University of Southern Queensland Faculty of Engineering and Surveying

# How to Establish Map Projections to Facilitate the use of GPS on Ground-Based Surveys

A dissertation submitted by

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# Abstract

With the increased use of RTK GPS, local ground-based coordinate systems are increasingly being used to represent ground distances on plans. Through their use, the incorrect representation of ground distances on plans can be minimised. However, many of the users of such systems do not know the limitations of these systems.

This dissertation developed and analysed two local ground-based coordinate systems based on Transverse Mercator and Tangent Plane projections. The method involved establishing local ground-based coordinate systems at an average project height at which grid distances approximately equalled ground distances over small areas. The testing focused on distance and angular errors caused purely by the process of projection. Other site dependent variables have also been assessed, including the effect of site height above and below projection level and the effect that the longitude of the site has on the distance accuracy of a site.

It was concluded that the major limiting factor, when using local ground-based coordinate systems, is the error in grid distances when compared to measured ground distances. The results obtained show a variation in error distribution between the coordinate systems, depending on the method of projection used. The results illustrate a number of accurate areas within which a number of defined measurement accuracies and magnitudes are not exceeded. The limits of the systems were found to be approximately 3km east/west in a Transverse Mercator projection and 19km in any direction in a plane system, from the central point of the site, before RTK GPS measurement accuracy is exceeded by projection distortion.

The need for quantification of errors in local ground-based coordinate systems is significant as they are used to produce plan distances when using RTK GPS. If used outside the limits defined in this dissertation, errors will occur in the plan distances resulting from the use of such systems.

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# ENG4111 Research Project Part 1 & ENG4112 Research Project Part 2

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I further certify that the work is original and has not been previously submitted for assessment in any other course or institution, except where specifically stated.

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# **Glossary of Terms**

- AHD = Australian Height Datum
- AMG = Australian Map Grid
- CAD = Computer Aided Design
- CSV = Comma-Separated Values
- DSF = Datum Scale Factor
- GDA = Geocentric Datum of Australia
- GPS = Global Positioning System
- h = Ellipsoidal Height of the Site
- ITRF = International Terrestrial Reference Frame
- MGA = Map Grid of Australia
- PPM = Part Per Million
- QLD = Queensland
- R = Geometric Mean Radius of Curvature
- RTK = Real Time Kinematic
- SMI Act = Survey and Mapping Infrastructure Act 2003
- TGO = Trimble Geomatics Office
- USQ = University of Southern Queensland
- UTM = Universal Transverse Mercator
- WGS84 = World Geodetic System 1984

# **Chapter 1: Introduction**

## 1.1 Project Background

In recent years a number of advances in electronic technology have allowed the proliferation of the use of GPS receivers and digital storage controllers in surveying practices. Their rapid adoption and ease of use have increased the use of local ground-based coordinate systems for surveying projects, by making it easier to set up and measure in such systems. Greater control point density and changing legislation also means that it is becoming easier and sometimes mandatory to connect surveys to the Australian Map Grid (AMG) and Map Grid of Australia (MGA) (*Geodetic Surveying B Study Book* 2007).

However, when using these map projections on a local site, scale factors need to be introduced to reduce measured distances to grid distances. This requires more time, knowledge and money to implement (*Geodetic Surveying B Study Book* 2007). Because local ground-based coordinate systems are set up to be the best fit for a particular site, they do not need corrections for scale. Because of this they are becoming more widely adopted in the field and it is important that the limitations of local ground-based systems be quantified and defined.

When using any local map projection to represent the Earth as a flat surface, errors will occur in the represented distances and angles because of the difference between a flat plane and the curved earth. These errors have traditionally been ignored by surveyors over small areas, but over larger areas the flat-Earth assumption is no longer valid. The area over which such a system is valid is often not quantified specifically with respect to measurement accuracy and plan requirements, but given as an approximation, such as a zone width restriction (*Geodetic Surveying B Study Book* 2007).

Differences in height between the projection surface and the elevation at which a distance is measured also induces errors into horizontal distances represented on a plan. This is because of the convergence of plumblines between differing level surfaces, on which the distances are calculated, see Figure 2.6. Often this difference

is ignored by surveyors, but over long lines and large differences in height, this correction can become significant.

A local projection is formed as an elevated reference surface that is the best fit for an area around a central point. It is able to minimise the errors in the local area by closely approximating the curved surface of the Earth in that area on a flat plane. Local ground-based coordinate systems can be constructed in such a way as to align a survey to an MGA meridian and eliminate negative coordinates on the local site. They are developed in such a manner that distances on the map projection will equal ground distances measured in the field (*Geodetic Surveying B Study Book* 2007).

