassland degradation, but the interactions between the ratio of C3 to C4 grasses and top soil moisture layer anomaly, the proportion of dryland agriculture and the ratio of C3 to C4 grasses, as well as top soil moisture layer anomaly and mid soil moisture layer anomaly were also important.

The models fit on the wetting dataset indicate that the interactions between PET anomaly and the ratio of C3 to C4 grasses, as well as between top soil moisture layer anomaly and the ratio of C3 to C4 grasses are the most important interactions of predictors of grassland recovery, but the interactions between PET anomaly and mid soil moisture layer trend as well as between mid soil moisture layer trend and top soil moisture layer trend were also important.

Section 3.3.2 also compared the prediction error statistics of the best BRT and BART models. It was determined that in general, the BART method produced models with more favourable prediction error statistics. In particular, the BART model fit on the drought dataset that had the largest R^2 value (selected as "best" by the R^2 statistic" explained an additional 10.17% of variation in the data than it's BRT counterpart, and BART model fit on the wetting dataset that has the largest R^2 values explained and additional 11.92% of variation in the data than it's BRT counterpart. Currently, BRTs are widely used in ecological studies (at the time of writing the Elith et al. (2008) paper, which outlines the use of BRTs in ecology, has been cited over 2000 times) but the models fit here indicate that BART models may offer improved predictive performance. However, the complexity of the BART method makes it more difficult to understand and implement than the BRT method (see section 3.3.2 for a more in-depth discussion), so BRT may still be preferable in some cases.

3.7 Future Work

More research is needed to more comprehensively assess the benefits, limitations, and differences between BRT and BART. Specifically, BRT and BART should be assessed and compared on additional datasets from a diverse range of ecological scenarios. In particular, the performances of BRT and BART on smaller datasets, and datasets with missing values require further research. Additionally, due to the similar performance of the random forest models in the reference analysis to the best BRT and BART models identified here, future work could also include a comparison of random forests with BRT and BART models.

Chapter 4

Conclusion

The key finding of this project is that BART models are likely superior in modelling ecological data than BRTs. As BRTs and BARTs have similar variable and interaction importance selection capabilities (research question 1), the choice of which model type to use becomes one of prediction error (research question 2), run time, ease of altering parameters, functionality of the software implementation, and (to a lesser extent) reasonableness of the defaults of said implementations (research question 3).

The BART method is preferable to the BRT method in all but one of the above categories. The BART models fitted on both datasets typically performed better or similarly (in terms of prediction error statistics) to their BRT model counterparts. BARTs also had shorter run times, more extensive functionality of the software implementation, and more appropriate defaults in the software implementation. The main drawback of BARTs compared to BRTs is that it is more complex, less user friendly, and therefore it is more difficult to learn how to use properly and effectively.

The findings of this project should be tempered with the knowledge that all models were fit on only two ecological datasets, from one scenario (grassland decline and recovery). Additional work should be conducted to determine if the results are consistent on other datasets and ecological scenarios.

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Appendix A

Appendix of Additional Results

Table A.1: Error Statisics of reference analysis models (linear regression, single tree, random forest) fit on each dataset (drought and wetting).

Regression Type	Data	Error Statistic	Value
Linear Regression	Drought	RMSE	5.35×10^{-04}
		MAE	4.07×10^{-04}
		R^2	0.3082
	Wetting	RMSE	6.00×10^{-04}
		MAE	4.65×10^{-04}
		R^2	0.2239
Single Tree	Drought	RMSE	5.18×10^{-04}
		MAE	3.93×10^{-04}
		R^2	0.3433
	Wetting	RMSE	6.01×10^{-04}
		MAE	4.59×10^{-04}
		R^2	0.3053
Random Forest	Drought	RMSE	4.60×10^{-04}
		MAE	3.41×10^{-04}
		R^2	0.5382
	Wetting	RMSE	5.36×10^{-04}
		MAE	4.02×10^{-04}
		R^2	0.4782

Table A.2: Error Statisics and number of trees of each BRT model fit on the drought dataset. Blank entries means that a model could not be built with that combination of parameter values.

Bag Fraction	Learning Rate	Tree Complexity	Number of Trees	RMSE	MAE	R^2
0.3	0.001	1	12050	5.08×10^{-04}	3.84×10^{-04}	0.3458
0.3	0.001	3	11750	4.83×10^{-04}	3.62×10^{-04}	0.4020
0.3	0.001	5	10450	4.74×10^{-04}	3.54×10^{-04}	0.4235
0.3	0.001	9	7850	4.67×10^{-04}	3.49×10^{-04}	0.4408
0.3	0.001	13	6450	4.64×10^{-04}	3.46×10^{-04}	0.4481
0.3	0.005	1	6250	4.97×10^{-04}	3.73×10^{-04}	0.3836
0.3	0.005	3	5900	4.68×10^{-04}	3.49×10^{-04}	0.4542
0.3	0.005	5	4300	4.64×10^{-04}	3.46×10^{-04}	0.4737
0.3	0.005	9	2100	4.63×10^{-04}	3.46×10^{-04}	0.4667
0.3	0.005	13	1950	4.62×10^{-04}	3.45×10^{-04}	0.4874
0.3	0.01	1	3800	4.95×10^{-04}	3.71×10^{-04}	0.3994
0.3	0.01	3	3400	4.70×10^{-04}	3.50×10^{-04}	0.4590
0.3	0.01	5	2550	4.62×10^{-04}	3.45×10^{-04}	0.4865
0.3	0.01	9	1400	4.63×10^{-04}	3.47×10^{-04}	0.4855
0.3	0.01	13	900	4.62×10^{-04}	3.45×10^{-04}	0.4814
0.3	0.05	1	3750	4.80×10^{-04}	3.61×10^{-04}	0.4821
0.3	0.05	3	950	4.71×10^{-04}	3.52×10^{-04}	0.4928
0.3	0.05	5	350	4.70×10^{-04}	3.49×10^{-04}	0.4669
0.3	0.05	9	300	4.66×10^{-04}	3.47×10^{-04}	0.4962
0.3	0.05	13	200	4.67×10^{-04}	3.52×10^{-04}	0.5078
0.3	0.1	1	2500	4.88×10^{-04}	3.67×10^{-04}	0.5028
0.3	0.1	3	550	4.76×10^{-04}	3.60×10^{-04}	0.5213
0.3	0.1	5				
0.3	0.1	9				
0.3	0.1	13				
0.6	0.001	1	12900	5.09×10^{-04}	3.84×10^{-04}	0.3344
0.6	0.001	3	8950	4.88×10^{-04}	3.66×10^{-04}	0.3842
0.6	0.001	5	9500	4.75×10^{-04}	3.55×10^{-04}	0.4141
0.6	0.001	9	7600	4.67×10^{-04}	3.48×10^{-04}	0.4369
0.6	0.001	13	5900	4.64×10^{-04}	3.46×10^{-04}	0.4421
0.6	0.005	1	6850	5.00×10^{-04}	3.75×10^{-04}	0.3852
0.6	0.005	3	5500	4.70×10^{-04}	3.49×10^{-04}	0.4538
0.6	0.005	5	4050	4.64×10^{-04}	3.45×10^{-04}	0.4747
0.6	0.005	9	2700	4.60×10^{-04}	3.42×10^{-04}	0.4859
0.6	0.005	13	1650	4.60×10^{-04}	3.43×10^{-04}	0.4747

Table A.2: (continued)

Bag Fraction	Learning Rate	Tree Complexity	Number of Trees	RMSE	MAE	R^2
0.6	0.01	1	4600	4.96×10^{-04}	3.71×10^{-04}	0.3951
0.6	0.01	3	3450	4.66×10^{-04}	3.45×10^{-04}	0.4655
0.6	0.01	5	1750	4.66×10^{-04}	3.47×10^{-04}	0.4603
0.6	0.01	9	1150	4.63×10^{-04}	3.45×10^{-04}	0.4719
0.6	0.01	13	800	4.60×10^{-04}	3.43×10^{-04}	0.4703
0.6	0.05	1	2750	4.83×10^{-04}	3.58×10^{-04}	0.4400
0.6	0.05	3	800	4.70×10^{-04}	3.49×10^{-04}	0.5017
0.6	0.05	5	550	4.64×10^{-04}	3.45×10^{-04}	0.4932
0.6	0.05	9	250	4.63×10^{-04}	3.46×10^{-04}	0.4835
0.6	0.05	13				
0.6	0.1	1	3050	4.79×10^{-04}	3.57×10^{-04}	0.5006
0.6	0.1	3	400	4.66×10^{-04}	3.46×10^{-04}	0.4834
0.6	0.1	5	200	4.69×10^{-04}	3.52×10^{-04}	0.4802
0.6	0.1	9				
0.6	0.1	13				
0.75	0.001	1	13150	5.09×10^{-04}	3.84×10^{-04}	0.3278
0.75	0.001	3	11300	4.85×10^{-04}	3.63×10^{-04}	0.3937
0.75	0.001	5	10250	4.74×10^{-04}	3.53×10^{-04}	0.4175
0.75	0.001	9	7100	4.67×10^{-04}	3.48×10^{-04}	0.4292
0.75	0.001	13	6250	4.63×10^{-04}	3.45×10^{-04}	0.4438
0.75	0.005	1	7050	5.01×10^{-04}	3.76×10^{-04}	0.3826
0.75	0.005	3	6100	4.67×10^{-04}	3.48×10^{-04}	0.4601
0.75	0.005	5	3950	4.64×10^{-04}	3.46×10^{-04}	0.4694
0.75	0.005	9	2550	4.59×10^{-04}	3.41×10^{-04}	0.4809
0.75	0.005	13	1550	4.60×10^{-04}	3.43×10^{-04}	0.4650
0.75	0.01	1	5500	4.95×10^{-04}	3.70×10^{-04}	0.4004
0.75	0.01	3	3750	4.66×10^{-04}	3.46×10^{-04}	0.4776
0.75	0.01	5	2200	4.64×10^{-04}	3.45×10^{-04}	0.4811
0.75	0.01	9	1150	4.61×10^{-04}	3.43×10^{-04}	0.4717
0.75	0.01	13	850	4.59×10^{-04}	3.41×10^{-04}	0.4772
0.75	0.05	1	2200	4.87×10^{-04}	3.61×10^{-04}	0.4196
0.75	0.05	3	750	4.66×10^{-04}	3.48×10^{-04}	0.4859
0.75	0.05	5	350	4.66×10^{-04}	3.46×10^{-04}	0.4608
0.75	0.05	9	250	4.63×10^{-04}	3.45×10^{-04}	0.4987
0.75	0.05	13	200	4.57×10^{-04}	3.42×10^{-04}	0.5035
0.75	0.1	1	2700	4.80×10^{-04}	3.58×10^{-04}	0.4792
0.75	0.1	3	450	4.70×10^{-04}	3.50×10^{-04}	0.5067

Table A.2: (continued)

Bag Fraction	Learning Rate	Tree Complexity	Number of Trees	RMSE	MAE	R^2
0.75	0.1	5	250	$4.65{\times}10^{-04}$	$3.48{\times}10^{-04}$	0.5141
0.75	0.1	9				
0.75	0.1	13				
0.9	0.001	1	13750	$5.09{\times}10^{-04}$	$3.84{\times}10^{-04}$	0.3231
0.9	0.001	3	11950	$4.85{\times}10^{-04}$	$3.63{\times}10^{-04}$	0.3900
0.9	0.001	5	10550	$4.73{\times}10^{-04}$	$3.53{\times}10^{-04}$	0.4146
0.9	0.001	9	8900	$4.64{\times}10^{-04}$	$3.45{\times}10^{-04}$	0.4451
0.9	0.001	13	6300	$4.62{\times}10^{-04}$	$3.45{\times}10^{-04}$	0.4425
0.9	0.005	1	7250	$5.03{\times}10^{-04}$	$3.78{\times}10^{-04}$	0.3793
0.9	0.005	3	6600	$4.68{\times}10^{-04}$	$3.48{\times}10^{-04}$	0.4641
0.9	0.005	5	4450	$4.63{\times}10^{-04}$	$3.44{\times}10^{-04}$	0.4797
0.9	0.005	9	3050	$4.58{\times}10^{-04}$	$3.40{\times}10^{-04}$	0.4953
0.9	0.005	13	2600	$4.57{\times}10^{-04}$	$3.40{\times}10^{-04}$	0.5166
0.9	0.01	1	4050	$5.02{\times}10^{-04}$	$3.76{\times}10^{-04}$	0.3844
0.9	0.01	3	4000	$4.67{\times}10^{-04}$	$3.47{\times}10^{-04}$	0.4815
0.9	0.01	5	1950	$4.65{\times}10^{-04}$	$3.45{\times}10^{-04}$	0.4654
0.9	0.01	9	1700	$4.58{\times}10^{-04}$	$3.41{\times}10^{-04}$	0.5071
0.9	0.01	13	1150	$4.56{\times}10^{-04}$	$3.40{\times}10^{-04}$	0.5036
0.9	0.05	1	2950	$4.86{\times}10^{-04}$	$3.61{\times}10^{-04}$	0.4282
0.9	0.05	3	1200	$4.63{\times}10^{-04}$	$3.43{\times}10^{-04}$	0.5161
0.9	0.05	5	550	$4.66{\times}10^{-04}$	$3.46{\times}10^{-04}$	0.5029
0.9	0.05	9	250	$4.59{\times}10^{-04}$	$3.42{\times}10^{-04}$	0.4890
0.9	0.05	13	200	$4.62{\times}10^{-04}$	$3.43{\times}10^{-04}$	0.4923
0.9	0.1	1	1650	$4.86{\times}10^{-04}$	$3.60{\times}10^{-04}$	0.4350
0.9	0.1	3	400	$4.71{\times}10^{-04}$	$3.49{\times}10^{-04}$	0.4854
0.9	0.1	5				
0.9	0.1	9				
0.9	0.1	13				

Table A.3: Error Statisics and number of trees of each BRT model fit on the wetting dataset. Blank entries means that a model could not be built with that combination of parameter values.

Bag Fraction	Learning Rate	Tree Complexity	Number of Trees	RMSE	MAE	R^2
0.3	0.001	1	12700	5.70×10^{-04}	4.40×10^{-04}	0.2592
0.3	0.001	3	10400	5.50×10^{-04}	4.20×10^{-04}	0.3205
0.3	0.001	5	8300	5.40×10^{-04}	4.20×10^{-04}	0.3406
0.3	0.001	9	7300	5.40×10^{-04}	4.10×10^{-04}	0.3746
0.3	0.001	13	6200	5.40×10^{-04}	4.10×10^{-04}	0.3861
0.3	0.005	1	4500	5.60×10^{-04}	4.30×10^{-04}	0.2986
0.3	0.005	3	6000	5.50×10^{-04}	4.10×10^{-04}	0.4006
0.3	0.005	5	3900	5.40×10^{-04}	4.10×10^{-04}	0.4138
0.3	0.005	9	1950	5.40×10^{-04}	4.10×10^{-04}	0.4011
0.3	0.005	13	1700	5.40×10^{-04}	4.10×10^{-04}	0.4213
0.3	0.01	1	5350	5.60×10^{-04}	4.30×10^{-04}	0.343
0.3	0.01	3	2350	5.40×10^{-04}	4.10×10^{-04}	0.3816
0.3	0.01	5	2000	5.40×10^{-04}	4.10×10^{-04}	0.4096
0.3	0.01	9	1400	5.40×10^{-04}	4.10×10^{-04}	0.4464
0.3	0.01	13	700	5.40×10^{-04}	4.10×10^{-04}	0.4003
0.3	0.05	1	1700	5.60×10^{-04}	4.20×10^{-04}	0.37
0.3	0.05	3	550	5.50×10^{-04}	4.20×10^{-04}	0.4107
0.3	0.05	5	650	5.50×10^{-04}	4.10×10^{-04}	0.4732
0.3	0.05	9	200	5.40×10^{-04}	4.10×10^{-04}	0.4137
0.3	0.05	13				
0.3	0.1	1	550	5.50×10^{-04}	4.30×10^{-04}	0.3468
0.3	0.1	3				
0.3	0.1	5				
0.3	0.1	9				
0.3	0.1	13				
0.6	0.001	1	13000	5.70×10^{-04}	4.40×10^{-04}	0.2404
0.6	0.001	3	11000	5.50×10^{-04}	4.20×10^{-04}	0.3182
0.6	0.001	5	8650	5.50×10^{-04}	4.20×10^{-04}	0.3388
0.6	0.001	9	7350	5.40×10^{-04}	4.10×10^{-04}	0.3714
0.6	0.001	13	5150	5.40×10^{-04}	4.10×10^{-04}	0.3649
0.6	0.005	1	4950	5.60×10^{-04}	4.30×10^{-04}	0.2993
0.6	0.005	3	5000	5.40×10^{-04}	4.10×10^{-04}	0.3821
0.6	0.005	5	3600	5.40×10^{-04}	4.10×10^{-04}	0.4121
0.6	0.005	9	2250	5.40×10^{-04}	4.10×10^{-04}	0.4155
0.6	0.005	13	1450	5.40×10^{-04}	4.10×10^{-04}	0.4058

Table A.3: (continued)