Through appropriate testing, errors in these local ground-based coordinate systems can be quantified. The errors determined from tests conducted on local ground-based coordinate systems can then be compared to measurement accuracies of current surveying instruments including RTK GPS and total stations and the required accuracy for Cadastral plan dimensions in QLD. Comparisons will allow the determination of the acceptable area over which local ground-based coordinate systems can be effectively used – defined by the area over which the errors associated with the map projection are less than normal survey measurement accuracy. Graphical presentation of the results will also allow potential users of local ground-based coordinate systems to make decisions about their own accuracy requirements.

A thorough, systematic analysis of these errors is required to facilitate future decision-making with respect to the possible uses, limitations of use and the accuracy of measurements made in local ground-based coordinate systems, for quality assurance purposes. Analysis of these errors will also help surveyors to exercise the due diligence that is expected from a surveying professional by better equipping them with information about the errors in such systems and providing a simple method of representation of ground distances on plans, where ground distances are required.

## 1.2 Research Aim and Objectives

#### 1.2.1 Research Aim

The aim of this project is to develop a procedure, determine the limitations and validate the use of local ground-based coordinate systems with respect to dimensions shown on plans when conducting Real Time Kinematic (RTK) GPS surveys.

#### 1.2.2 Research Objectives

This project's objectives are as follows:

- Analyse errors in the GPS system
- Develop a set of procedures to define local based coordinate systems
- Develop methods for defining errors associated with local coordinate systems
- Review and discuss the errors and their effect on dimensions
- Validate the procedures through the use of test data
- Discuss and make recommendations from the results

## 1.3 Justification

The justification of this project stems from the incorrect representation of ground distances on survey plans. This occurs when grid distances are shown that differ from the ground distances that are required. The incorrect representation of ground distances has become more evident with the increased need to join surveys to survey control networks such as the MGA network. However, in using the MGA map projection, scale factors must be introduced on a local project to output ground distances on a local plane. The problem arises, however, that not all users of map projections such as MGA are aware of this or they are unable to apply the corrections properly. As a result, distances other than ground distances such as MGA grid distances are being shown on survey plans. To illustrate this problem, consider the 400m line measured in the MGA system in Figure 1.1.

Endpoint MGA coordinates: 499800.000 East 6946945.938 North MGA Grid Distance: 400.000m

Endpoint MGA coordinates: 500200.000 East 6946945.938 North

Figure 1.1: Ground and MGA distance comparison

In Figure 1.1 the line is at an elevation of 700m, crossing the centre of MGA zone 56. It can be seen that the ground distance (the distance measured in the field) is different from the MGA grid distance. This is representative of the type of error that is being made when presenting MGA grid distances as ground distances on survey plans.

In current software it is possible to interrogate each line individually to obtain a ground distance. However, it is time consuming and cumbersome to do. This is especially evident in tasks that have lots of lines and require design to be completed at ground level, such as a subdivision layout design. A local ground-based coordinate system set up within the software will allow the use of ground distances for measurement and design on all lines and thus remove this problem, saving time and increasing productivity.

To minimise the use of incorrect grid distances on survey plans that require ground distances, local ground-based coordinate systems can be used. Their popularity has increased with the increased use of digital data recorders and RTK GPS to conduct surveys. However, not all new users of local ground-based coordinate systems understand the limitations of local ground-based coordinate systems regarding the accuracy obtainable from such systems and how this relates to the required plan accuracy. For surveyors to achieve acceptable quality for surveys conducted with local ground-based coordinate systems, it is necessary to have an understanding of the errors within the system. Once these limitations are known, the surveyor is in a position to implement limitations on the use of local ground-based coordinate systems to ensure the accuracy of data collected and presented on plans.

## 1.4 Scope

The testing undertaken by the author will focus on local ground-based coordinate systems developed using Transverse Mercator and tangent plane projections. These two systems are the focus of this project because they are considered by the author to be the two most commonly used projection methods in Australia and are therefore the most relevant for testing. The testing will cover the distance and angular errors within local ground-based coordinate systems due to projection distortion. Testing will also cover site-specific effects on local ground-based coordinate system accuracy, including changes in height from the project site elevation and changes in project longitude. The distribution and magnitude of these errors will be represented graphically and in tables. Subsequently, they will be related to the measurement accuracy obtainable with current RTK GPS and total stations and the accuracy required on cadastral survey plans in Queensland. Testing will be completed using a range of software packages and will focus on the theoretical errors within the system. Validation of results will be carried out through the use of test data, generated within these systems. This data will consist of a number of figures that will be used to make comparisons between the systems' accuracy.

## 1.5 Conclusion

This dissertation aims to test the errors associated with local ground-based coordinate systems developed using various map projection methods. It is important to conduct such testing so the limitations of the use of local ground-based coordinate systems can be established with respect to measurement accuracy and plan requirements. The visualisation of errors within local ground-based coordinate systems will also help in making recommendations about their use.

To establish the type and nature of errors within local ground-based coordinate systems, a literature review will be conducted. The literature review will provide background information on errors that occur within map projections, how to measure these errors and any previous work completed into these measuring errors. The literature review will present previous research and identify current shortcomings in the previously published literature on the topic of local ground-

based coordinate systems. The literature review will also provide the background for methods used to represent map distortion and provide the basis for the representation of projection errors in this project.

# **Chapter 2: Literature Review**

## 2.1 Introduction

This chapter will review the current literature that is available and provide a basis for the development of methods to test and quantify the errors associated with local ground-based coordinate systems and their suitability for use in preparing plans. It will also define the extent of previous research on local ground-based coordinate systems and define a number of measurement accuracies and legal requirements against which to compare the accuracy of local ground-based coordinate systems.

The chapter aims to provide an outline and important background information about local ground-based coordinate systems and how they are developed. It also aims to provide relevant information on their use and the errors associated with the map projection process. A review of previous research in the area of local ground-based ground coordinate systems will also be conducted and gaps in previous research presented.

The chapter will begin by introducing the concept of local ground-based coordinate systems and common projections used in their formation. Suitable methods for the display of map projection-distortion will also be covered. The chapter will conclude by providing a brief overview of the accuracy achievable with current RTK GPS, current total stations and the Queensland cadastral surveying requirements.

## 2.2 Local Ground-Based Coordinate Systems

#### 2.2.1 Background

The Earth is a curved surface that approximates an ellipsoid, but for most surveying applications it is acceptable to assume that the surface of the Earth is flat (*Geodetic Surveying B Study Book* 2007). The process of representing the Earth's surface on a two-dimensional flat plane is known as map projection and causes errors in the representation of features (Estopinal 1992). It is preferable from a surveyor's perspective to use a system that presents the Earth as flat because this is what is

shown on cadastral and engineering construction drawings. The flat representation of the Earth also allows the use of simple plane geometry and avoids the use of curvature corrections (Maling 1992).

Over large areas, regional projections provide a good approximation for mapping purposes (*Geodetic Surveying B Study Book* 2007). Over smaller areas, however, the grid and ground distances in small scale systems, such as MGA, are not equal and these small scale systems are unsuitable for large scale purposes, such as Cadastral and engineering construction surveying. Local ground-based coordinate systems are developed over smaller areas to minimise projection distortion & provide a method of obtaining coordinates at ground level, instead of projection level (Wisconsin State Cartographers Office 2004). The result is that grid distances presented on a map are the same as ground distances measured in the field (*Geodetic Surveying B Study Book* 2007) see Figure 2.1.



Figure 2.1: Difference between grid and ground (measured) distances

Previous work by the Wisconsin State Cartographers Office (2004) and Burkholder (1993) has presented the use of three common projections used to form local

ground-based coordinate systems. The projections include the Lambert conic conformal, the Transverse Mercator and the tangent plane. These projections are developed using a known ellipsoid and the origin placed such that the grid/ground difference in the local area is minimised. False coordinates may also be assigned to the origin to avoid negative coordinates in the coordinate system (Wisconsin State Cartographers Office 2004). It must be noted that the Lambert conic conformal projection is outside the scope of this project and will not be discussed further.

#### 2.2.2 Tangent Plane Projections

Tangent plane projections are formed by bringing a plane into contact with an ellipsoid and transferring the features from one surface to the other (Iliffe 2002). This type of projection utilises a flat plane as the projection surface that is brought into contact with the ellipsoid at a point of tangency, see Figure 2.2. To establish a projection, the point of tangency must be defined and the orientation of the projection must be specified (Estopinal 1992).