Bag Fraction	Learning Rate	Tree Complexity	Number of Trees	RMSE	MAE	R^2
0.6	0.01	1	5000	5.60×10^{-04}	4.30×10^{-04}	0.3337
0.6	0.01	3	2850	5.40×10^{-04}	4.10×10^{-04}	0.3915
0.6	0.01	5	1600	5.40×10^{-04}	4.10×10^{-04}	0.3918
0.6	0.01	9	1400	5.40×10^{-04}	4.10×10^{-04}	0.4371
0.6	0.01	13	700	5.40×10^{-04}	4.10×10^{-04}	0.4026
0.6	0.05	1	3150	5.50×10^{-04}	4.20×10^{-04}	0.3883
0.6	0.05	3	550	5.50×10^{-04}	4.20×10^{-04}	0.39
0.6	0.05	5	300	5.50×10^{-04}	4.10×10^{-04}	0.3982
0.6	0.05	9				
0.6	0.05	13				
0.6	0.1	1	1150	5.50×10^{-04}	4.20×10^{-04}	0.3734
0.6	0.1	3	350	5.50×10^{-04}	4.10×10^{-04}	0.4266
0.6	0.1	5	200	5.50×10^{-04}	4.10×10^{-04}	0.4337
0.6	0.1	9				
0.6	0.1	13				
0.75	0.001	1	14350	5.70×10^{-04}	4.40×10^{-04}	0.2416
0.75	0.001	3	11800	5.50×10^{-04}	4.20×10^{-04}	0.3185
0.75	0.001	5	9250	5.50×10^{-04}	4.20×10^{-04}	0.3394
0.75	0.001	9	6900	5.40×10^{-04}	4.10×10^{-04}	0.3608
0.75	0.001	13	5650	5.40×10^{-04}	4.10×10^{-04}	0.3747
0.75	0.005	1	4500	5.60×10^{-04}	4.30×10^{-04}	0.2841
0.75	0.005	3	4900	5.40×10^{-04}	4.10×10^{-04}	0.3793
0.75	0.005	5	3800	5.50×10^{-04}	4.10×10^{-04}	0.4041
0.75	0.005	9	2250	5.40×10^{-04}	4.10×10^{-04}	0.4128
0.75	0.005	13	1300	5.40×10^{-04}	4.10×10^{-04}	0.3906
0.75	0.01	1	4800	5.60×10^{-04}	4.30×10^{-04}	0.3273
0.75	0.01	3	2350	5.50×10^{-04}	4.10×10^{-04}	0.3722
0.75	0.01	5	2100	5.50×10^{-04}	4.10×10^{-04}	0.4168
0.75	0.01	9	1050	5.40×10^{-04}	4.10×10^{-04}	0.4062
0.75	0.01	13	700	5.40×10^{-04}	4.10×10^{-04}	0.3988
0.75	0.05	1	1600	5.50×10^{-04}	4.20×10^{-04}	0.3511
0.75	0.05	3	500	5.40×10^{-04}	4.10×10^{-04}	0.3801
0.75	0.05	5	350	5.40×10^{-04}	4.10×10^{-04}	0.4065
0.75	0.05	9	300	5.40×10^{-04}	4.10×10^{-04}	0.4436
0.75	0.05	13				
0.75	0.1	1	1700	5.60×10^{-04}	4.20×10^{-04}	0.3953
0.75	0.1	3	300	5.40×10^{-04}	4.10×10^{-04}	0.3994

Table A.3: (continued)

Bag Fraction	Learning Rate	Tree Complexity	Number of Trees	RMSE	MAE	R^2
0.75	0.1	5	200	$5.50{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.4232
0.75	0.1	9				
0.75	0.1	13				
0.9	0.001	1	13800	$5.70{\times}10^{-04}$	$4.40{\times}10^{-04}$	0.2234
0.9	0.001	3	14600	$5.50{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.3277
0.9	0.001	5	9200	$5.50{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.3303
0.9	0.001	9	7150	$5.40{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3577
0.9	0.001	13	6650	$5.40{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3869
0.9	0.005	1	5600	$5.60{\times}10^{-04}$	$4.30{\times}10^{-04}$	0.2907
0.9	0.005	3	5600	$5.40{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3844
0.9	0.005	5	3150	$5.40{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3809
0.9	0.005	9	2150	$5.40{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3998
0.9	0.005	13	1450	$5.40{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3977
0.9	0.01	1	3550	$5.60{\times}10^{-04}$	$4.30{\times}10^{-04}$	0.3022
0.9	0.01	3	3200	$5.40{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3965
0.9	0.01	5	1300	$5.50{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3625
0.9	0.01	9	950	$5.50{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3625
0.9	0.01	13	750	$5.40{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.4031
0.9	0.05	1	3050	$5.50{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.3715
0.9	0.05	3	600	$5.50{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.3953
0.9	0.05	5	450	$5.50{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.4179
0.9	0.05	9	250	$5.50{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.4183
0.9	0.05	13	200	$5.40{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.4357
0.9	0.1	1	600	$5.60{\times}10^{-04}$	$4.30{\times}10^{-04}$	0.331
0.9	0.1	3	250	$5.50{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.3782
0.9	0.1	5	250	$5.50{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.4489
0.9	0.1	9				
0.9	0.1	13				

Table A.4: BRT models that could not be trained on the drought dataset.

Bag Fraction	Learning Rate	Tree Complexity
0.3	0.1	5
0.3	0.1	9
0.3	0.1	13
0.6	0.05	13
0.6	0.1	9
0.6	0.1	13
0.75	0.1	9
0.75	0.1	13
0.9	0.1	5
0.9	0.1	9
0.9	0.1	13

Table A.5: BRT models that could not be trained on the wetting dataset.

Bag Fraction	Learning Rate	Tree Complexity
0.3	0.05	13
0.3	0.1	3
0.3	0.1	5
0.3	0.1	9
0.3	0.1	13
0.6	0.05	9
0.6	0.05	13
0.6	0.1	9
0.6	0.1	13
0.75	0.05	13
0.75	0.1	9

Table A.6: Error Statisics of each BART model fit on the drought dataset.

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.5	50	0.25	3	0.8	4.94×10^{-04}	3.72×10^{-04}	0.3906
0.5	0.5	50	0.25	3	0.9	5.03×10^{-04}	3.75×10^{-04}	0.3978
0.5	0.5	50	0.25	3	0.99	4.98×10^{-04}	3.73×10^{-04}	0.4081
0.5	0.5	50	0.25	7	0.8	4.96×10^{-04}	3.73×10^{-04}	0.4070
0.5	0.5	50	0.25	7	0.9	5.01×10^{-04}	3.79×10^{-04}	0.3905
0.5	0.5	50	0.25	7	0.99	4.99×10^{-04}	3.74×10^{-04}	0.3951
0.5	0.5	50	0.25	10	0.8	5.05×10^{-04}	3.79×10^{-04}	0.4237
0.5	0.5	50	0.25	10	0.9	3.71×10^{-04}	4.92×10^{-04}	0.4027
0.5	0.5	50	0.25	10	0.99	4.92×10^{-04}	3.73×10^{-04}	0.3992
0.5	0.5	50	1	3	0.8	4.85×10^{-04}	3.62×10^{-04}	0.4391
0.5	0.5	50	1	3	0.9	4.89×10^{-04}	3.64×10^{-04}	0.4309
0.5	0.5	50	1	3	0.99	4.79×10^{-04}	3.59×10^{-04}	0.4424
0.5	0.5	50	1	7	0.8	4.87×10^{-04}	3.63×10^{-04}	0.4287
0.5	0.5	50	1	7	0.9	4.86×10^{-04}	3.65×10^{-04}	0.4399
0.5	0.5	50	1	7	0.99	4.83×10^{-04}	3.59×10^{-04}	0.4375
0.5	0.5	50	1	10	0.8	4.80×10^{-04}	3.59×10^{-04}	0.4534
0.5	0.5	50	1	10	0.9	4.84×10^{-04}	3.64×10^{-04}	0.4406
0.5	0.5	50	1	10	0.99	4.79×10^{-04}	3.61×10^{-04}	0.4416
0.5	0.5	50	2	3	0.8	4.78×10^{-04}	3.58×10^{-04}	0.4481
0.5	0.5	50	2	3	0.9	4.73×10^{-04}	3.55×10^{-04}	0.4378
0.5	0.5	50	2	3	0.99	4.75×10^{-04}	3.56×10^{-04}	0.4410
0.5	0.5	50	2	7	0.8	4.73×10^{-04}	3.55×10^{-04}	0.4507
0.5	0.5	50	2	7	0.9	4.77×10^{-04}	3.59×10^{-04}	0.4370
0.5	0.5	50	2	7	0.99	4.78×10^{-04}	3.58×10^{-04}	0.4446
0.5	0.5	50	2	10	0.8	4.77×10^{-04}	3.57×10^{-04}	0.4309
0.5	0.5	50	2	10	0.9	4.77×10^{-04}	3.58×10^{-04}	0.4504
0.5	0.5	50	2	10	0.99	4.84×10^{-04}	3.60×10^{-04}	0.4381
0.5	0.5	100	0.25	3	0.8	4.82×10^{-04}	3.57×10^{-04}	0.4188
0.5	0.5	100	0.25	3	0.9	4.87×10^{-04}	3.63×10^{-04}	0.4296
0.5	0.5	100	0.25	3	0.99	4.99×10^{-04}	3.72×10^{-04}	0.4531
0.5	0.5	100	0.25	7	0.8	4.89×10^{-04}	3.64×10^{-04}	0.4356
0.5	0.5	100	0.25	7	0.9	4.86×10^{-04}	3.63×10^{-04}	0.4559
0.5	0.5	100	0.25	7	0.99	5.04×10^{-04}	3.75×10^{-04}	0.4302
0.5	0.5	100	0.25	10	0.8	4.82×10^{-04}	3.63×10^{-04}	0.4257
0.5	0.5	100	0.25	10	0.9	4.85×10^{-04}	3.63×10^{-04}	0.4316
0.5	0.5	100	0.25	10	0.99	4.90×10^{-04}	3.67×10^{-04}	0.4403
0.5	0.5	100	1	3	0.8	4.72×10^{-04}	3.51×10^{-04}	0.4904
0.5	0.5	100	1	3	0.9	4.70×10^{-04}	3.51×10^{-04}	0.4822

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.5	100	1	3	0.99	4.70×10^{-04}	3.53×10^{-04}	0.4817
0.5	0.5	100	1	7	0.8	4.70×10^{-04}	3.54×10^{-04}	0.4808
0.5	0.5	100	1	7	0.9	4.75×10^{-04}	3.53×10^{-04}	0.4818
0.5	0.5	100	1	7	0.99	4.72×10^{-04}	3.53×10^{-04}	0.4930
0.5	0.5	100	1	10	0.8	4.74×10^{-04}	3.56×10^{-04}	0.4860
0.5	0.5	100	1	10	0.9	4.73×10^{-04}	3.54×10^{-04}	0.4966
0.5	0.5	100	1	10	0.99	4.82×10^{-04}	3.58×10^{-04}	0.4803
0.5	0.5	100	2	3	0.8	4.74×10^{-04}	3.55×10^{-04}	0.4692
0.5	0.5	100	2	3	0.9	4.70×10^{-04}	3.51×10^{-04}	0.4786
0.5	0.5	100	2	3	0.99	4.72×10^{-04}	3.52×10^{-04}	0.4733
0.5	0.5	100	2	7	0.8	4.73×10^{-04}	3.56×10^{-04}	0.4902
0.5	0.5	100	2	7	0.9	4.72×10^{-04}	3.53×10^{-04}	0.4747
0.5	0.5	100	2	7	0.99	4.65×10^{-04}	3.48×10^{-04}	0.4842
0.5	0.5	100	2	10	0.8	4.68×10^{-04}	3.52×10^{-04}	0.4869
0.5	0.5	100	2	10	0.9	4.72×10^{-04}	3.55×10^{-04}	0.4759
0.5	0.5	100	2	10	0.99	4.77×10^{-04}	3.54×10^{-04}	0.4804
0.5	0.5	200	0.25	3	0.8	4.80×10^{-04}	3.59×10^{-04}	0.4914
0.5	0.5	200	0.25	3	0.9	4.73×10^{-04}	3.53×10^{-04}	0.4817
0.5	0.5	200	0.25	3	0.99	4.70×10^{-04}	3.54×10^{-04}	0.5122
0.5	0.5	200	0.25	7	0.8	4.81×10^{-04}	3.61×10^{-04}	0.5010
0.5	0.5	200	0.25	7	0.9	4.74×10^{-04}	3.55×10^{-04}	0.4948
0.5	0.5	200	0.25	7	0.99	4.77×10^{-04}	3.58×10^{-04}	0.4807
0.5	0.5	200	0.25	10	0.8	4.80×10^{-04}	3.57×10^{-04}	0.5026
0.5	0.5	200	0.25	10	0.9	4.76×10^{-04}	3.59×10^{-04}	0.5161
0.5	0.5	200	0.25	10	0.99	4.65×10^{-04}	3.47×10^{-04}	0.4715
0.5	0.5	200	1	3	0.8	4.70×10^{-04}	3.54×10^{-04}	0.5310
0.5	0.5	200	1	3	0.9	4.68×10^{-04}	3.51×10^{-04}	0.5262
0.5	0.5	200	1	3	0.99	4.67×10^{-04}	3.52×10^{-04}	0.5376
0.5	0.5	200	1	7	0.8	4.70×10^{-04}	3.54×10^{-04}	0.5457
0.5	0.5	200	1	7	0.9	4.71×10^{-04}	3.54×10^{-04}	0.5399
0.5	0.5	200	1	7	0.99	4.70×10^{-04}	3.53×10^{-04}	0.5401
0.5	0.5	200	1	10	0.8	4.70×10^{-04}	3.53×10^{-04}	0.5445
0.5	0.5	200	1	10	0.9	4.69×10^{-04}	3.52×10^{-04}	0.5237
0.5	0.5	200	1	10	0.99	4.71×10^{-04}	3.56×10^{-04}	0.5331
0.5	0.5	200	2	3	0.8	4.66×10^{-04}	3.49×10^{-04}	0.4919
0.5	0.5	200	2	3	0.9	4.69×10^{-04}	3.52×10^{-04}	0.5115
0.5	0.5	200	2	3	0.99	4.68×10^{-04}	3.53×10^{-04}	0.4933
0.5	0.5	200	2	7	0.8	4.67×10^{-04}	3.51×10^{-04}	0.5009

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.5	200	2	7	0.9	4.73×10^{-04}	3.54×10^{-04}	0.4982
0.5	0.5	200	2	7	0.99	4.70×10^{-04}	3.52×10^{-04}	0.5029
0.5	0.5	200	2	10	0.8	4.65×10^{-04}	3.50×10^{-04}	0.4972
0.5	0.5	200	2	10	0.9	4.67×10^{-04}	3.53×10^{-04}	0.5021
0.5	0.5	200	2	10	0.99	4.70×10^{-04}	3.52×10^{-04}	0.4999
0.5	0.95	50	0.25	3	0.8	4.92×10^{-04}	3.72×10^{-04}	0.3952
0.5	0.95	50	0.25	3	0.9	5.03×10^{-04}	3.77×10^{-04}	0.3998
0.5	0.95	50	0.25	3	0.99	4.94×10^{-04}	3.71×10^{-04}	0.3980
0.5	0.95	50	0.25	7	0.8	4.95×10^{-04}	3.71×10^{-04}	0.4017
0.5	0.95	50	0.25	7	0.9	5.01×10^{-04}	3.78×10^{-04}	0.3956
0.5	0.95	50	0.25	7	0.99	4.95×10^{-04}	3.76×10^{-04}	0.3967
0.5	0.95	50	0.25	10	0.8	4.93×10^{-04}	3.71×10^{-04}	0.4030
0.5	0.95	50	0.25	10	0.9	5.00×10^{-04}	3.71×10^{-04}	0.3899
0.5	0.95	50	0.25	10	0.99	4.94×10^{-04}	3.76×10^{-04}	0.4000
0.5	0.95	50	1	3	0.8	4.79×10^{-04}	3.61×10^{-04}	0.4351
0.5	0.95	50	1	3	0.9	4.88×10^{-04}	3.65×10^{-04}	0.4298
0.5	0.95	50	1	3	0.99	4.81×10^{-04}	3.62×10^{-04}	0.4486
0.5	0.95	50	1	7	0.8	4.84×10^{-04}	3.61×10^{-04}	0.4349
0.5	0.95	50	1	7	0.9	4.82×10^{-04}	3.59×10^{-04}	0.4273
0.5	0.95	50	1	7	0.99	4.84×10^{-04}	3.64×10^{-04}	0.4373
0.5	0.95	50	1	10	0.8	4.86×10^{-04}	3.63×10^{-04}	0.4306
0.5	0.95	50	1	10	0.9	4.86×10^{-04}	3.62×10^{-04}	0.4355
0.5	0.95	50	1	10	0.99	4.89×10^{-04}	3.66×10^{-04}	0.4259
0.5	0.95	50	2	3	0.8	4.82×10^{-04}	3.59×10^{-04}	0.4376
0.5	0.95	50	2	3	0.9	4.77×10^{-04}	3.61×10^{-04}	0.4352
0.5	0.95	50	2	3	0.99	4.79×10^{-04}	3.60×10^{-04}	0.4217
0.5	0.95	50	2	7	0.8	4.85×10^{-04}	3.65×10^{-04}	0.4418
0.5	0.95	50	2	7	0.9	4.81×10^{-04}	3.63×10^{-04}	0.4353
0.5	0.95	50	2	7	0.99	4.77×10^{-04}	3.56×10^{-04}	0.4349
0.5	0.95	50	2	10	0.8	4.79×10^{-04}	3.60×10^{-04}	0.4394
0.5	0.95	50	2	10	0.9	4.82×10^{-04}	3.63×10^{-04}	0.4438
0.5	0.95	50	2	10	0.99	4.76×10^{-04}	3.57×10^{-04}	0.4430
0.5	0.95	100	0.25	3	0.8	4.88×10^{-04}	3.68×10^{-04}	0.4372
0.5	0.95	100	0.25	3	0.9	4.83×10^{-04}	3.60×10^{-04}	0.4258
0.5	0.95	100	0.25	3	0.99	4.89×10^{-04}	3.63×10^{-04}	0.4273
0.5	0.95	100	0.25	7	0.8	4.85×10^{-04}	3.65×10^{-04}	0.4269
0.5	0.95	100	0.25	7	0.9	4.85×10^{-04}	3.61×10^{-04}	0.4164
0.5	0.95	100	0.25	7	0.99	4.84×10^{-04}	3.62×10^{-04}	0.4280