Figure 2.2: Visualisation of a Tangent Plane Projection (Source: Estopinal, 1992)

An advantage of tangent plane projections is the ease with which a surveyor is able to establish the system. The surveyor has the choice of the position of the point of tangency and the choice of direction (Estopinal 1992). Because the shape of the Earth is ignored, the surveyor does not need knowledge of the geodetic positions of points to map the area. Nor does the geodetic position of points need to be collected. Simple geometry and plane mathematics can also be used in the projection (Estopinal 1992).

Tangent plane surveys, unless referenced to the same point of tangency, are free of each other and have no common tie or reference. Surveys done in such systems do not contain information about the survey's relationship to any other work not completed within the survey. This means that directions and distances of common lines between two surveys of differing projection points will disagree. Directions between two identical points on two projections will also differ (Estopinal 1992).

#### 2.2.3 Transverse Mercator Projections

The Transverse Mercator system projects geodetic coordinates onto a concentric cylinder which is tangential to the equator and makes contact along one meridian (*Geodetic Surveying B Study Book* 2007). To minimise distortion, the Earth may be rotated to bring different meridians into contact with the cylinder for different areas around the globe. The true origin for each zone is the intersection of the equator and the contacting meridian (POSC 1997).



Figure 2.3: Transverse Mercator Projection (Source: Natural Resources Canada, 2005)

Transverse Mercator projections are conformal, but all rhumb lines are not plotted as straight lines on the projection. Scale exaggeration in the projection increases away from the central meridian in an east-west direction and the projection is usually limited to zones. These generally extend two or three degrees either side of the central meridian (Robinson et al. 1995). Because of the scale exaggeration, the zones generally have a large north-south extent and a limited east-west extent (Wisconsin State Cartographers Office 2004). In order to minimise scale factor distortion across the zone as a whole, the central scale factor may also be reduced to less than one (*Geodetic Surveying B Study Book* 2007).

The location of the natural origin in Transverse Mercator coordinate systems is at the intersection of a chosen parallel (usually the equator) and the central meridian. Depending on the location of the natural origin, the coordinates derived from the projection may be negative. False coordinates can be assigned to the natural origin to prevent the occurrence of negative coordinates. They can also be assigned to a specific convenient location such as a meridian/parallel intersection (POSC 1997).

To unambiguously define a coordinate system using the Transverse Mercator projection method, a number of parameters are used (POSC 1997). In summary these include:

- Longitude of the natural origin
- Latitude of the natural origin
- Scale factor at the natural origin
- False easting
- False northing

The Transverse Mercator projection is commonly used in the Universal Transverse Mercator (UTM) system. The system consists of 60, six degree mapping zones around the Earth. Zone numbers start at a longitude of 180° east and increase in an easterly direction (Iliffe 2002, p. 77). The system is used for mapping between 84° north and 80° south and the resulting map is known as the universal Transverse Mercator grid (Wolf & Brinker 1994 p. 472).

## 2.3 Distortion in Map Projections

When transforming the surface of the Earth onto a plane, the geometrical relationships on the spheroid cannot be exactly duplicated. The geometrical difference between features on the Earth's surface and their flat representation is known as map projection distortion. The major distortions that occur within map projections relate to the representation of angles, areas and distances (Robinson et al. 1995).

On a sphere, the scale can be thought of as in unity everywhere, apart from at the poles. The process of projection changes the uniformity of scale between points on the sphere, relative to points presented on the plane. This change is referred to as distortion (Brainerd & Pang 1998).

Length distortions between points can also be caused by changes in elevation over which a length is measured. This is because the measured slope distance differs from the distance on a level surface, because of the convergence of plumb lines (Burkholder 1991). A review of the distance and angular distortions that occur in map projections will be presented below.

#### 2.3.1 Distance Distortions

Distance distortions result from the varying of scale along a line between two points in a projection (Brainerd & Pang 1998). If distance is to be represented correctly on a map projection, a uniform scale must occur along the line that is the same as the principle scale on the globe (Robinson et al. 1995). If this does not occur, there will be a difference between the distance on the globe's surface and the plane distance on the resulting plan.

Distortions to the distance resulting from projection vary depending on the location and length of a line within a projection (Burkholder 1993). Distance distortions in a line can be visualised as the difference between the plane distance shown on the plan and the spheroidal distance projected onto the plane (grid distance), see Figure 2.4.



Figure 2.4: Difference between Plane and Projected Grid Distance

The measure of linear distortion that has been mathematically imposed on an ellipsoidal distance, so that they can be represented on the plane, is known as the grid scale factor or point scale factor (Stem 1989). This factor is the ratio of an infinitesimal distance at a point on the grid and its corresponding distance on the spheroid. The grid scale factor varies from point to point in a projection and in conformal projections it is independent of azimuth (*Geodetic Surveying B Study Book* 2007).

Grid scale factor varies along the length of a line and only represents the linear distortion at an infinitesimally small distance on the projection. Therefore, another method must be used to determine the distortions along a line within a projection. A convenient method used to represent the scale factor along a line quantitatively is the line scale factor: also known as the grid scale factor of a line (Allan, Hollwey & Maynes 1968). The line scale factor is the ratio of a plane distance on the grid to the corresponding ellipsoidal distance (*Geodetic Surveying B Study Book* 2007).

Methods used to compute the line scale factor vary and include averaging of the grid scale factors at the end points of a line or using Simpson's rule (Iliffe 2002). The averaging of endpoint grid scale factors is only suitable over small lines. Therefore, the preferred method for accurate computation of the line scale factor is the Simpson rule method (Allan, Hollwey & Maynes 1968).

#### 2.3.2 Angular Distortions

Angular distortions develop because a straight line observed between two points does not plot as a straight line in a projection (Iliffe 2002). This difference arises because the shortest distance between two points on an ellipsoid plots as a curved line concave toward the central meridian when projected onto the mapping plane. Some projections are able to arrange the scale factor distribution to show rhumb lines or arcs of great circles as straight lines. However, no projection can plot the direction of all great circles as straight lines so that the angular relationship between the map graticule and the globe graticule is the same (Robinson et al. 1995).

A grid bearing is the angle between grid north and the tangent to the arc at the point, measured clockwise from north (*Geodetic Surveying B Study Book* 2007, p. 5.24). A plane bearing is the angle between grid north and a straight line drawn between the ends of a projected arc, formed by the projection of the ellipsoidal distance (*Geodetic Surveying B Study Book* 2007, p. 5). The difference between the plane bearing and the grid bearing represents the angular distortion that occurs in projections and is known as the arc-to-chord correction ( $\delta$ ). The arc-to-chord correction is represented as the correction angle in Figure 2.5.



Figure 2.5: The Arc-to-Chord Correction – the difference between a plane line and the projected line (Source: Iliffe, 2002)

The arc-to-chord correction is an angular quantity and differs in magnitude at either end of a line (*Geodetic Surveying B Study Book* 2007). The nature of the arc-tochord correction varies depending on the length on the line, its position in the projection and its position relative to the central meridian (Maling 1992).

#### 2.3.3 Errors due to Elevation

A horizontal distance can be defined as "the chord distance between two plumb lines. The two end points have the same elevation and the chord is perpendicular to the vertical (plumb line) only at the chord midpoint" (Burkholder 1991, p. 105).

A level surface is perpendicular at all points to the local plumb line, but a horizontal plane is perpendicular to a plumb line at a point. Due to the Earth's curvature, the horizontal plane will diverge increasingly from the level surface when moving away from the surfaces coincident point (Burkholder 1991).

Horizontal distances in plane surveying are obtained using the right angle component of slope distance. This is not strictly correct, as horizontal distances are dependent on elevation. This is due to the convergence of plumb lines and any slope distances will contain systematic errors due to convergence (Burkholder 1991). Horizontal distance will therefore vary depending on the height above a datum at which it is measured and the difference in height over which it is measured. This means that distances measured above the height of the datum will be longer than if they were measured on the datum and heights measured below the level of the datum will be shorter than that measured in the datum. See Figure 2.6.



Figure 2.6: The effect that height has on horizontal distances

Heights measured in the field using vertical datums related to the geoid, such as the Australian Height Datum (AHD), must be reduced to ellipsoidal heights before they are reduced to distances related to an ellipsoid. This will involve the application of the relevant geoid-ellipsoid separation value to any heights measured (see Figure 2.7). If this is not done and orthometric heights are used as ellipsoidal heights, an error of approximately 1ppm for every 6.5 meters of geoid-ellipsoid separation will be introduced (Inter-Governmental Committee on Surveying and Mapping 2008).



Figure 2.7: Distance reductions compared to the ellipsoid (Source Inter-Governmental Committee on Surveying and Mapping, 2008)

In determining what is truly a horizontal distance, the definition used above has been found to be adequate for distance reductions when compared to a number of other methods (Burkholder 1991). There are two techniques used to reduce slope distances, including the endpoint elevation and zenith vertical angle methods. The method most suitable for testing in this project is the endpoint elevation method, as it easily allows testing and analysis between points of differing elevation.