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.95	100	0.25	10	0.8	4.88×10^{-04}	3.65×10^{-04}	0.4314
0.5	0.95	100	0.25	10	0.9	4.97×10^{-04}	3.71×10^{-04}	0.4384
0.5	0.95	100	0.25	10	0.99	4.91×10^{-04}	3.66×10^{-04}	0.4221
0.5	0.95	100	1	3	0.8	4.78×10^{-04}	3.60×10^{-04}	0.4745
0.5	0.95	100	1	3	0.9	4.82×10^{-04}	3.57×10^{-04}	0.4514
0.5	0.95	100	1	3	0.99	4.77×10^{-04}	3.58×10^{-04}	0.4758
0.5	0.95	100	1	7	0.8	4.82×10^{-04}	3.58×10^{-04}	0.4962
0.5	0.95	100	1	7	0.9	4.82×10^{-04}	3.59×10^{-04}	0.4758
0.5	0.95	100	1	7	0.99	4.77×10^{-04}	3.59×10^{-04}	0.4817
0.5	0.95	100	1	10	0.8	4.76×10^{-04}	3.55×10^{-04}	0.4747
0.5	0.95	100	1	10	0.9	4.75×10^{-04}	3.54×10^{-04}	0.4797
0.5	0.95	100	1	10	0.99	4.77×10^{-04}	3.57×10^{-04}	0.4712
0.5	0.95	100	2	3	0.8	4.69×10^{-04}	3.53×10^{-04}	0.4540
0.5	0.95	100	2	3	0.9	4.72×10^{-04}	3.54×10^{-04}	0.4654
0.5	0.95	100	2	3	0.99	4.72×10^{-04}	3.55×10^{-04}	0.4722
0.5	0.95	100	2	7	0.8	4.75×10^{-04}	3.55×10^{-04}	0.4651
0.5	0.95	100	2	7	0.9	4.75×10^{-04}	3.57×10^{-04}	0.4547
0.5	0.95	100	2	7	0.99	4.74×10^{-04}	3.54×10^{-04}	0.4618
0.5	0.95	100	2	10	0.8	4.71×10^{-04}	3.53×10^{-04}	0.4781
0.5	0.95	100	2	10	0.9	4.76×10^{-04}	3.56×10^{-04}	0.4591
0.5	0.95	100	2	10	0.99	4.71×10^{-04}	3.55×10^{-04}	0.4713
0.5	0.95	200	0.25	3	0.8	4.83×10^{-04}	3.62×10^{-04}	0.4977
0.5	0.95	200	0.25	3	0.9	4.93×10^{-04}	3.65×10^{-04}	0.5328
0.5	0.95	200	0.25	3	0.99	4.81×10^{-04}	3.61×10^{-04}	0.5009
0.5	0.95	200	0.25	7	0.8	4.83×10^{-04}	3.61×10^{-04}	0.4923
0.5	0.95	200	0.25	7	0.9	4.77×10^{-04}	3.53×10^{-04}	0.4866
0.5	0.95	200	0.25	7	0.99	4.82×10^{-04}	3.61×10^{-04}	0.5105
0.5	0.95	200	0.25	10	0.8	4.80×10^{-04}	3.59×10^{-04}	0.4924
0.5	0.95	200	0.25	10	0.9	4.78×10^{-04}	3.57×10^{-04}	0.5232
0.5	0.95	200	0.25	10	0.99	4.78×10^{-04}	3.57×10^{-04}	0.4922
0.5	0.95	200	1	3	0.8	4.70×10^{-04}	3.54×10^{-04}	0.5335
0.5	0.95	200	1	3	0.9	4.82×10^{-04}	3.61×10^{-04}	0.5461
0.5	0.95	200	1	3	0.99	4.76×10^{-04}	3.58×10^{-04}	0.5234
0.5	0.95	200	1	7	0.8	4.68×10^{-04}	3.51×10^{-04}	0.5081
0.5	0.95	200	1	7	0.9	4.71×10^{-04}	3.54×10^{-04}	0.5324
0.5	0.95	200	1	7	0.99	4.78×10^{-04}	3.58×10^{-04}	0.5201
0.5	0.95	200	1	10	0.8	4.72×10^{-04}	3.57×10^{-04}	0.5353
0.5	0.95	200	1	10	0.9	4.75×10^{-04}	3.54×10^{-04}	0.5244

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.95	200	1	10	0.99	4.73×10^{-04}	3.52×10^{-04}	0.5243
0.5	0.95	200	2	3	0.8	4.69×10^{-04}	3.51×10^{-04}	0.4845
0.5	0.95	200	2	3	0.9	4.67×10^{-04}	3.51×10^{-04}	0.4785
0.5	0.95	200	2	3	0.99	4.72×10^{-04}	3.54×10^{-04}	0.4784
0.5	0.95	200	2	7	0.8	4.73×10^{-04}	3.54×10^{-04}	0.4828
0.5	0.95	200	2	7	0.9	4.69×10^{-04}	3.51×10^{-04}	0.4913
0.5	0.95	200	2	7	0.99	4.70×10^{-04}	3.54×10^{-04}	0.4799
0.5	0.95	200	2	10	0.8	4.70×10^{-04}	3.53×10^{-04}	0.4930
0.5	0.95	200	2	10	0.9	4.68×10^{-04}	3.51×10^{-04}	0.4789
0.5	0.95	200	2	10	0.99	4.70×10^{-04}	3.54×10^{-04}	0.4822
0.5	2	50	0.25	3	0.8	4.96×10^{-04}	3.75×10^{-04}	0.3944
0.5	2	50	0.25	3	0.9	4.95×10^{-04}	3.74×10^{-04}	0.3873
0.5	2	50	0.25	3	0.99	5.02×10^{-04}	3.75×10^{-04}	0.4030
0.5	2	50	0.25	7	0.8	4.99×10^{-04}	3.77×10^{-04}	0.3985
0.5	2	50	0.25	7	0.9	4.97×10^{-04}	3.74×10^{-04}	0.3942
0.5	2	50	0.25	7	0.99	4.97×10^{-04}	3.76×10^{-04}	0.4013
0.5	2	50	0.25	10	0.8	4.90×10^{-04}	3.69×10^{-04}	0.3953
0.5	2	50	0.25	10	0.9	4.95×10^{-04}	3.74×10^{-04}	0.3998
0.5	2	50	0.25	10	0.99	5.00×10^{-04}	3.75×10^{-04}	0.4074
0.5	2	50	1	3	0.8	4.86×10^{-04}	3.66×10^{-04}	0.4216
0.5	2	50	1	3	0.9	4.94×10^{-04}	3.70×10^{-04}	0.4113
0.5	2	50	1	3	0.99	4.91×10^{-04}	3.68×10^{-04}	0.4171
0.5	2	50	1	7	0.8	4.91×10^{-04}	3.71×10^{-04}	0.4163
0.5	2	50	1	7	0.9	4.91×10^{-04}	3.66×10^{-04}	0.4139
0.5	2	50	1	7	0.99	4.89×10^{-04}	3.66×10^{-04}	0.4160
0.5	2	50	1	10	0.8	4.90×10^{-04}	3.66×10^{-04}	0.4153
0.5	2	50	1	10	0.9	4.95×10^{-04}	3.72×10^{-04}	0.4137
0.5	2	50	1	10	0.99	4.90×10^{-04}	3.70×10^{-04}	0.4276
0.5	2	50	2	3	0.8	4.88×10^{-04}	3.68×10^{-04}	0.4212
0.5	2	50	2	3	0.9	4.88×10^{-04}	3.67×10^{-04}	0.4187
0.5	2	50	2	3	0.99	4.80×10^{-04}	3.62×10^{-04}	0.4203
0.5	2	50	2	7	0.8	4.88×10^{-04}	3.65×10^{-04}	0.4216
0.5	2	50	2	7	0.9	4.86×10^{-04}	3.64×10^{-04}	0.4177
0.5	2	50	2	7	0.99	4.89×10^{-04}	3.68×10^{-04}	0.4179
0.5	2	50	2	10	0.8	4.86×10^{-04}	3.66×10^{-04}	0.4136
0.5	2	50	2	10	0.9	4.84×10^{-04}	3.66×10^{-04}	0.4226
0.5	2	50	2	10	0.99	4.87×10^{-04}	3.63×10^{-04}	0.4195
0.5	2	100	0.25	3	0.8	4.89×10^{-04}	3.69×10^{-04}	0.4313

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	2	100	0.25	3	0.9	4.86×10^{-04}	3.66×10^{-04}	0.4296
0.5	2	100	0.25	3	0.99	4.85×10^{-04}	3.64×10^{-04}	0.4239
0.5	2	100	0.25	7	0.8	4.93×10^{-04}	3.71×10^{-04}	0.4298
0.5	2	100	0.25	7	0.9	4.86×10^{-04}	3.64×10^{-04}	0.4401
0.5	2	100	0.25	7	0.99	4.93×10^{-04}	3.70×10^{-04}	0.4222
0.5	2	100	0.25	10	0.8	4.93×10^{-04}	3.69×10^{-04}	0.4387
0.5	2	100	0.25	10	0.9	4.92×10^{-04}	3.72×10^{-04}	0.4399
0.5	2	100	0.25	10	0.99	4.86×10^{-04}	3.60×10^{-04}	0.4242
0.5	2	100	1	3	0.8	4.82×10^{-04}	3.62×10^{-04}	0.4627
0.5	2	100	1	3	0.9	4.85×10^{-04}	3.65×10^{-04}	0.4637
0.5	2	100	1	3	0.99	4.83×10^{-04}	3.60×10^{-04}	0.4538
0.5	2	100	1	7	0.8	4.82×10^{-04}	3.63×10^{-04}	0.4644
0.5	2	100	1	7	0.9	4.83×10^{-04}	3.63×10^{-04}	0.4512
0.5	2	100	1	7	0.99	4.77×10^{-04}	3.58×10^{-04}	0.4510
0.5	2	100	1	10	0.8	4.83×10^{-04}	3.62×10^{-04}	0.4706
0.5	2	100	1	10	0.9	4.80×10^{-04}	3.58×10^{-04}	0.4573
0.5	2	100	1	10	0.99	4.77×10^{-04}	3.58×10^{-04}	0.4655
0.5	2	100	2	3	0.8	4.79×10^{-04}	3.59×10^{-04}	0.4412
0.5	2	100	2	3	0.9	4.83×10^{-04}	3.63×10^{-04}	0.4448
0.5	2	100	2	3	0.99	4.79×10^{-04}	3.57×10^{-04}	0.4457
0.5	2	100	2	7	0.8	4.82×10^{-04}	3.63×10^{-04}	0.4448
0.5	2	100	2	7	0.9	4.80×10^{-04}	3.61×10^{-04}	0.4506
0.5	2	100	2	7	0.99	4.80×10^{-04}	3.61×10^{-04}	0.4502
0.5	2	100	2	10	0.8	4.80×10^{-04}	3.60×10^{-04}	0.4561
0.5	2	100	2	10	0.9	4.79×10^{-04}	3.61×10^{-04}	0.4518
0.5	2	100	2	10	0.99	4.76×10^{-04}	3.56×10^{-04}	0.4477
0.5	2	200	0.25	3	0.8	4.86×10^{-04}	3.64×10^{-04}	0.4989
0.5	2	200	0.25	3	0.9	4.86×10^{-04}	3.64×10^{-04}	0.5063
0.5	2	200	0.25	3	0.99	4.79×10^{-04}	3.60×10^{-04}	0.5035
0.5	2	200	0.25	7	0.8	4.87×10^{-04}	3.65×10^{-04}	0.4749
0.5	2	200	0.25	7	0.9	4.78×10^{-04}	3.59×10^{-04}	0.4818
0.5	2	200	0.25	7	0.99	4.85×10^{-04}	3.61×10^{-04}	0.4875
0.5	2	200	0.25	10	0.8	4.87×10^{-04}	3.61×10^{-04}	0.4868
0.5	2	200	0.25	10	0.9	4.77×10^{-04}	3.59×10^{-04}	0.4871
0.5	2	200	0.25	10	0.99	4.78×10^{-04}	3.60×10^{-04}	0.4804
0.5	2	200	1	3	0.8	4.75×10^{-04}	3.58×10^{-04}	0.5107
0.5	2	200	1	3	0.9	4.80×10^{-04}	3.61×10^{-04}	0.4952
0.5	2	200	1	3	0.99	4.81×10^{-04}	3.64×10^{-04}	0.5182

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	2	200	1	7	0.8	4.77×10^{-04}	3.57×10^{-04}	0.5029
0.5	2	200	1	7	0.9	4.74×10^{-04}	3.57×10^{-04}	0.5111
0.5	2	200	1	7	0.99	4.84×10^{-04}	3.61×10^{-04}	0.4984
0.5	2	200	1	10	0.8	4.74×10^{-04}	3.57×10^{-04}	0.4989
0.5	2	200	1	10	0.9	4.76×10^{-04}	3.54×10^{-04}	0.4991
0.5	2	200	1	10	0.99	4.78×10^{-04}	3.60×10^{-04}	0.5061
0.5	2	200	2	3	0.8	4.76×10^{-04}	3.57×10^{-04}	0.4689
0.5	2	200	2	3	0.9	4.73×10^{-04}	3.54×10^{-04}	0.4688
0.5	2	200	2	3	0.99	4.75×10^{-04}	3.57×10^{-04}	0.4720
0.5	2	200	2	7	0.8	4.71×10^{-04}	3.55×10^{-04}	0.4657
0.5	2	200	2	7	0.9	4.75×10^{-04}	3.55×10^{-04}	0.4665
0.5	2	200	2	7	0.99	4.74×10^{-04}	3.56×10^{-04}	0.4710
0.5	2	200	2	10	0.8	4.77×10^{-04}	3.57×10^{-04}	0.4603
0.5	2	200	2	10	0.9	4.74×10^{-04}	3.58×10^{-04}	0.4611
0.5	2	200	2	10	0.99	4.77×10^{-04}	3.58×10^{-04}	0.4625
0.95	0.5	50	0.25	3	0.8	4.93×10^{-04}	3.67×10^{-04}	0.4237
0.95	0.5	50	0.25	3	0.9	4.82×10^{-04}	3.62×10^{-04}	0.4111
0.95	0.5	50	0.25	3	0.99	4.89×10^{-04}	3.65×10^{-04}	0.4399
0.95	0.5	50	0.25	7	0.8	4.89×10^{-04}	3.68×10^{-04}	0.4295
0.95	0.5	50	0.25	7	0.9	4.80×10^{-04}	3.62×10^{-04}	0.4221
0.95	0.5	50	0.25	7	0.99	4.89×10^{-04}	3.67×10^{-04}	0.4337
0.95	0.5	50	0.25	10	0.8	4.98×10^{-04}	3.70×10^{-04}	0.4300
0.95	0.5	50	0.25	10	0.9	4.86×10^{-04}	3.66×10^{-04}	0.4394
0.95	0.5	50	0.25	10	0.99	4.90×10^{-04}	3.66×10^{-04}	0.4189
0.95	0.5	50	1	3	0.8	4.74×10^{-04}	3.50×10^{-04}	0.4885
0.95	0.5	50	1	3	0.9	4.71×10^{-04}	3.51×10^{-04}	0.5034
0.95	0.5	50	1	3	0.99	4.71×10^{-04}	3.51×10^{-04}	0.4937
0.95	0.5	50	1	7	0.8	4.69×10^{-04}	3.51×10^{-04}	0.4831
0.95	0.5	50	1	7	0.9	4.71×10^{-04}	3.56×10^{-04}	0.4950
0.95	0.5	50	1	7	0.99	4.74×10^{-04}	3.53×10^{-04}	0.5063
0.95	0.5	50	1	10	0.8	4.73×10^{-04}	3.54×10^{-04}	0.4900
0.95	0.5	50	1	10	0.9	4.66×10^{-04}	3.49×10^{-04}	0.4816
0.95	0.5	50	1	10	0.99	4.74×10^{-04}	3.53×10^{-04}	0.4872
0.95	0.5	50	2	3	0.8	4.68×10^{-04}	3.49×10^{-04}	0.4953
0.95	0.5	50	2	3	0.9	4.68×10^{-04}	3.46×10^{-04}	0.4765
0.95	0.5	50	2	3	0.99	4.69×10^{-04}	3.54×10^{-04}	0.4771
0.95	0.5	50	2	7	0.8	4.72×10^{-04}	3.53×10^{-04}	0.4858
0.95	0.5	50	2	7	0.9	4.71×10^{-04}	3.53×10^{-04}	0.4914

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	0.5	50	2	7	0.99	4.68×10^{-04}	3.51×10^{-04}	0.4968
0.95	0.5	50	2	10	0.8	4.75×10^{-04}	3.55×10^{-04}	0.4975
0.95	0.5	50	2	10	0.9	4.69×10^{-04}	3.52×10^{-04}	0.4805
0.95	0.5	50	2	10	0.99	4.74×10^{-04}	3.55×10^{-04}	0.5016
0.95	0.5	100	0.25	3	0.8	4.82×10^{-04}	3.60×10^{-04}	0.5014
0.95	0.5	100	0.25	3	0.9	4.84×10^{-04}	3.60×10^{-04}	0.4828
0.95	0.5	100	0.25	3	0.99	4.82×10^{-04}	3.60×10^{-04}	0.4996
0.95	0.5	100	0.25	7	0.8	4.84×10^{-04}	3.64×10^{-04}	0.4770
0.95	0.5	100	0.25	7	0.9	4.72×10^{-04}	3.52×10^{-04}	0.4791
0.95	0.5	100	0.25	7	0.99	4.88×10^{-04}	3.64×10^{-04}	0.4964
0.95	0.5	100	0.25	10	0.8	4.77×10^{-04}	3.54×10^{-04}	0.4985
0.95	0.5	100	0.25	10	0.9	4.77×10^{-04}	3.57×10^{-04}	0.4814
0.95	0.5	100	0.25	10	0.99	4.91×10^{-04}	3.65×10^{-04}	0.4916
0.95	0.5	100	1	3	0.8	4.71×10^{-04}	3.52×10^{-04}	0.5274
0.95	0.5	100	1	3	0.9	4.74×10^{-04}	3.55×10^{-04}	0.5346
0.95	0.5	100	1	3	0.99	4.66×10^{-04}	3.50×10^{-04}	0.5526
0.95	0.5	100	1	7	0.8	4.72×10^{-04}	3.54×10^{-04}	0.5277
0.95	0.5	100	1	7	0.9	4.72×10^{-04}	3.54×10^{-04}	0.5342
0.95	0.5	100	1	7	0.99	4.77×10^{-04}	3.56×10^{-04}	0.5392
0.95	0.5	100	1	10	0.8	4.63×10^{-04}	3.47×10^{-04}	0.5338
0.95	0.5	100	1	10	0.9	4.74×10^{-04}	3.56×10^{-04}	0.5441
0.95	0.5	100	1	10	0.99	4.64×10^{-04}	3.48×10^{-04}	0.5344
0.95	0.5	100	2	3	0.8	4.70×10^{-04}	3.52×10^{-04}	0.5319
0.95	0.5	100	2	3	0.9	4.65×10^{-04}	3.50×10^{-04}	0.5302
0.95	0.5	100	2	3	0.99	4.67×10^{-04}	3.51×10^{-04}	0.5212
0.95	0.5	100	2	7	0.8	4.68×10^{-04}	3.50×10^{-04}	0.5416
0.95	0.5	100	2	7	0.9	4.70×10^{-04}	3.52×10^{-04}	0.5133
0.95	0.5	100	2	7	0.99	4.67×10^{-04}	3.49×10^{-04}	0.5094
0.95	0.5	100	2	10	0.8	4.67×10^{-04}	3.50×10^{-04}	0.5207
0.95	0.5	100	2	10	0.9	4.66×10^{-04}	3.49×10^{-04}	0.5322
0.95	0.5	100	2	10	0.99	4.70×10^{-04}	3.51×10^{-04}	0.5303
0.95	0.5	200	0.25	3	0.8	4.70×10^{-04}	3.51×10^{-04}	0.5303
0.95	0.5	200	0.25	3	0.9	4.76×10^{-04}	3.57×10^{-04}	0.5890
0.95	0.5	200	0.25	3	0.99	4.74×10^{-04}	3.58×10^{-04}	0.5590
0.95	0.5	200	0.25	7	0.8	4.81×10^{-04}	3.63×10^{-04}	0.5977
0.95	0.5	200	0.25	7	0.9	4.75×10^{-04}	3.59×10^{-04}	0.6103
0.95	0.5	200	0.25	7	0.99	4.88×10^{-04}	3.62×10^{-04}	0.5821
0.95	0.5	200	0.25	10	0.8	4.77×10^{-04}	3.56×10^{-04}	0.5816

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	0.5	200	0.25	10	0.9	4.81×10^{-04}	3.62×10^{-04}	0.5911
0.95	0.5	200	0.25	10	0.99	4.83×10^{-04}	3.63×10^{-04}	0.5682
0.95	0.5	200	1	3	0.8	4.75×10^{-04}	3.57×10^{-04}	0.6180
0.95	0.5	200	1	3	0.9	4.75×10^{-04}	3.59×10^{-04}	0.6203
0.95	0.5	200	1	3	0.99	4.73×10^{-04}	3.55×10^{-04}	0.6106
0.95	0.5	200	1	7	0.8	4.76×10^{-04}	3.58×10^{-04}	0.6013
0.95	0.5	200	1	7	0.9	4.68×10^{-04}	3.54×10^{-04}	0.6195
0.95	0.5	200	1	7	0.99	4.68×10^{-04}	3.51×10^{-04}	0.5951
0.95	0.5	200	1	10	0.8	4.78×10^{-04}	3.58×10^{-04}	0.5949
0.95	0.5	200	1	10	0.9	4.70×10^{-04}	3.54×10^{-04}	0.5979
0.95	0.5	200	1	10	0.99	4.74×10^{-04}	3.61×10^{-04}	0.6216
0.95	0.5	200	2	3	0.8	4.66×10^{-04}	3.50×10^{-04}	0.5440
0.95	0.5	200	2	3	0.9	4.66×10^{-04}	3.50×10^{-04}	0.5600
0.95	0.5	200	2	3	0.99	4.67×10^{-04}	3.51×10^{-04}	0.5465
0.95	0.5	200	2	7	0.8	4.61×10^{-04}	3.46×10^{-04}	0.5584
0.95	0.5	200	2	7	0.9	4.71×10^{-04}	3.53×10^{-04}	0.5604
0.95	0.5	200	2	7	0.99	4.71×10^{-04}	3.54×10^{-04}	0.5554
0.95	0.5	200	2	10	0.8	4.67×10^{-04}	3.49×10^{-04}	0.5483
0.95	0.5	200	2	10	0.9	4.66×10^{-04}	3.48×10^{-04}	0.5502
0.95	0.5	200	2	10	0.99	4.67×10^{-04}	3.49×10^{-04}	0.5498
0.95	0.95	50	0.25	3	0.8	4.91×10^{-04}	3.70×10^{-04}	0.4452
0.95	0.95	50	0.25	3	0.9	4.83×10^{-04}	3.61×10^{-04}	0.4363
0.95	0.95	50	0.25	3	0.99	4.82×10^{-04}	3.61×10^{-04}	0.4290
0.95	0.95	50	0.25	7	0.8	4.81×10^{-04}	3.62×10^{-04}	0.4420
0.95	0.95	50	0.25	7	0.9	4.86×10^{-04}	3.64×10^{-04}	0.4238
0.95	0.95	50	0.25	7	0.99	4.89×10^{-04}	3.65×10^{-04}	0.4339
0.95	0.95	50	0.25	10	0.8	4.92×10^{-04}	3.68×10^{-04}	0.4408
0.95	0.95	50	0.25	10	0.9	4.86×10^{-04}	3.64×10^{-04}	0.4440
0.95	0.95	50	0.25	10	0.99	4.83×10^{-04}	3.62×10^{-04}	0.4324
0.95	0.95	50	1	3	0.8	4.77×10^{-04}	3.58×10^{-04}	0.4808
0.95	0.95	50	1	3	0.9	4.74×10^{-04}	3.54×10^{-04}	0.4601
0.95	0.95	50	1	3	0.99	4.75×10^{-04}	3.51×10^{-04}	0.4817
0.95	0.95	50	1	7	0.8	4.81×10^{-04}	3.62×10^{-04}	0.4772
0.95	0.95	50	1	7	0.9	4.75×10^{-04}	3.57×10^{-04}	0.4755
0.95	0.95	50	1	7	0.99	4.77×10^{-04}	3.58×10^{-04}	0.4677
0.95	0.95	50	1	10	0.8	4.72×10^{-04}	3.54×10^{-04}	0.4672
0.95	0.95	50	1	10	0.9	4.76×10^{-04}	3.57×10^{-04}	0.4589
0.95	0.95	50	1	10	0.99	4.79×10^{-04}	3.59×10^{-04}	0.4745

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	0.95	50	2	3	0.8	4.73×10^{-04}	3.54×10^{-04}	0.4669
0.95	0.95	50	2	3	0.9	4.75×10^{-04}	3.54×10^{-04}	0.4650
0.95	0.95	50	2	3	0.99	4.71×10^{-04}	3.53×10^{-04}	0.4768
0.95	0.95	50	2	7	0.8	4.66×10^{-04}	3.50×10^{-04}	0.4635
0.95	0.95	50	2	7	0.9	4.69×10^{-04}	3.55×10^{-04}	0.4686
0.95	0.95	50	2	7	0.99	4.76×10^{-04}	3.55×10^{-04}	0.4723
0.95	0.95	50	2	10	0.8	4.73×10^{-04}	3.55×10^{-04}	0.4745
0.95	0.95	50	2	10	0.9	4.76×10^{-04}	3.54×10^{-04}	0.4909
0.95	0.95	50	2	10	0.99	4.73×10^{-04}	3.54×10^{-04}	0.4715
0.95	0.95	100	0.25	3	0.8	4.80×10^{-04}	3.60×10^{-04}	0.4972
0.95	0.95	100	0.25	3	0.9	4.81×10^{-04}	3.57×10^{-04}	0.5002
0.95	0.95	100	0.25	3	0.99	4.85×10^{-04}	3.58×10^{-04}	0.4847
0.95	0.95	100	0.25	7	0.8	4.77×10^{-04}	3.57×10^{-04}	0.4955
0.95	0.95	100	0.25	7	0.9	4.77×10^{-04}	3.55×10^{-04}	0.4753
0.95	0.95	100	0.25	7	0.99	4.78×10^{-04}	3.55×10^{-04}	0.4833
0.95	0.95	100	0.25	10	0.8	4.81×10^{-04}	3.61×10^{-04}	0.5233
0.95	0.95	100	0.25	10	0.9	4.81×10^{-04}	3.61×10^{-04}	0.4696
0.95	0.95	100	0.25	10	0.99	4.85×10^{-04}	3.62×10^{-04}	0.5011
0.95	0.95	100	1	3	0.8	4.71×10^{-04}	3.53×10^{-04}	0.5267
0.95	0.95	100	1	3	0.9	4.68×10^{-04}	3.54×10^{-04}	0.5376
0.95	0.95	100	1	3	0.99	4.76×10^{-04}	3.54×10^{-04}	0.5345
0.95	0.95	100	1	7	0.8	4.71×10^{-04}	3.55×10^{-04}	0.5026
0.95	0.95	100	1	7	0.9	4.77×10^{-04}	3.60×10^{-04}	0.5104
0.95	0.95	100	1	7	0.99	4.68×10^{-04}	3.49×10^{-04}	0.5354
0.95	0.95	100	1	10	0.8	4.70×10^{-04}	3.53×10^{-04}	0.5217
0.95	0.95	100	1	10	0.9	4.73×10^{-04}	3.55×10^{-04}	0.5202
0.95	0.95	100	1	10	0.99	4.74×10^{-04}	3.56×10^{-04}	0.5210
0.95	0.95	100	2	3	0.8	4.68×10^{-04}	3.51×10^{-04}	0.5196
0.95	0.95	100	2	3	0.9	4.69×10^{-04}	3.53×10^{-04}	0.5113
0.95	0.95	100	2	3	0.99	4.72×10^{-04}	3.52×10^{-04}	0.5012
0.95	0.95	100	2	7	0.8	4.69×10^{-04}	3.51×10^{-04}	0.5059
0.95	0.95	100	2	7	0.9	4.71×10^{-04}	3.53×10^{-04}	0.4966
0.95	0.95	100	2	7	0.99	4.70×10^{-04}	3.53×10^{-04}	0.5074
0.95	0.95	100	2	10	0.8	4.71×10^{-04}	3.54×10^{-04}	0.5159
0.95	0.95	100	2	10	0.9	4.70×10^{-04}	3.55×10^{-04}	0.5019
0.95	0.95	100	2	10	0.99	4.65×10^{-04}	3.49×10^{-04}	0.5146
0.95	0.95	200	0.25	3	0.8	4.78×10^{-04}	3.59×10^{-04}	0.5887
0.95	0.95	200	0.25	3	0.9	4.85×10^{-04}	3.68×10^{-04}	0.5997

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	0.95	200	0.25	3	0.99	4.87×10^{-04}	3.63×10^{-04}	0.5968
0.95	0.95	200	0.25	7	0.8	4.73×10^{-04}	3.57×10^{-04}	0.5689
0.95	0.95	200	0.25	7	0.9	4.77×10^{-04}	3.57×10^{-04}	0.5721
0.95	0.95	200	0.25	7	0.99	4.84×10^{-04}	3.63×10^{-04}	0.6165
0.95	0.95	200	0.25	10	0.8	4.74×10^{-04}	3.55×10^{-04}	0.5836
0.95	0.95	200	0.25	10	0.9	4.87×10^{-04}	3.67×10^{-04}	0.5737
0.95	0.95	200	0.25	10	0.99	4.92×10^{-04}	3.65×10^{-04}	0.5778
0.95	0.95	200	1	3	0.8	4.75×10^{-04}	3.57×10^{-04}	0.5999
0.95	0.95	200	1	3	0.9	4.72×10^{-04}	3.55×10^{-04}	0.5837
0.95	0.95	200	1	3	0.99	4.82×10^{-04}	3.64×10^{-04}	0.6027
0.95	0.95	200	1	7	0.8	4.72×10^{-04}	3.56×10^{-04}	0.5704
0.95	0.95	200	1	7	0.9	4.75×10^{-04}	3.58×10^{-04}	0.5933
0.95	0.95	200	1	7	0.99	4.76×10^{-04}	3.58×10^{-04}	0.5825
0.95	0.95	200	1	10	0.8	4.74×10^{-04}	3.57×10^{-04}	0.5727
0.95	0.95	200	1	10	0.9	4.80×10^{-04}	3.61×10^{-04}	0.5858
0.95	0.95	200	1	10	0.99	4.78×10^{-04}	3.59×10^{-04}	0.6044
0.95	0.95	200	2	3	0.8	4.72×10^{-04}	3.54×10^{-04}	0.5378
0.95	0.95	200	2	3	0.9	4.65×10^{-04}	3.49×10^{-04}	0.5197
0.95	0.95	200	2	3	0.99	4.65×10^{-04}	3.52×10^{-04}	0.5293
0.95	0.95	200	2	7	0.8	4.66×10^{-04}	3.48×10^{-04}	0.5192
0.95	0.95	200	2	7	0.9	4.66×10^{-04}	3.50×10^{-04}	0.5296
0.95	0.95	200	2	7	0.99	4.66×10^{-04}	3.51×10^{-04}	0.5083
0.95	0.95	200	2	10	0.8	4.72×10^{-04}	3.55×10^{-04}	0.5259
0.95	0.95	200	2	10	0.9	4.63×10^{-04}	3.49×10^{-04}	0.5251
0.95	0.95	200	2	10	0.99	4.65×10^{-04}	3.49×10^{-04}	0.5332
0.95	2	50	0.25	3	0.8	4.89×10^{-04}	3.65×10^{-04}	0.4416
0.95	2	50	0.25	3	0.9	4.91×10^{-04}	3.71×10^{-04}	0.4247
0.95	2	50	0.25	3	0.99	4.89×10^{-04}	3.64×10^{-04}	0.4241
0.95	2	50	0.25	7	0.8	4.83×10^{-04}	3.65×10^{-04}	0.4388
0.95	2	50	0.25	7	0.9	4.87×10^{-04}	3.64×10^{-04}	0.4294
0.95	2	50	0.25	7	0.99	4.97×10^{-04}	3.73×10^{-04}	0.4390
0.95	2	50	0.25	10	0.8	4.87×10^{-04}	3.63×10^{-04}	0.4245
0.95	2	50	0.25	10	0.9	4.94×10^{-04}	3.71×10^{-04}	0.4274
0.95	2	50	0.25	10	0.99	4.89×10^{-04}	3.65×10^{-04}	0.4181
0.95	2	50	1	3	0.8	4.81×10^{-04}	3.63×10^{-04}	0.4370
0.95	2	50	1	3	0.9	4.80×10^{-04}	3.60×10^{-04}	0.4410
0.95	2	50	1	3	0.99	4.82×10^{-04}	3.59×10^{-04}	0.4420
0.95	2	50	1	7	0.8	4.85×10^{-04}	3.64×10^{-04}	0.4311

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	2	50	1	7	0.9	4.82×10^{-04}	3.63×10^{-04}	0.4567
0.95	2	50	1	7	0.99	4.83×10^{-04}	3.60×10^{-04}	0.4425
0.95	2	50	1	10	0.8	4.89×10^{-04}	3.64×10^{-04}	0.4380
0.95	2	50	1	10	0.9	4.83×10^{-04}	3.59×10^{-04}	0.4252
0.95	2	50	1	10	0.99	4.84×10^{-04}	3.63×10^{-04}	0.4441
0.95	2	50	2	3	0.8	4.79×10^{-04}	3.61×10^{-04}	0.4329
0.95	2	50	2	3	0.9	4.87×10^{-04}	3.63×10^{-04}	0.4390
0.95	2	50	2	3	0.99	4.79×10^{-04}	3.59×10^{-04}	0.4493
0.95	2	50	2	7	0.8	4.75×10^{-04}	3.56×10^{-04}	0.4488
0.95	2	50	2	7	0.9	4.80×10^{-04}	3.60×10^{-04}	0.4255
0.95	2	50	2	7	0.99	4.83×10^{-04}	3.62×10^{-04}	0.4430
0.95	2	50	2	10	0.8	4.82×10^{-04}	3.64×10^{-04}	0.4355
0.95	2	50	2	10	0.9	4.78×10^{-04}	3.61×10^{-04}	0.4325
0.95	2	50	2	10	0.99	4.82×10^{-04}	3.61×10^{-04}	0.4388
0.95	2	100	0.25	3	0.8	4.83×10^{-04}	3.57×10^{-04}	0.4757
0.95	2	100	0.25	3	0.9	4.81×10^{-04}	3.60×10^{-04}	0.4949
0.95	2	100	0.25	3	0.99	4.94×10^{-04}	3.68×10^{-04}	0.4556
0.95	2	100	0.25	7	0.8	4.90×10^{-04}	3.69×10^{-04}	0.4727
0.95	2	100	0.25	7	0.9	4.86×10^{-04}	3.62×10^{-04}	0.4893
0.95	2	100	0.25	7	0.99	4.92×10^{-04}	3.67×10^{-04}	0.4840
0.95	2	100	0.25	10	0.8	4.87×10^{-04}	3.66×10^{-04}	0.4730
0.95	2	100	0.25	10	0.9	4.81×10^{-04}	3.56×10^{-04}	0.4753
0.95	2	100	0.25	10	0.99	4.88×10^{-04}	3.66×10^{-04}	0.4772
0.95	2	100	1	3	0.8	4.76×10^{-04}	3.59×10^{-04}	0.4996
0.95	2	100	1	3	0.9	4.81×10^{-04}	3.61×10^{-04}	0.4929
0.95	2	100	1	3	0.99	4.74×10^{-04}	3.56×10^{-04}	0.5045
0.95	2	100	1	7	0.8	4.84×10^{-04}	3.62×10^{-04}	0.4851
0.95	2	100	1	7	0.9	4.76×10^{-04}	3.57×10^{-04}	0.4856
0.95	2	100	1	7	0.99	4.77×10^{-04}	3.58×10^{-04}	0.4854
0.95	2	100	1	10	0.8	4.78×10^{-04}	3.57×10^{-04}	0.5080
0.95	2	100	1	10	0.9	4.68×10^{-04}	3.51×10^{-04}	0.4986
0.95	2	100	1	10	0.99	4.79×10^{-04}	3.57×10^{-04}	0.5040
0.95	2	100	2	3	0.8	4.78×10^{-04}	3.55×10^{-04}	0.4686
0.95	2	100	2	3	0.9	4.73×10^{-04}	3.56×10^{-04}	0.4826
0.95	2	100	2	3	0.99	4.75×10^{-04}	3.58×10^{-04}	0.4736
0.95	2	100	2	7	0.8	4.74×10^{-04}	3.54×10^{-04}	0.4790
0.95	2	100	2	7	0.9	4.72×10^{-04}	3.56×10^{-04}	0.4681
0.95	2	100	2	7	0.99	4.70×10^{-04}	3.52×10^{-04}	0.4703

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	2	100	2	10	0.8	4.73×10^{-04}	3.55×10^{-04}	0.4766
0.95	2	100	2	10	0.9	4.74×10^{-04}	3.56×10^{-04}	0.4832
0.95	2	100	2	10	0.99	4.76×10^{-04}	3.55×10^{-04}	0.4652
0.95	2	200	0.25	3	0.8	4.74×10^{-04}	3.55×10^{-04}	0.5408
0.95	2	200	0.25	3	0.9	4.97×10^{-04}	3.71×10^{-04}	0.6230
0.95	2	200	0.25	3	0.99	4.82×10^{-04}	3.64×10^{-04}	0.5726
0.95	2	200	0.25	7	0.8	4.81×10^{-04}	3.62×10^{-04}	0.5750
0.95	2	200	0.25	7	0.9	4.74×10^{-04}	3.58×10^{-04}	0.5453
0.95	2	200	0.25	7	0.99	4.84×10^{-04}	3.65×10^{-04}	0.5677
0.95	2	200	0.25	10	0.8	4.70×10^{-04}	3.57×10^{-04}	0.5799
0.95	2	200	0.25	10	0.9	4.85×10^{-04}	3.64×10^{-04}	0.5467
0.95	2	200	0.25	10	0.99	4.76×10^{-04}	3.60×10^{-04}	0.5551
0.95	2	200	1	3	0.8	4.73×10^{-04}	3.56×10^{-04}	0.5410
0.95	2	200	1	3	0.9	4.70×10^{-04}	3.54×10^{-04}	0.5346
0.95	2	200	1	3	0.99	4.69×10^{-04}	3.51×10^{-04}	0.5339
0.95	2	200	1	7	0.8	4.69×10^{-04}	3.51×10^{-04}	0.5222
0.95	2	200	1	7	0.9	4.76×10^{-04}	3.58×10^{-04}	0.5477
0.95	2	200	1	7	0.99	4.73×10^{-04}	3.55×10^{-04}	0.5356
0.95	2	200	1	10	0.8	4.77×10^{-04}	3.58×10^{-04}	0.5437
0.95	2	200	1	10	0.9	4.69×10^{-04}	3.53×10^{-04}	0.5330
0.95	2	200	1	10	0.99	4.72×10^{-04}	3.54×10^{-04}	0.5316
0.95	2	200	2	3	0.8	4.68×10^{-04}	3.53×10^{-04}	0.4874
0.95	2	200	2	3	0.9	4.69×10^{-04}	3.51×10^{-04}	0.4931
0.95	2	200	2	3	0.99	4.67×10^{-04}	3.51×10^{-04}	0.4965
0.95	2	200	2	7	0.8	4.72×10^{-04}	3.54×10^{-04}	0.4940
0.95	2	200	2	7	0.9	4.71×10^{-04}	3.52×10^{-04}	0.4818
0.95	2	200	2	7	0.99	4.71×10^{-04}	3.53×10^{-04}	0.4939
0.95	2	200	2	10	0.8	4.69×10^{-04}	3.53×10^{-04}	0.4927
0.95	2	200	2	10	0.9	4.67×10^{-04}	3.53×10^{-04}	0.4926
0.95	2	200	2	10	0.99	4.70×10^{-04}	3.53×10^{-04}	0.4964
2	0.5	50	0.25	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0023
2	0.5	50	0.25	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	0.25	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	50	0.25	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	50	0.25	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	0.5	50	0.25	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	0.25	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	0.25	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.5	50	0.25	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0023
2	0.5	50	1	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	1	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	1	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	50	1	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	50	1	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0023
2	0.5	50	1	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	1	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	1	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	1	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	50	2	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	2	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	2	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	2	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	50	2	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	50	2	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	50	2	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	50	2	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	50	2	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	0.25	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	0.25	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	0.25	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	0.25	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	0.5	100	0.25	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	0.25	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	0.25	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	0.25	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	0.25	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	1	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	1	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	1	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	1	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	1	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	1	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	1	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	1	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	1	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	2	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0020

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.5	100	2	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	2	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	2	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	2	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	100	2	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	2	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	2	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	100	2	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	0.25	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	0.25	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	0.25	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	200	0.25	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	0.25	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	0.5	200	0.25	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	0.25	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	0.5	200	0.25	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	200	0.25	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	1	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	0.5	200	1	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	1	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	200	1	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	1	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	1	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	1	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	200	1	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	1	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	2	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	0.5	200	2	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	200	2	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	2	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	200	2	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	200	2	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	200	2	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.5	200	2	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.5	200	2	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	0.25	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	0.25	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	0.25	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.95	50	0.25	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	0.25	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	0.25	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	0.25	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	0.25	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	0.25	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	1	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	1	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	1	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	1	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	1	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	0.95	50	1	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	1	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	1	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	1	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	2	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	2	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	2	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	2	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	2	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	2	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	2	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	50	2	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	50	2	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	0.95	100	0.25	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	100	0.25	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	100	0.25	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	100	0.25	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	100	0.25	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0023
2	0.95	100	0.25	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	100	0.25	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	100	0.25	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	100	0.25	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	0.95	100	1	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	100	1	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	100	1	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	100	1	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	0.95	100	1	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.95	100	1	7	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	100	1	10	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	100	1	10	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	100	1	10	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	100	2	3	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0020
2	0.95	100	2	3	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	100	2	3	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	100	2	7	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	100	2	7	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	100	2	7	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	100	2	10	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0023
2	0.95	100	2	10	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	100	2	10	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	0.25	3	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	0.25	3	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	0.25	3	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	0.25	7	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	0.25	7	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0020
2	0.95	200	0.25	7	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	200	0.25	10	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	200	0.25	10	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0020
2	0.95	200	0.25	10	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0020
2	0.95	200	1	3	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	200	1	3	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	1	3	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	1	7	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	1	7	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	1	7	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	200	1	10	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	200	1	10	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	200	1	10	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	2	3	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	2	3	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	200	2	3	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	2	7	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	2	7	0.9	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0021
2	0.95	200	2	7	0.99	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022
2	0.95	200	2	10	0.8	$6.50{\times}10^{-04}$	$5.11{\times}10^{-04}$	0.0022

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.95	200	2	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	0.95	200	2	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	0.25	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	0.25	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	0.25	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	0.25	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	0.25	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	2	50	0.25	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	0.25	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	0.25	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	0.25	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	1	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	1	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	1	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	1	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0023
2	2	50	1	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	2	50	1	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	1	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	1	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	1	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	2	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	2	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	2	50	2	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	2	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	2	50	2	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	2	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	2	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	50	2	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	50	2	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	0.25	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	100	0.25	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0023
2	2	100	0.25	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	100	0.25	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	0.25	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	0.25	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	0.25	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	0.25	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	0.25	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	2	100	1	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	1	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	1	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	1	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	1	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	100	1	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	1	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0023
2	2	100	1	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	1	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	2	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	100	2	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	100	2	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	2	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	2	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	2	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	2	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	100	2	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	100	2	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	0.25	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	0.25	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	0.25	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0023
2	2	200	0.25	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	0.25	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	0.25	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	0.25	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	2	200	0.25	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	0.25	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0020
2	2	200	1	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	1	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	1	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	1	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	1	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	1	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	1	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	1	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	1	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	2	3	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	2	3	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021

Table A.6: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	2	200	2	3	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	2	7	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	2	7	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	2	7	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	2	10	0.8	6.50×10^{-04}	5.11×10^{-04}	0.0022
2	2	200	2	10	0.9	6.50×10^{-04}	5.11×10^{-04}	0.0021
2	2	200	2	10	0.99	6.50×10^{-04}	5.11×10^{-04}	0.0022

Table A.7: Error Statisics of each BART model fit on the wetting dataset.

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.5	50	0.25	3	0.8	5.61×10^{-04}	4.27×10^{-04}	0.3478
0.5	0.5	50	0.25	3	0.9	5.63×10^{-04}	4.29×10^{-04}	0.3386
0.5	0.5	50	0.25	3	0.99	5.61×10^{-04}	4.28×10^{-04}	0.3346
0.5	0.5	50	0.25	7	0.8	5.55×10^{-04}	4.27×10^{-04}	0.3428
0.5	0.5	50	0.25	7	0.9	5.52×10^{-04}	4.19×10^{-04}	0.3352
0.5	0.5	50	0.25	7	0.99	5.60×10^{-04}	4.26×10^{-04}	0.3332
0.5	0.5	50	0.25	10	0.8	5.56×10^{-04}	4.22×10^{-04}	0.3332
0.5	0.5	50	0.25	10	0.9	5.57×10^{-04}	4.24×10^{-04}	0.3446
0.5	0.5	50	0.25	10	0.99	5.58×10^{-04}	4.23×10^{-04}	0.3413
0.5	0.5	50	1	3	0.8	5.50×10^{-04}	4.13×10^{-04}	0.3869
0.5	0.5	50	1	3	0.9	5.53×10^{-04}	4.19×10^{-04}	0.3818
0.5	0.5	50	1	3	0.99	5.61×10^{-04}	4.23×10^{-04}	0.3893
0.5	0.5	50	1	7	0.8	5.56×10^{-04}	4.22×10^{-04}	0.3922
0.5	0.5	50	1	7	0.9	5.52×10^{-04}	4.20×10^{-04}	0.3821
0.5	0.5	50	1	7	0.99	5.51×10^{-04}	4.14×10^{-04}	0.3784
0.5	0.5	50	1	10	0.8	5.51×10^{-04}	4.16×10^{-04}	0.3824
0.5	0.5	50	1	10	0.9	5.48×10^{-04}	4.15×10^{-04}	0.3927
0.5	0.5	50	1	10	0.99	5.51×10^{-04}	4.17×10^{-04}	0.3942
0.5	0.5	50	2	3	0.8	5.46×10^{-04}	4.11×10^{-04}	0.3951
0.5	0.5	50	2	3	0.9	5.54×10^{-04}	4.19×10^{-04}	0.3888
0.5	0.5	50	2	3	0.99	5.52×10^{-04}	4.18×10^{-04}	0.388
0.5	0.5	50	2	7	0.8	5.54×10^{-04}	4.20×10^{-04}	0.3879
0.5	0.5	50	2	7	0.9	5.46×10^{-04}	4.14×10^{-04}	0.3853
0.5	0.5	50	2	7	0.99	5.47×10^{-04}	4.16×10^{-04}	0.3848
0.5	0.5	50	2	10	0.8	5.49×10^{-04}	4.16×10^{-04}	0.3865
0.5	0.5	50	2	10	0.9	5.52×10^{-04}	4.17×10^{-04}	0.3965
0.5	0.5	50	2	10	0.99	5.47×10^{-04}	4.16×10^{-04}	0.3832
0.5	0.5	100	0.25	3	0.8	5.54×10^{-04}	4.23×10^{-04}	0.3773
0.5	0.5	100	0.25	3	0.9	5.67×10^{-04}	4.30×10^{-04}	0.3991
0.5	0.5	100	0.25	3	0.99	5.53×10^{-04}	4.18×10^{-04}	0.3929
0.5	0.5	100	0.25	7	0.8	5.55×10^{-04}	4.22×10^{-04}	0.3771
0.5	0.5	100	0.25	7	0.9	5.55×10^{-04}	4.25×10^{-04}	0.3876
0.5	0.5	100	0.25	7	0.99	5.59×10^{-04}	4.21×10^{-04}	0.372
0.5	0.5	100	0.25	10	0.8	5.55×10^{-04}	4.19×10^{-04}	0.3628
0.5	0.5	100	0.25	10	0.9	5.57×10^{-04}	4.25×10^{-04}	0.3876
0.5	0.5	100	0.25	10	0.99	5.60×10^{-04}	4.25×10^{-04}	0.3801
0.5	0.5	100	1	3	0.8	5.48×10^{-04}	4.16×10^{-04}	0.4333
0.5	0.5	100	1	3	0.9	5.48×10^{-04}	4.12×10^{-04}	0.4233

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.5	100	1	3	0.99	5.51×10^{-04}	4.13×10^{-04}	0.4165
0.5	0.5	100	1	7	0.8	5.49×10^{-04}	4.13×10^{-04}	0.4275
0.5	0.5	100	1	7	0.9	5.50×10^{-04}	4.14×10^{-04}	0.4458
0.5	0.5	100	1	7	0.99	5.58×10^{-04}	4.21×10^{-04}	0.4327
0.5	0.5	100	1	10	0.8	5.59×10^{-04}	4.20×10^{-04}	0.4361
0.5	0.5	100	1	10	0.9	5.53×10^{-04}	4.17×10^{-04}	0.431
0.5	0.5	100	1	10	0.99	5.48×10^{-04}	4.12×10^{-04}	0.4265
0.5	0.5	100	2	3	0.8	5.50×10^{-04}	4.18×10^{-04}	0.4289
0.5	0.5	100	2	3	0.9	5.45×10^{-04}	4.10×10^{-04}	0.4292
0.5	0.5	100	2	3	0.99	5.47×10^{-04}	4.14×10^{-04}	0.4165
0.5	0.5	100	2	7	0.8	5.50×10^{-04}	4.16×10^{-04}	0.4224
0.5	0.5	100	2	7	0.9	5.47×10^{-04}	4.11×10^{-04}	0.4369
0.5	0.5	100	2	7	0.99	5.49×10^{-04}	4.14×10^{-04}	0.4248
0.5	0.5	100	2	10	0.8	5.46×10^{-04}	4.13×10^{-04}	0.4201
0.5	0.5	100	2	10	0.9	5.47×10^{-04}	4.12×10^{-04}	0.4295
0.5	0.5	100	2	10	0.99	5.44×10^{-04}	4.11×10^{-04}	0.4296
0.5	0.5	200	0.25	3	0.8	5.57×10^{-04}	4.19×10^{-04}	0.4397
0.5	0.5	200	0.25	3	0.9	5.54×10^{-04}	4.17×10^{-04}	0.4382
0.5	0.5	200	0.25	3	0.99	5.51×10^{-04}	4.14×10^{-04}	0.4489
0.5	0.5	200	0.25	7	0.8	5.49×10^{-04}	4.15×10^{-04}	0.4359
0.5	0.5	200	0.25	7	0.9	5.59×10^{-04}	4.23×10^{-04}	0.4306
0.5	0.5	200	0.25	7	0.99	5.53×10^{-04}	4.19×10^{-04}	0.4263
0.5	0.5	200	0.25	10	0.8	5.57×10^{-04}	4.17×10^{-04}	0.45
0.5	0.5	200	0.25	10	0.9	5.48×10^{-04}	4.16×10^{-04}	0.4334
0.5	0.5	200	0.25	10	0.99	5.62×10^{-04}	4.20×10^{-04}	0.4207
0.5	0.5	200	1	3	0.8	5.57×10^{-04}	4.19×10^{-04}	0.4908
0.5	0.5	200	1	3	0.9	5.54×10^{-04}	4.16×10^{-04}	0.4949
0.5	0.5	200	1	3	0.99	5.57×10^{-04}	4.18×10^{-04}	0.4987
0.5	0.5	200	1	7	0.8	5.62×10^{-04}	4.20×10^{-04}	0.5086
0.5	0.5	200	1	7	0.9	5.56×10^{-04}	4.16×10^{-04}	0.4954
0.5	0.5	200	1	7	0.99	5.54×10^{-04}	4.17×10^{-04}	0.4894
0.5	0.5	200	1	10	0.8	5.55×10^{-04}	4.14×10^{-04}	0.4951
0.5	0.5	200	1	10	0.9	5.60×10^{-04}	4.19×10^{-04}	0.489
0.5	0.5	200	1	10	0.99	5.60×10^{-04}	4.24×10^{-04}	0.4885
0.5	0.5	200	2	3	0.8	5.50×10^{-04}	4.16×10^{-04}	0.448
0.5	0.5	200	2	3	0.9	5.47×10^{-04}	4.14×10^{-04}	0.4402
0.5	0.5	200	2	3	0.99	5.47×10^{-04}	4.13×10^{-04}	0.4471
0.5	0.5	200	2	7	0.8	5.46×10^{-04}	4.12×10^{-04}	0.4549

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.5	200	2	7	0.9	5.50×10^{-04}	4.14×10^{-04}	0.447
0.5	0.5	200	2	7	0.99	5.46×10^{-04}	4.13×10^{-04}	0.4397
0.5	0.5	200	2	10	0.8	5.45×10^{-04}	4.12×10^{-04}	0.4474
0.5	0.5	200	2	10	0.9	5.48×10^{-04}	4.14×10^{-04}	0.451
0.5	0.5	200	2	10	0.99	5.49×10^{-04}	4.11×10^{-04}	0.4493
0.5	0.95	50	0.25	3	0.8	5.58×10^{-04}	4.27×10^{-04}	0.3452
0.5	0.95	50	0.25	3	0.9	5.59×10^{-04}	4.27×10^{-04}	0.3344
0.5	0.95	50	0.25	3	0.99	5.60×10^{-04}	4.29×10^{-04}	0.3412
0.5	0.95	50	0.25	7	0.8	5.56×10^{-04}	4.26×10^{-04}	0.3296
0.5	0.95	50	0.25	7	0.9	5.62×10^{-04}	4.32×10^{-04}	0.3329
0.5	0.95	50	0.25	7	0.99	5.65×10^{-04}	4.28×10^{-04}	0.3482
0.5	0.95	50	0.25	10	0.8	5.57×10^{-04}	4.25×10^{-04}	0.3558
0.5	0.95	50	0.25	10	0.9	5.56×10^{-04}	4.25×10^{-04}	0.3312
0.5	0.95	50	0.25	10	0.99	5.58×10^{-04}	4.25×10^{-04}	0.3349
0.5	0.95	50	1	3	0.8	5.52×10^{-04}	4.18×10^{-04}	0.3806
0.5	0.95	50	1	3	0.9	5.57×10^{-04}	4.23×10^{-04}	0.3731
0.5	0.95	50	1	3	0.99	5.56×10^{-04}	4.21×10^{-04}	0.3693
0.5	0.95	50	1	7	0.8	5.61×10^{-04}	4.24×10^{-04}	0.3816
0.5	0.95	50	1	7	0.9	5.54×10^{-04}	4.20×10^{-04}	0.3679
0.5	0.95	50	1	7	0.99	5.51×10^{-04}	4.19×10^{-04}	0.3759
0.5	0.95	50	1	10	0.8	5.48×10^{-04}	4.16×10^{-04}	0.381
0.5	0.95	50	1	10	0.9	5.48×10^{-04}	4.12×10^{-04}	0.3786
0.5	0.95	50	1	10	0.99	5.50×10^{-04}	4.17×10^{-04}	0.3726
0.5	0.95	50	2	3	0.8	5.51×10^{-04}	4.18×10^{-04}	0.3668
0.5	0.95	50	2	3	0.9	5.52×10^{-04}	4.16×10^{-04}	0.3842
0.5	0.95	50	2	3	0.99	5.51×10^{-04}	4.17×10^{-04}	0.3962
0.5	0.95	50	2	7	0.8	5.47×10^{-04}	4.14×10^{-04}	0.3828
0.5	0.95	50	2	7	0.9	5.54×10^{-04}	4.19×10^{-04}	0.3823
0.5	0.95	50	2	7	0.99	5.54×10^{-04}	4.21×10^{-04}	0.3813
0.5	0.95	50	2	10	0.8	5.46×10^{-04}	4.15×10^{-04}	0.3844
0.5	0.95	50	2	10	0.9	5.49×10^{-04}	4.16×10^{-04}	0.3784
0.5	0.95	50	2	10	0.99	5.50×10^{-04}	4.17×10^{-04}	0.376
0.5	0.95	100	0.25	3	0.8	5.54×10^{-04}	4.22×10^{-04}	0.3543
0.5	0.95	100	0.25	3	0.9	5.54×10^{-04}	4.21×10^{-04}	0.3838
0.5	0.95	100	0.25	3	0.99	5.58×10^{-04}	4.24×10^{-04}	0.3873
0.5	0.95	100	0.25	7	0.8	5.52×10^{-04}	4.20×10^{-04}	0.3805
0.5	0.95	100	0.25	7	0.9	5.48×10^{-04}	4.17×10^{-04}	0.372
0.5	0.95	100	0.25	7	0.99	5.50×10^{-04}	4.21×10^{-04}	0.3685

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.95	100	0.25	10	0.8	5.60×10^{-04}	4.22×10^{-04}	0.373
0.5	0.95	100	0.25	10	0.9	5.66×10^{-04}	4.29×10^{-04}	0.3745
0.5	0.95	100	0.25	10	0.99	5.63×10^{-04}	4.26×10^{-04}	0.3879
0.5	0.95	100	1	3	0.8	5.49×10^{-04}	4.15×10^{-04}	0.4102
0.5	0.95	100	1	3	0.9	5.48×10^{-04}	4.10×10^{-04}	0.417
0.5	0.95	100	1	3	0.99	5.52×10^{-04}	4.21×10^{-04}	0.4248
0.5	0.95	100	1	7	0.8	5.56×10^{-04}	4.18×10^{-04}	0.4254
0.5	0.95	100	1	7	0.9	5.47×10^{-04}	4.13×10^{-04}	0.4196
0.5	0.95	100	1	7	0.99	5.46×10^{-04}	4.11×10^{-04}	0.4115
0.5	0.95	100	1	10	0.8	5.54×10^{-04}	4.19×10^{-04}	0.4323
0.5	0.95	100	1	10	0.9	5.55×10^{-04}	4.16×10^{-04}	0.4166
0.5	0.95	100	1	10	0.99	5.55×10^{-04}	4.14×10^{-04}	0.4214
0.5	0.95	100	2	3	0.8	5.51×10^{-04}	4.13×10^{-04}	0.4174
0.5	0.95	100	2	3	0.9	5.55×10^{-04}	4.19×10^{-04}	0.406
0.5	0.95	100	2	3	0.99	5.54×10^{-04}	4.18×10^{-04}	0.4318
0.5	0.95	100	2	7	0.8	5.44×10^{-04}	4.15×10^{-04}	0.405
0.5	0.95	100	2	7	0.9	5.52×10^{-04}	4.16×10^{-04}	0.3998
0.5	0.95	100	2	7	0.99	5.48×10^{-04}	4.13×10^{-04}	0.4136
0.5	0.95	100	2	10	0.8	5.50×10^{-04}	4.19×10^{-04}	0.4077
0.5	0.95	100	2	10	0.9	5.45×10^{-04}	4.12×10^{-04}	0.3938
0.5	0.95	100	2	10	0.99	5.55×10^{-04}	4.19×10^{-04}	0.413
0.5	0.95	200	0.25	3	0.8	5.62×10^{-04}	4.26×10^{-04}	0.4636
0.5	0.95	200	0.25	3	0.9	5.57×10^{-04}	4.22×10^{-04}	0.4299
0.5	0.95	200	0.25	3	0.99	5.59×10^{-04}	4.19×10^{-04}	0.4373
0.5	0.95	200	0.25	7	0.8	5.59×10^{-04}	4.17×10^{-04}	0.4603
0.5	0.95	200	0.25	7	0.9	5.55×10^{-04}	4.17×10^{-04}	0.4288
0.5	0.95	200	0.25	7	0.99	5.53×10^{-04}	4.18×10^{-04}	0.4623
0.5	0.95	200	0.25	10	0.8	5.57×10^{-04}	4.22×10^{-04}	0.4377
0.5	0.95	200	0.25	10	0.9	5.52×10^{-04}	4.12×10^{-04}	0.4521
0.5	0.95	200	0.25	10	0.99	5.55×10^{-04}	4.17×10^{-04}	0.4408
0.5	0.95	200	1	3	0.8	5.51×10^{-04}	4.17×10^{-04}	0.4599
0.5	0.95	200	1	3	0.9	5.53×10^{-04}	4.14×10^{-04}	0.4949
0.5	0.95	200	1	3	0.99	5.58×10^{-04}	4.20×10^{-04}	0.4811
0.5	0.95	200	1	7	0.8	5.55×10^{-04}	4.16×10^{-04}	0.4848
0.5	0.95	200	1	7	0.9	5.55×10^{-04}	4.18×10^{-04}	0.4758
0.5	0.95	200	1	7	0.99	5.62×10^{-04}	4.22×10^{-04}	0.4783
0.5	0.95	200	1	10	0.8	5.54×10^{-04}	4.16×10^{-04}	0.473
0.5	0.95	200	1	10	0.9	5.50×10^{-04}	4.12×10^{-04}	0.4779

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	0.95	200	1	10	0.99	5.52×10^{-04}	4.13×10^{-04}	0.4671
0.5	0.95	200	2	3	0.8	5.48×10^{-04}	4.13×10^{-04}	0.43
0.5	0.95	200	2	3	0.9	5.52×10^{-04}	4.16×10^{-04}	0.4261
0.5	0.95	200	2	3	0.99	5.49×10^{-04}	4.14×10^{-04}	0.4243
0.5	0.95	200	2	7	0.8	5.43×10^{-04}	4.11×10^{-04}	0.4318
0.5	0.95	200	2	7	0.9	5.49×10^{-04}	4.15×10^{-04}	0.429
0.5	0.95	200	2	7	0.99	5.54×10^{-04}	4.18×10^{-04}	0.4307
0.5	0.95	200	2	10	0.8	5.51×10^{-04}	4.16×10^{-04}	0.4208
0.5	0.95	200	2	10	0.9	5.52×10^{-04}	4.19×10^{-04}	0.4329
0.5	0.95	200	2	10	0.99	5.48×10^{-04}	4.13×10^{-04}	0.4341
0.5	2	50	0.25	3	0.8	5.59×10^{-04}	4.26×10^{-04}	0.3195
0.5	2	50	0.25	3	0.9	5.57×10^{-04}	4.25×10^{-04}	0.3314
0.5	2	50	0.25	3	0.99	5.60×10^{-04}	4.27×10^{-04}	0.3271
0.5	2	50	0.25	7	0.8	5.61×10^{-04}	4.29×10^{-04}	0.3314
0.5	2	50	0.25	7	0.9	5.66×10^{-04}	4.31×10^{-04}	0.3298
0.5	2	50	0.25	7	0.99	5.60×10^{-04}	4.25×10^{-04}	0.3327
0.5	2	50	0.25	10	0.8	5.62×10^{-04}	4.28×10^{-04}	0.3399
0.5	2	50	0.25	10	0.9	5.61×10^{-04}	4.28×10^{-04}	0.3392
0.5	2	50	0.25	10	0.99	5.70×10^{-04}	4.35×10^{-04}	0.3226
0.5	2	50	1	3	0.8	5.60×10^{-04}	4.25×10^{-04}	0.3565
0.5	2	50	1	3	0.9	5.56×10^{-04}	4.24×10^{-04}	0.3602
0.5	2	50	1	3	0.99	5.53×10^{-04}	4.22×10^{-04}	0.3577
0.5	2	50	1	7	0.8	5.55×10^{-04}	4.20×10^{-04}	0.3792
0.5	2	50	1	7	0.9	5.57×10^{-04}	4.22×10^{-04}	0.3597
0.5	2	50	1	7	0.99	5.61×10^{-04}	4.29×10^{-04}	0.365
0.5	2	50	1	10	0.8	5.48×10^{-04}	4.19×10^{-04}	0.3623
0.5	2	50	1	10	0.9	5.51×10^{-04}	4.21×10^{-04}	0.3501
0.5	2	50	1	10	0.99	5.54×10^{-04}	4.20×10^{-04}	0.3659
0.5	2	50	2	3	0.8	5.54×10^{-04}	4.20×10^{-04}	0.3659
0.5	2	50	2	3	0.9	5.54×10^{-04}	4.18×10^{-04}	0.3846
0.5	2	50	2	3	0.99	5.54×10^{-04}	4.22×10^{-04}	0.3719
0.5	2	50	2	7	0.8	5.55×10^{-04}	4.23×10^{-04}	0.3591
0.5	2	50	2	7	0.9	5.55×10^{-04}	4.25×10^{-04}	0.3844
0.5	2	50	2	7	0.99	5.52×10^{-04}	4.19×10^{-04}	0.362
0.5	2	50	2	10	0.8	5.48×10^{-04}	4.15×10^{-04}	0.3536
0.5	2	50	2	10	0.9	5.53×10^{-04}	4.21×10^{-04}	0.361
0.5	2	50	2	10	0.99	5.51×10^{-04}	4.18×10^{-04}	0.3718
0.5	2	100	0.25	3	0.8	5.61×10^{-04}	4.24×10^{-04}	0.3734

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	2	100	0.25	3	0.9	5.59×10^{-04}	4.26×10^{-04}	0.3705
0.5	2	100	0.25	3	0.99	5.65×10^{-04}	4.30×10^{-04}	0.3748
0.5	2	100	0.25	7	0.8	5.53×10^{-04}	4.18×10^{-04}	0.3755
0.5	2	100	0.25	7	0.9	5.56×10^{-04}	4.22×10^{-04}	0.3646
0.5	2	100	0.25	7	0.99	5.57×10^{-04}	4.24×10^{-04}	0.3704
0.5	2	100	0.25	10	0.8	5.57×10^{-04}	4.24×10^{-04}	0.3704
0.5	2	100	0.25	10	0.9	5.59×10^{-04}	4.25×10^{-04}	0.381
0.5	2	100	0.25	10	0.99	5.53×10^{-04}	4.21×10^{-04}	0.3774
0.5	2	100	1	3	0.8	5.49×10^{-04}	4.15×10^{-04}	0.3913
0.5	2	100	1	3	0.9	5.52×10^{-04}	4.16×10^{-04}	0.4076
0.5	2	100	1	3	0.99	5.51×10^{-04}	4.16×10^{-04}	0.3998
0.5	2	100	1	7	0.8	5.55×10^{-04}	4.20×10^{-04}	0.408
0.5	2	100	1	7	0.9	5.55×10^{-04}	4.20×10^{-04}	0.4142
0.5	2	100	1	7	0.99	5.55×10^{-04}	4.21×10^{-04}	0.414
0.5	2	100	1	10	0.8	5.48×10^{-04}	4.17×10^{-04}	0.3952
0.5	2	100	1	10	0.9	5.57×10^{-04}	4.20×10^{-04}	0.4131
0.5	2	100	1	10	0.99	5.51×10^{-04}	4.15×10^{-04}	0.4024
0.5	2	100	2	3	0.8	5.50×10^{-04}	4.18×10^{-04}	0.3845
0.5	2	100	2	3	0.9	5.47×10^{-04}	4.18×10^{-04}	0.3845
0.5	2	100	2	3	0.99	5.52×10^{-04}	4.19×10^{-04}	0.3966
0.5	2	100	2	7	0.8	5.51×10^{-04}	4.17×10^{-04}	0.3875
0.5	2	100	2	7	0.9	5.52×10^{-04}	4.19×10^{-04}	0.3888
0.5	2	100	2	7	0.99	5.56×10^{-04}	4.21×10^{-04}	0.3863
0.5	2	100	2	10	0.8	5.53×10^{-04}	4.21×10^{-04}	0.3893
0.5	2	100	2	10	0.9	5.47×10^{-04}	4.14×10^{-04}	0.3808
0.5	2	100	2	10	0.99	5.53×10^{-04}	4.20×10^{-04}	0.3877
0.5	2	200	0.25	3	0.8	5.60×10^{-04}	4.18×10^{-04}	0.4379
0.5	2	200	0.25	3	0.9	5.58×10^{-04}	4.22×10^{-04}	0.4366
0.5	2	200	0.25	3	0.99	5.60×10^{-04}	4.19×10^{-04}	0.4258
0.5	2	200	0.25	7	0.8	5.58×10^{-04}	4.19×10^{-04}	0.4363
0.5	2	200	0.25	7	0.9	5.58×10^{-04}	4.19×10^{-04}	0.4363
0.5	2	200	0.25	7	0.99	5.59×10^{-04}	4.17×10^{-04}	0.4349
0.5	2	200	0.25	10	0.8	5.65×10^{-04}	4.25×10^{-04}	0.4431
0.5	2	200	0.25	10	0.9	5.59×10^{-04}	4.21×10^{-04}	0.4284
0.5	2	200	0.25	10	0.99	5.66×10^{-04}	4.24×10^{-04}	0.4471
0.5	2	200	1	3	0.8	5.53×10^{-04}	4.18×10^{-04}	0.4505
0.5	2	200	1	3	0.9	5.54×10^{-04}	4.17×10^{-04}	0.4589
0.5	2	200	1	3	0.99	5.63×10^{-04}	4.26×10^{-04}	0.4626

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.5	2	200	1	7	0.8	5.58×10^{-04}	4.21×10^{-04}	0.4523
0.5	2	200	1	7	0.9	5.54×10^{-04}	4.19×10^{-04}	0.4424
0.5	2	200	1	7	0.99	5.57×10^{-04}	4.20×10^{-04}	0.4459
0.5	2	200	1	10	0.8	5.57×10^{-04}	4.21×10^{-04}	0.4576
0.5	2	200	1	10	0.9	5.54×10^{-04}	4.20×10^{-04}	0.4451
0.5	2	200	1	10	0.99	5.49×10^{-04}	4.15×10^{-04}	0.4518
0.5	2	200	2	3	0.8	5.52×10^{-04}	4.18×10^{-04}	0.4095
0.5	2	200	2	3	0.9	5.52×10^{-04}	4.20×10^{-04}	0.4072
0.5	2	200	2	3	0.99	5.53×10^{-04}	4.18×10^{-04}	0.4129
0.5	2	200	2	7	0.8	5.51×10^{-04}	4.20×10^{-04}	0.3973
0.5	2	200	2	7	0.9	5.52×10^{-04}	4.19×10^{-04}	0.4139
0.5	2	200	2	7	0.99	5.51×10^{-04}	4.18×10^{-04}	0.4065
0.5	2	200	2	10	0.8	5.51×10^{-04}	4.18×10^{-04}	0.4006
0.5	2	200	2	10	0.9	5.52×10^{-04}	4.19×10^{-04}	0.4068
0.5	2	200	2	10	0.99	5.49×10^{-04}	4.16×10^{-04}	0.402
0.95	0.5	50	0.25	3	0.8	5.57×10^{-04}	4.20×10^{-04}	0.3803
0.95	0.5	50	0.25	3	0.9	5.50×10^{-04}	4.18×10^{-04}	0.3643
0.95	0.5	50	0.25	3	0.99	5.65×10^{-04}	4.29×10^{-04}	0.3861
0.95	0.5	50	0.25	7	0.8	5.60×10^{-04}	4.25×10^{-04}	0.3752
0.95	0.5	50	0.25	7	0.9	5.57×10^{-04}	4.21×10^{-04}	0.3673
0.95	0.5	50	0.25	7	0.99	5.55×10^{-04}	4.25×10^{-04}	0.3678
0.95	0.5	50	0.25	10	0.8	5.52×10^{-04}	4.20×10^{-04}	0.3824
0.95	0.5	50	0.25	10	0.9	5.66×10^{-04}	4.27×10^{-04}	0.3923
0.95	0.5	50	0.25	10	0.99	5.57×10^{-04}	4.23×10^{-04}	0.3778
0.95	0.5	50	1	3	0.8	5.48×10^{-04}	4.14×10^{-04}	0.4244
0.95	0.5	50	1	3	0.9	5.44×10^{-04}	4.15×10^{-04}	0.4338
0.95	0.5	50	1	3	0.99	5.48×10^{-04}	4.15×10^{-04}	0.4119
0.95	0.5	50	1	7	0.8	5.51×10^{-04}	4.17×10^{-04}	0.4455
0.95	0.5	50	1	7	0.9	5.51×10^{-04}	4.16×10^{-04}	0.4277
0.95	0.5	50	1	7	0.99	5.52×10^{-04}	4.13×10^{-04}	0.446
0.95	0.5	50	1	10	0.8	5.45×10^{-04}	4.08×10^{-04}	0.4469
0.95	0.5	50	1	10	0.9	5.55×10^{-04}	4.20×10^{-04}	0.4503
0.95	0.5	50	1	10	0.99	5.55×10^{-04}	4.19×10^{-04}	0.4272
0.95	0.5	50	2	3	0.8	5.39×10^{-04}	4.11×10^{-04}	0.4459
0.95	0.5	50	2	3	0.9	5.45×10^{-04}	4.13×10^{-04}	0.4352
0.95	0.5	50	2	3	0.99	5.42×10^{-04}	4.08×10^{-04}	0.4328
0.95	0.5	50	2	7	0.8	5.48×10^{-04}	4.14×10^{-04}	0.4322
0.95	0.5	50	2	7	0.9	5.43×10^{-04}	4.07×10^{-04}	0.4375

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	0.5	50	2	7	0.99	5.43×10^{-04}	4.10×10^{-04}	0.425
0.95	0.5	50	2	10	0.8	5.49×10^{-04}	4.15×10^{-04}	0.4479
0.95	0.5	50	2	10	0.9	5.48×10^{-04}	4.14×10^{-04}	0.4382
0.95	0.5	50	2	10	0.99	5.46×10^{-04}	4.10×10^{-04}	0.4347
0.95	0.5	100	0.25	3	0.8	5.61×10^{-04}	4.23×10^{-04}	0.4383
0.95	0.5	100	0.25	3	0.9	5.56×10^{-04}	4.20×10^{-04}	0.4235
0.95	0.5	100	0.25	3	0.99	5.62×10^{-04}	4.21×10^{-04}	0.4632
0.95	0.5	100	0.25	7	0.8	5.58×10^{-04}	4.26×10^{-04}	0.4425
0.95	0.5	100	0.25	7	0.9	5.69×10^{-04}	4.26×10^{-04}	0.4517
0.95	0.5	100	0.25	7	0.99	5.60×10^{-04}	4.22×10^{-04}	0.461
0.95	0.5	100	0.25	10	0.8	5.51×10^{-04}	4.19×10^{-04}	0.4587
0.95	0.5	100	0.25	10	0.9	5.49×10^{-04}	4.12×10^{-04}	0.4515
0.95	0.5	100	0.25	10	0.99	5.57×10^{-04}	4.14×10^{-04}	0.4281
0.95	0.5	100	1	3	0.8	5.55×10^{-04}	4.16×10^{-04}	0.5027
0.95	0.5	100	1	3	0.9	5.51×10^{-04}	4.14×10^{-04}	0.4976
0.95	0.5	100	1	3	0.99	5.50×10^{-04}	4.12×10^{-04}	0.4908
0.95	0.5	100	1	7	0.8	5.57×10^{-04}	4.15×10^{-04}	0.5104
0.95	0.5	100	1	7	0.9	5.49×10^{-04}	4.14×10^{-04}	0.4937
0.95	0.5	100	1	7	0.99	5.60×10^{-04}	4.20×10^{-04}	0.5146
0.95	0.5	100	1	10	0.8	5.56×10^{-04}	4.21×10^{-04}	0.5022
0.95	0.5	100	1	10	0.9	5.55×10^{-04}	4.18×10^{-04}	0.52
0.95	0.5	100	1	10	0.99	5.52×10^{-04}	4.12×10^{-04}	0.506
0.95	0.5	100	2	3	0.8	5.52×10^{-04}	4.12×10^{-04}	0.506
0.95	0.5	100	2	3	0.9	5.48×10^{-04}	4.12×10^{-04}	0.4954
0.95	0.5	100	2	3	0.99	5.54×10^{-04}	4.19×10^{-04}	0.4752
0.95	0.5	100	2	7	0.8	5.50×10^{-04}	4.15×10^{-04}	0.4843
0.95	0.5	100	2	7	0.9	5.52×10^{-04}	4.15×10^{-04}	0.483
0.95	0.5	100	2	7	0.99	5.47×10^{-04}	4.13×10^{-04}	0.475
0.95	0.5	100	2	10	0.8	5.48×10^{-04}	4.10×10^{-04}	0.4712
0.95	0.5	100	2	10	0.9	5.51×10^{-04}	4.14×10^{-04}	0.4981
0.95	0.5	100	2	10	0.99	5.50×10^{-04}	4.11×10^{-04}	0.4813
0.95	0.5	200	0.25	3	0.8	5.72×10^{-04}	4.22×10^{-04}	0.5364
0.95	0.5	200	0.25	3	0.9	5.67×10^{-04}	4.27×10^{-04}	0.5305
0.95	0.5	200	0.25	3	0.99	5.59×10^{-04}	4.17×10^{-04}	0.5315
0.95	0.5	200	0.25	7	0.8	5.70×10^{-04}	4.23×10^{-04}	0.5281
0.95	0.5	200	0.25	7	0.9	5.65×10^{-04}	4.23×10^{-04}	0.5331
0.95	0.5	200	0.25	7	0.99	5.71×10^{-04}	4.24×10^{-04}	0.5309
0.95	0.5	200	0.25	10	0.8	5.64×10^{-04}	4.19×10^{-04}	0.5316

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	0.5	200	0.25	10	0.9	5.76×10^{-04}	4.33×10^{-04}	0.5456
0.95	0.5	200	0.25	10	0.99	5.69×10^{-04}	4.23×10^{-04}	0.5289
0.95	0.5	200	1	3	0.8	5.71×10^{-04}	4.25×10^{-04}	0.5766
0.95	0.5	200	1	3	0.9	5.62×10^{-04}	4.19×10^{-04}	0.5818
0.95	0.5	200	1	3	0.99	5.66×10^{-04}	4.21×10^{-04}	0.5906
0.95	0.5	200	1	7	0.8	5.65×10^{-04}	4.21×10^{-04}	0.5768
0.95	0.5	200	1	7	0.9	5.67×10^{-04}	4.21×10^{-04}	0.5898
0.95	0.5	200	1	7	0.99	5.68×10^{-04}	4.26×10^{-04}	0.5829
0.95	0.5	200	1	10	0.8	5.75×10^{-04}	4.26×10^{-04}	0.5618
0.95	0.5	200	1	10	0.9	5.67×10^{-04}	4.20×10^{-04}	0.5924
0.95	0.5	200	1	10	0.99	5.67×10^{-04}	4.22×10^{-04}	0.5837
0.95	0.5	200	2	3	0.8	5.53×10^{-04}	4.15×10^{-04}	0.5039
0.95	0.5	200	2	3	0.9	5.53×10^{-04}	4.15×10^{-04}	0.5271
0.95	0.5	200	2	3	0.99	5.51×10^{-04}	4.12×10^{-04}	0.5225
0.95	0.5	200	2	7	0.8	5.49×10^{-04}	4.12×10^{-04}	0.5098
0.95	0.5	200	2	7	0.9	5.51×10^{-04}	4.17×10^{-04}	0.534
0.95	0.5	200	2	7	0.99	5.48×10^{-04}	4.14×10^{-04}	0.5234
0.95	0.5	200	2	10	0.8	5.57×10^{-04}	4.17×10^{-04}	0.5251
0.95	0.5	200	2	10	0.9	5.49×10^{-04}	4.09×10^{-04}	0.5211
0.95	0.5	200	2	10	0.99	5.56×10^{-04}	4.16×10^{-04}	0.5287
0.95	0.95	50	0.25	3	0.8	5.53×10^{-04}	4.20×10^{-04}	0.3795
0.95	0.95	50	0.25	3	0.9	5.58×10^{-04}	4.22×10^{-04}	0.3829
0.95	0.95	50	0.25	3	0.99	5.56×10^{-04}	4.22×10^{-04}	0.3868
0.95	0.95	50	0.25	7	0.8	5.45×10^{-04}	4.15×10^{-04}	0.4034
0.95	0.95	50	0.25	7	0.9	5.56×10^{-04}	4.21×10^{-04}	0.3835
0.95	0.95	50	0.25	7	0.99	5.54×10^{-04}	4.22×10^{-04}	0.3668
0.95	0.95	50	0.25	10	0.8	5.61×10^{-04}	4.26×10^{-04}	0.3638
0.95	0.95	50	0.25	10	0.9	5.55×10^{-04}	4.22×10^{-04}	0.3856
0.95	0.95	50	0.25	10	0.99	5.52×10^{-04}	4.18×10^{-04}	0.3908
0.95	0.95	50	1	3	0.8	5.51×10^{-04}	4.18×10^{-04}	0.4245
0.95	0.95	50	1	3	0.9	5.46×10^{-04}	4.12×10^{-04}	0.4158
0.95	0.95	50	1	3	0.99	5.56×10^{-04}	4.18×10^{-04}	0.411
0.95	0.95	50	1	7	0.8	5.46×10^{-04}	4.10×10^{-04}	0.4332
0.95	0.95	50	1	7	0.9	5.48×10^{-04}	4.15×10^{-04}	0.4248
0.95	0.95	50	1	7	0.99	5.54×10^{-04}	4.20×10^{-04}	0.4191
0.95	0.95	50	1	10	0.8	5.55×10^{-04}	4.17×10^{-04}	0.4116
0.95	0.95	50	1	10	0.9	5.48×10^{-04}	4.17×10^{-04}	0.4107
0.95	0.95	50	1	10	0.99	5.58×10^{-04}	4.25×10^{-04}	0.4247

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	0.95	50	2	3	0.8	5.43×10^{-04}	4.10×10^{-04}	0.421
0.95	0.95	50	2	3	0.9	5.55×10^{-04}	4.18×10^{-04}	0.4195
0.95	0.95	50	2	3	0.99	5.45×10^{-04}	4.16×10^{-04}	0.402
0.95	0.95	50	2	7	0.8	5.44×10^{-04}	4.10×10^{-04}	0.4096
0.95	0.95	50	2	7	0.9	5.48×10^{-04}	4.16×10^{-04}	0.4172
0.95	0.95	50	2	7	0.99	5.54×10^{-04}	4.21×10^{-04}	0.4219
0.95	0.95	50	2	10	0.8	5.44×10^{-04}	4.12×10^{-04}	0.4218
0.95	0.95	50	2	10	0.9	5.41×10^{-04}	4.11×10^{-04}	0.4039
0.95	0.95	50	2	10	0.99	5.50×10^{-04}	4.15×10^{-04}	0.4076
0.95	0.95	100	0.25	3	0.8	5.55×10^{-04}	4.19×10^{-04}	0.4407
0.95	0.95	100	0.25	3	0.9	5.70×10^{-04}	4.27×10^{-04}	0.47
0.95	0.95	100	0.25	3	0.99	5.51×10^{-04}	4.17×10^{-04}	0.4451
0.95	0.95	100	0.25	7	0.8	5.60×10^{-04}	4.22×10^{-04}	0.4568
0.95	0.95	100	0.25	7	0.9	5.54×10^{-04}	4.14×10^{-04}	0.4567
0.95	0.95	100	0.25	7	0.99	5.58×10^{-04}	4.21×10^{-04}	0.4581
0.95	0.95	100	0.25	10	0.8	5.51×10^{-04}	4.13×10^{-04}	0.4556
0.95	0.95	100	0.25	10	0.9	5.62×10^{-04}	4.22×10^{-04}	0.4669
0.95	0.95	100	0.25	10	0.99	5.59×10^{-04}	4.18×10^{-04}	0.4427
0.95	0.95	100	1	3	0.8	5.58×10^{-04}	4.19×10^{-04}	0.4742
0.95	0.95	100	1	3	0.9	5.64×10^{-04}	4.21×10^{-04}	0.4872
0.95	0.95	100	1	3	0.99	5.56×10^{-04}	4.16×10^{-04}	0.4785
0.95	0.95	100	1	7	0.8	5.69×10^{-04}	4.30×10^{-04}	0.5029
0.95	0.95	100	1	7	0.9	5.48×10^{-04}	4.14×10^{-04}	0.4713
0.95	0.95	100	1	7	0.99	5.51×10^{-04}	4.14×10^{-04}	0.4764
0.95	0.95	100	1	10	0.8	5.67×10^{-04}	4.23×10^{-04}	0.478
0.95	0.95	100	1	10	0.9	5.56×10^{-04}	4.17×10^{-04}	0.4793
0.95	0.95	100	1	10	0.99	5.52×10^{-04}	4.14×10^{-04}	0.4772
0.95	0.95	100	2	3	0.8	5.50×10^{-04}	4.14×10^{-04}	0.4638
0.95	0.95	100	2	3	0.9	5.54×10^{-04}	4.16×10^{-04}	0.4619
0.95	0.95	100	2	3	0.99	5.54×10^{-04}	4.16×10^{-04}	0.4555
0.95	0.95	100	2	7	0.8	5.50×10^{-04}	4.14×10^{-04}	0.4582
0.95	0.95	100	2	7	0.9	5.50×10^{-04}	4.13×10^{-04}	0.4684
0.95	0.95	100	2	7	0.99	5.55×10^{-04}	4.17×10^{-04}	0.4756
0.95	0.95	100	2	10	0.8	5.50×10^{-04}	4.12×10^{-04}	0.4799
0.95	0.95	100	2	10	0.9	5.49×10^{-04}	4.13×10^{-04}	0.4617
0.95	0.95	100	2	10	0.99	5.47×10^{-04}	4.13×10^{-04}	0.4452
0.95	0.95	200	0.25	3	0.8	5.62×10^{-04}	4.14×10^{-04}	0.5468
0.95	0.95	200	0.25	3	0.9	5.64×10^{-04}	4.18×10^{-04}	0.5079

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	0.95	200	0.25	3	0.99	5.63×10^{-04}	4.18×10^{-04}	0.5354
0.95	0.95	200	0.25	7	0.8	5.69×10^{-04}	4.22×10^{-04}	0.5352
0.95	0.95	200	0.25	7	0.9	5.73×10^{-04}	4.23×10^{-04}	0.5352
0.95	0.95	200	0.25	7	0.99	5.64×10^{-04}	4.18×10^{-04}	0.5037
0.95	0.95	200	0.25	10	0.8	5.69×10^{-04}	4.21×10^{-04}	0.5418
0.95	0.95	200	0.25	10	0.9	5.70×10^{-04}	4.27×10^{-04}	0.5206
0.95	0.95	200	0.25	10	0.99	5.60×10^{-04}	4.19×10^{-04}	0.5262
0.95	0.95	200	1	3	0.8	5.64×10^{-04}	4.21×10^{-04}	0.5411
0.95	0.95	200	1	3	0.9	5.54×10^{-04}	4.13×10^{-04}	0.5617
0.95	0.95	200	1	3	0.99	5.71×10^{-04}	4.25×10^{-04}	0.5474
0.95	0.95	200	1	7	0.8	5.65×10^{-04}	4.22×10^{-04}	0.5505
0.95	0.95	200	1	7	0.9	5.57×10^{-04}	4.19×10^{-04}	0.5543
0.95	0.95	200	1	7	0.99	5.69×10^{-04}	4.24×10^{-04}	0.5663
0.95	0.95	200	1	10	0.8	5.63×10^{-04}	4.17×10^{-04}	0.5381
0.95	0.95	200	1	10	0.9	5.57×10^{-04}	4.16×10^{-04}	0.5303
0.95	0.95	200	1	10	0.99	5.59×10^{-04}	4.17×10^{-04}	0.5548
0.95	0.95	200	2	3	0.8	5.46×10^{-04}	4.10×10^{-04}	0.483
0.95	0.95	200	2	3	0.9	5.56×10^{-04}	4.16×10^{-04}	0.4908
0.95	0.95	200	2	3	0.99	5.51×10^{-04}	4.12×10^{-04}	0.493
0.95	0.95	200	2	7	0.8	5.55×10^{-04}	4.17×10^{-04}	0.472
0.95	0.95	200	2	7	0.9	5.51×10^{-04}	4.16×10^{-04}	0.4916
0.95	0.95	200	2	7	0.99	5.49×10^{-04}	4.13×10^{-04}	0.4902
0.95	0.95	200	2	10	0.8	5.46×10^{-04}	4.08×10^{-04}	0.4786
0.95	0.95	200	2	10	0.9	5.52×10^{-04}	4.16×10^{-04}	0.4824
0.95	0.95	200	2	10	0.99	5.52×10^{-04}	4.12×10^{-04}	0.4795
0.95	2	50	0.25	3	0.8	5.60×10^{-04}	4.26×10^{-04}	0.3758
0.95	2	50	0.25	3	0.9	5.54×10^{-04}	4.20×10^{-04}	0.3719
0.95	2	50	0.25	3	0.99	5.52×10^{-04}	4.18×10^{-04}	0.3908
0.95	2	50	0.25	7	0.8	5.56×10^{-04}	4.23×10^{-04}	0.3649
0.95	2	50	0.25	7	0.9	5.74×10^{-04}	4.33×10^{-04}	0.3709
0.95	2	50	0.25	7	0.99	5.57×10^{-04}	4.25×10^{-04}	0.3666
0.95	2	50	0.25	10	0.8	5.60×10^{-04}	4.23×10^{-04}	0.3629
0.95	2	50	0.25	10	0.9	5.55×10^{-04}	4.20×10^{-04}	0.3672
0.95	2	50	0.25	10	0.99	5.51×10^{-04}	4.18×10^{-04}	0.3557
0.95	2	50	1	3	0.8	5.52×10^{-04}	4.16×10^{-04}	0.3848
0.95	2	50	1	3	0.9	5.47×10^{-04}	4.16×10^{-04}	0.3829
0.95	2	50	1	3	0.99	5.56×10^{-04}	4.19×10^{-04}	0.3957
0.95	2	50	1	7	0.8	5.56×10^{-04}	4.22×10^{-04}	0.3957

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	2	50	1	7	0.9	$5.47{\times}10^{-04}$	$4.14{\times}10^{-04}$	0.3968
0.95	2	50	1	7	0.99	$5.61{\times}10^{-04}$	$4.27{\times}10^{-04}$	0.3825
0.95	2	50	1	10	0.8	$5.46{\times}10^{-04}$	$4.17{\times}10^{-04}$	0.3862
0.95	2	50	1	10	0.9	$5.59{\times}10^{-04}$	$4.25{\times}10^{-04}$	0.3925
0.95	2	50	1	10	0.99	$5.48{\times}10^{-04}$	$4.16{\times}10^{-04}$	0.3759
0.95	2	50	2	3	0.8	$5.48{\times}10^{-04}$	$4.14{\times}10^{-04}$	0.3845
0.95	2	50	2	3	0.9	$5.55{\times}10^{-04}$	$4.21{\times}10^{-04}$	0.3926
0.95	2	50	2	3	0.99	$5.52{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.3754
0.95	2	50	2	7	0.8	$5.54{\times}10^{-04}$	$4.19{\times}10^{-04}$	0.4054
0.95	2	50	2	7	0.9	$5.52{\times}10^{-04}$	$4.19{\times}10^{-04}$	0.3906
0.95	2	50	2	7	0.99	$5.45{\times}10^{-04}$	$4.14{\times}10^{-04}$	0.3745
0.95	2	50	2	10	0.8	$5.49{\times}10^{-04}$	$4.16{\times}10^{-04}$	0.3762
0.95	2	50	2	10	0.9	$5.50{\times}10^{-04}$	$4.16{\times}10^{-04}$	0.3955
0.95	2	50	2	10	0.99	$5.59{\times}10^{-04}$	$4.25{\times}10^{-04}$	0.3774
0.95	2	100	0.25	3	0.8	$5.52{\times}10^{-04}$	$4.17{\times}10^{-04}$	0.4466
0.95	2	100	0.25	3	0.9	$5.59{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.435
0.95	2	100	0.25	3	0.99	$5.50{\times}10^{-04}$	$4.16{\times}10^{-04}$	0.4256
0.95	2	100	0.25	7	0.8	$5.56{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.4389
0.95	2	100	0.25	7	0.9	$5.56{\times}10^{-04}$	$4.17{\times}10^{-04}$	0.4204
0.95	2	100	0.25	7	0.99	$5.61{\times}10^{-04}$	$4.19{\times}10^{-04}$	0.4452
0.95	2	100	0.25	10	0.8	$5.61{\times}10^{-04}$	$4.21{\times}10^{-04}$	0.4441
0.95	2	100	0.25	10	0.9	$5.53{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.439
0.95	2	100	0.25	10	0.99	$5.53{\times}10^{-04}$	$4.14{\times}10^{-04}$	0.4358
0.95	2	100	1	3	0.8	$5.53{\times}10^{-04}$	$4.19{\times}10^{-04}$	0.4451
0.95	2	100	1	3	0.9	$5.58{\times}10^{-04}$	$4.24{\times}10^{-04}$	0.445
0.95	2	100	1	3	0.99	$5.61{\times}10^{-04}$	$4.22{\times}10^{-04}$	0.4466
0.95	2	100	1	7	0.8	$5.61{\times}10^{-04}$	$4.22{\times}10^{-04}$	0.4466
0.95	2	100	1	7	0.9	$5.60{\times}10^{-04}$	$4.20{\times}10^{-04}$	0.4378
0.95	2	100	1	7	0.99	$5.49{\times}10^{-04}$	$4.13{\times}10^{-04}$	0.4505
0.95	2	100	1	10	0.8	$5.52{\times}10^{-04}$	$4.15{\times}10^{-04}$	0.4477
0.95	2	100	1	10	0.9	$5.53{\times}10^{-04}$	$4.15{\times}10^{-04}$	0.4487
0.95	2	100	1	10	0.99	$5.56{\times}10^{-04}$	$4.18{\times}10^{-04}$	0.4361
0.95	2	100	2	3	0.8	$5.51{\times}10^{-04}$	$4.19{\times}10^{-04}$	0.4257
0.95	2	100	2	3	0.9	$5.52{\times}10^{-04}$	$4.16{\times}10^{-04}$	0.4248
0.95	2	100	2	3	0.99	$5.52{\times}10^{-04}$	$4.18{\times}10^{-04}$	0.4156
0.95	2	100	2	7	0.8	$5.47{\times}10^{-04}$	$4.12{\times}10^{-04}$	0.4189
0.95	2	100	2	7	0.9	$5.45{\times}10^{-04}$	$4.10{\times}10^{-04}$	0.4201
0.95	2	100	2	7	0.99	$5.59{\times}10^{-04}$	$4.22{\times}10^{-04}$	0.4245

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
0.95	2	100	2	10	0.8	5.53×10^{-04}	4.21×10^{-04}	0.4183
0.95	2	100	2	10	0.9	5.49×10^{-04}	4.16×10^{-04}	0.4223
0.95	2	100	2	10	0.99	5.49×10^{-04}	4.14×10^{-04}	0.4137
0.95	2	200	0.25	3	0.8	5.70×10^{-04}	4.30×10^{-04}	0.5139
0.95	2	200	0.25	3	0.9	5.59×10^{-04}	4.18×10^{-04}	0.4951
0.95	2	200	0.25	3	0.99	5.59×10^{-04}	4.18×10^{-04}	0.4951
0.95	2	200	0.25	7	0.8	5.78×10^{-04}	4.29×10^{-04}	0.5256
0.95	2	200	0.25	7	0.9	5.56×10^{-04}	4.18×10^{-04}	0.4846
0.95	2	200	0.25	7	0.99	5.66×10^{-04}	4.24×10^{-04}	0.4984
0.95	2	200	0.25	10	0.8	5.58×10^{-04}	4.17×10^{-04}	0.505
0.95	2	200	0.25	10	0.9	5.84×10^{-04}	4.31×10^{-04}	0.5349
0.95	2	200	0.25	10	0.99	5.67×10^{-04}	4.24×10^{-04}	0.4947
0.95	2	200	1	3	0.8	5.60×10^{-04}	4.18×10^{-04}	0.5021
0.95	2	200	1	3	0.9	5.52×10^{-04}	4.18×10^{-04}	0.4867
0.95	2	200	1	3	0.99	5.59×10^{-04}	4.20×10^{-04}	0.4918
0.95	2	200	1	7	0.8	5.58×10^{-04}	4.20×10^{-04}	0.5039
0.95	2	200	1	7	0.9	5.61×10^{-04}	4.23×10^{-04}	0.4948
0.95	2	200	1	7	0.99	5.59×10^{-04}	4.21×10^{-04}	0.4932
0.95	2	200	1	10	0.8	5.56×10^{-04}	4.20×10^{-04}	0.4924
0.95	2	200	1	10	0.9	5.52×10^{-04}	4.15×10^{-04}	0.49
0.95	2	200	1	10	0.99	5.57×10^{-04}	4.19×10^{-04}	0.5007
0.95	2	200	2	3	0.8	5.49×10^{-04}	4.16×10^{-04}	0.4399
0.95	2	200	2	3	0.9	5.50×10^{-04}	4.14×10^{-04}	0.4521
0.95	2	200	2	3	0.99	5.52×10^{-04}	4.16×10^{-04}	0.442
0.95	2	200	2	7	0.8	5.53×10^{-04}	4.19×10^{-04}	0.4367
0.95	2	200	2	7	0.9	5.44×10^{-04}	4.09×10^{-04}	0.4461
0.95	2	200	2	7	0.99	5.53×10^{-04}	4.16×10^{-04}	0.434
0.95	2	200	2	10	0.8	5.53×10^{-04}	4.16×10^{-04}	0.434
0.95	2	200	2	10	0.9	5.47×10^{-04}	4.15×10^{-04}	0.4407
0.95	2	200	2	10	0.99	5.56×10^{-04}	4.19×10^{-04}	0.4434
2	0.5	50	0.25	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	50	0.25	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.5	50	0.25	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	50	0.25	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.5	50	0.25	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	50	0.25	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.5	50	0.25	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.5	50	0.25	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.5	50	0.25	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.5	50	1	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	50	1	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	50	1	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	50	1	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	50	1	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.5	50	1	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.014
2	0.5	50	1	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	50	1	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.5	50	1	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	50	2	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	50	2	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	50	2	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.5	50	2	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	50	2	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	50	2	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	50	2	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0149
2	0.5	50	2	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	50	2	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.5	100	0.25	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.5	100	0.25	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.5	100	0.25	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0147
2	0.5	100	0.25	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.014
2	0.5	100	0.25	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.5	100	0.25	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	100	0.25	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0138
2	0.5	100	0.25	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	100	0.25	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.5	100	1	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.5	100	1	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	100	1	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	100	1	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	100	1	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	100	1	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.5	100	1	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0147
2	0.5	100	1	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	100	1	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.5	100	2	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0145

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.5	100	2	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.014
2	0.5	100	2	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	100	2	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.5	100	2	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	100	2	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	100	2	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	100	2	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	100	2	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	0.25	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.5	200	0.25	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.5	200	0.25	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.5	200	0.25	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.5	200	0.25	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.5	200	0.25	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.5	200	0.25	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	200	0.25	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	200	0.25	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.5	200	1	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	200	1	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	200	1	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.5	200	1	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	1	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	1	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	1	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	1	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	1	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.5	200	2	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.5	200	2	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	2	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	2	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.5	200	2	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	2	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	2	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.5	200	2	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.5	200	2	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	50	0.25	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	50	0.25	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	50	0.25	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.95	50	0.25	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	50	0.25	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.95	50	0.25	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.014
2	0.95	50	0.25	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.95	50	0.25	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	50	0.25	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0147
2	0.95	50	1	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	50	1	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	50	1	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.95	50	1	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	50	1	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.95	50	1	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	50	1	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	50	1	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0139
2	0.95	50	1	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	50	2	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.95	50	2	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.95	50	2	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	50	2	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	50	2	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0147
2	0.95	50	2	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	50	2	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	50	2	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	50	2	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	100	0.25	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	100	0.25	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	100	0.25	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.95	100	0.25	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.95	100	0.25	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	100	0.25	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	100	0.25	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0147
2	0.95	100	0.25	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	100	0.25	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	100	1	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	100	1	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	100	1	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	100	1	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0147
2	0.95	100	1	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.95	100	1	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.95	100	1	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.95	100	1	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	100	1	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.95	100	2	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.95	100	2	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	100	2	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.95	100	2	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	100	2	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	100	2	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	100	2	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	100	2	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	100	2	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	200	0.25	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.95	200	0.25	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	200	0.25	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	200	0.25	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.95	200	0.25	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	200	0.25	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.95	200	0.25	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	200	0.25	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	200	0.25	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	200	1	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	200	1	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.95	200	1	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	0.95	200	1	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.95	200	1	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.014
2	0.95	200	1	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	0.95	200	1	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	200	1	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	200	1	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	200	2	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	200	2	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	200	2	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0147
2	0.95	200	2	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	0.95	200	2	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	0.95	200	2	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	0.95	200	2	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	0.95	200	2	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	0.95	200	2	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	50	0.25	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.014
2	2	50	0.25	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	50	0.25	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	50	0.25	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	50	0.25	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	50	0.25	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	50	0.25	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	50	0.25	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	50	0.25	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	50	1	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	50	1	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	50	1	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0148
2	2	50	1	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.014
2	2	50	1	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	50	1	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	50	1	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	50	1	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	50	1	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	50	2	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	50	2	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	50	2	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	50	2	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	50	2	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	50	2	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	50	2	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	50	2	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	50	2	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	100	0.25	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	100	0.25	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0147
2	2	100	0.25	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	100	0.25	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	100	0.25	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	100	0.25	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.014
2	2	100	0.25	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	100	0.25	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	100	0.25	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	2	100	1	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	100	1	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	100	1	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	100	1	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	100	1	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	100	1	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	100	1	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	100	1	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	2	100	1	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	100	2	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	100	2	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	100	2	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	100	2	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	100	2	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	100	2	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	100	2	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	100	2	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	100	2	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	200	0.25	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	200	0.25	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	200	0.25	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	200	0.25	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	200	0.25	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	2	200	0.25	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	200	0.25	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	200	0.25	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	200	0.25	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	200	1	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	200	1	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0139
2	2	200	1	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	200	1	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	200	1	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	200	1	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	200	1	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	200	1	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	200	1	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0144
2	2	200	2	3	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	200	2	3	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0147

Table A.7: (continued)

α	β	m	k	v	q	RMSE	MAE	R^2
2	2	200	2	3	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141
2	2	200	2	7	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	200	2	7	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0146
2	2	200	2	7	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0142
2	2	200	2	10	0.8	6.89×10^{-04}	5.41×10^{-04}	0.0143
2	2	200	2	10	0.9	6.89×10^{-04}	5.41×10^{-04}	0.0145
2	2	200	2	10	0.99	6.89×10^{-04}	5.41×10^{-04}	0.0141

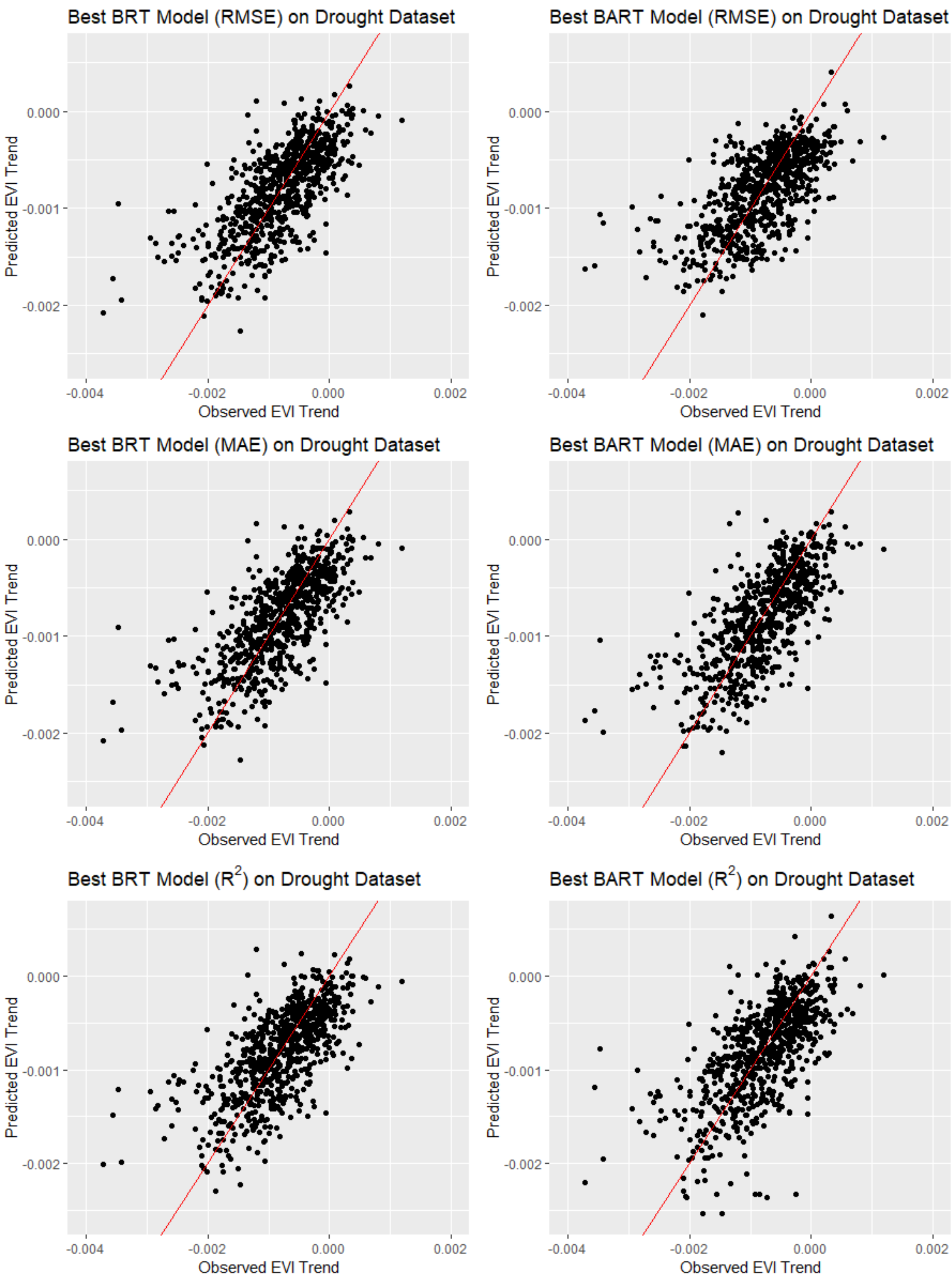

Figure A.1: Predicted EVI trend against observed EVI trend for the best performing BRT (left) and BART (right) models on the drought dataset. The reference line where the observed EVI trend equals the predicted EVI trend is shown in red.

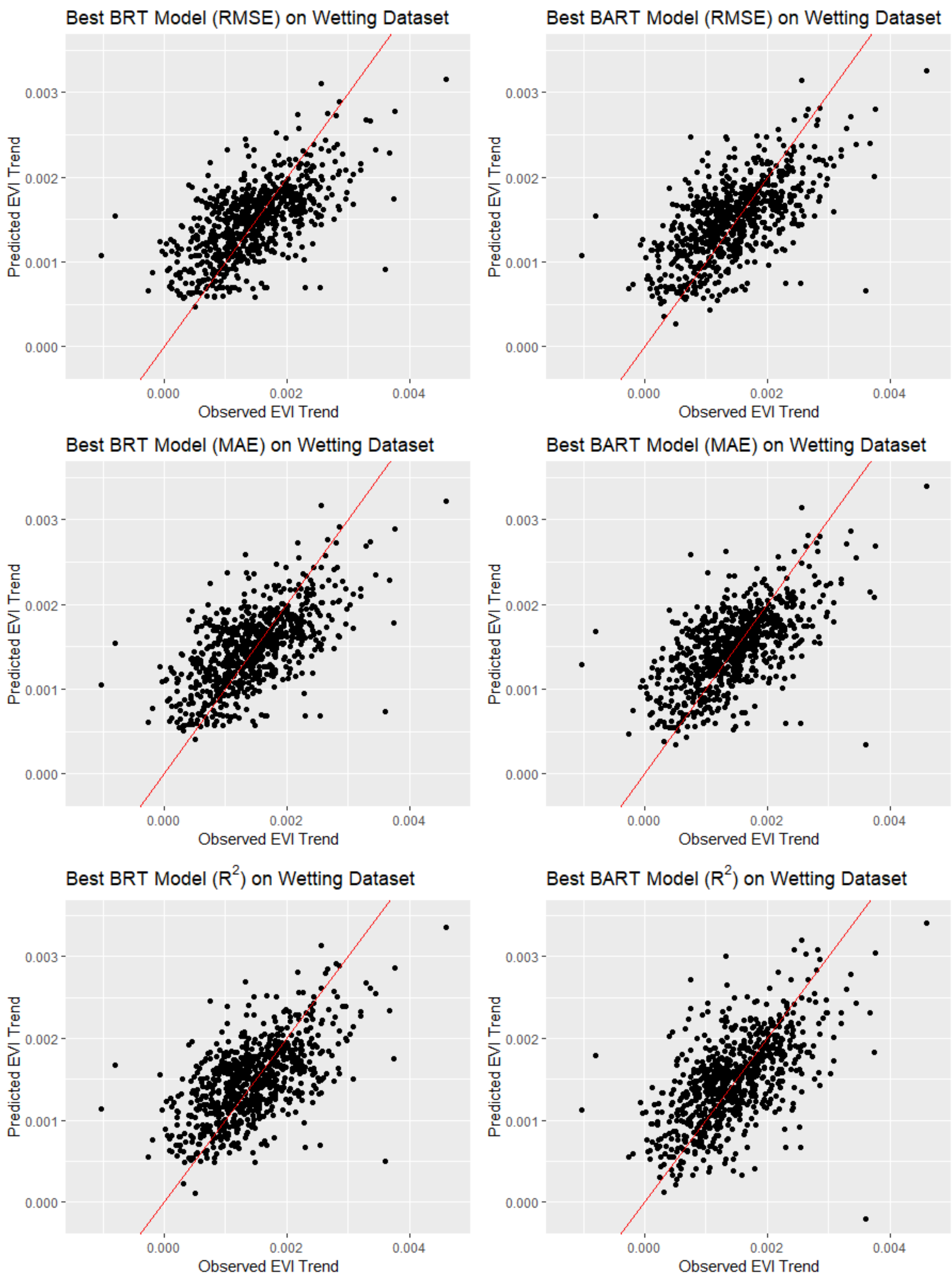

Figure A.2: Predicted EVI trend against observed EVI trend for the best performing BRT (left) and BART (right) models on the wetting dataset. The reference line where the observed EVI trend equals the predicted EVI trend is shown in red.

Table A.8: Error Statistics of best performing BRT and BART models calculated on each dataset.

		BRT	BART
Drought Dataset	Best RMSE Model	4.560×10^{-04}	3.713×10^{-04}
	Best MAE Model	3.398×10^{-04}	3.457×10^{-04}
	Best R^2 Model	0.5213	0.623
Wetting Dataset	Best RMSE Model	5.353×10^{-04}	5.386×10^{-04}
	Best MAE Model	4.051×10^{-04}	4.066×10^{-04}
	Best R^2 Model	0.4732	0.5924