#### 2.3.4 Distortion Representation

Map projection distortions can be represented through the use of numerous methods including interactive computer programs, colours, isolines, Tissot's indicatrix & familiar shapes (Mulcahy & Clarke 2001). For this project, the isoline method is suitable for the visualisation of projection distortion, because of its simplicity, ability to display a range of distortions and the ease with which it can be generated.

Isolines are lines that connect points of equal value and can be assumed to be continuous, such as contour lines on elevation maps. The isoline visualisation method uses lines on the map projection to represent the magnitude and distribution of distortion, by connecting points of equal distortion value. Shading in between lines may also be used to help identify the distribution of distortions within the map projection (Mulcahy & Clarke 2001).

The isoline method can display both angular and distance distortions within a projection (Brainerd & Pang 1998). This method quantitatively symbolises map projection distortion and provides absolute values of distortion. One strength of this method is its ability to portray the amount and distribution of distortion. Another is its ability to determine absolute error values (Mulcahy & Clarke 2001).



Figure 2.8: Example of the Isoline method used to display angular deformation (Source: Mulcahy & Clarke, 2001)

# 2.4 Real Time Kinematic Global Positioning Systems (RTK GPS)

#### 2.4.1 Overview

RTK GPS can provide instantaneous, precise positions of a roving unit in the field (Dion 2002). To do this, the system uses two receivers. One is set up on a known point (the base) and the other is free to move around (the rover). For the system to operate, both receivers must be observing at least four of the same satellites

simultaneously (Wolf & Ghilani 2002). The observations at the base are then transmitted in real time, using a radio link or a mobile phone connection to the rover. The roving receiver then uses a double differencing technique, using its own observations and base data to determine its position (Dion 2002).

The global positioning system is susceptible to a number of errors, including clock bias, ionospheric and tropospheric refraction, orbital errors, multipath, operator error, satellite geometry and selective availability (Wolf & Ghilani 2002). Not all of these errors affect RTK GPS, because some are removed through the use of the differencing technique. The remaining errors in the RTK system include multipath, orbital errors, operator error and unmodelled atmospheric errors (Lemmon & Gerdan 1999).

## 2.4.2 Accuracy

The accuracy of RTK GPS is generally stated by the manufacturer of the equipment. By looking at a number of current systems, it should be possible to get an estimate of the accuracies that can be obtained from modern RTK GPS. This can be used as a benchmark for measurement errors within the system. Some current systems and their errors include:

- The Topcon GR-3 receiver is a next-generation multi-constellation receiver. The system has a stated RTK Horizontal vector accuracy between the rover and the base of 10mm + 1ppm (Topcon Positioning Systems Inc 2008).
- The Trimble 5800 is a dual frequency GPS receiver (Trimble Navigation Limited 2006) and the Trimble R8 is a multi-frequency GNSS receiver (Trimble Navigation Limited 2007b). Both systems have a stated kinematic horizontal accuracy of 10mm + 1ppm RMS.
- The Leica ATX1230 GG is a GNSS compatible antenna and has a stated horizontal accuracy in RTK moving mode of 10mm + 1ppm horizontal (Leica Geosystems AG 2007).

It should be noted that these stated accuracies are dependent on various factors such as satellite geometry, observation times and multipath. However, they provide an adequate estimate of the performance of the current systems, to allow the measurement accuracy to be compared to map projection distortion. From this it is concluded that current receivers on the market today are able to obtain a horizontal accuracy in RTK mode of 10mm + 1ppm.

## 2.5 Total Station Measurement Accuracy

#### 2.5.1 Overview

Accuracy of total stations consists of two parts; the accuracy of distance measurement and the accuracy of the instruments angular measurement. These accuracies are generally stated by the manufacturer of the equipment. By looking at a number of current instruments' accuracies, it should be possible to get an idea of the accuracies obtainable with modern equipment.

#### 2.5.2 Accuracy

The Trimble S6 DR300+ instrument is a robotic total station manufactured by Trimble. The instrument has a stated angle measurement accuracy ranging from 2, 3 or 5 seconds. The stated distance measurement accuracy is  $\pm$  3 mm + 2 ppm with a range of 2500m in standard clear conditions using 1 prism (Trimble Navigation Limited 2007b).

The Topcon GPT-9003A/903A is a robotic total station that has a stated angle measurement accuracy of 3 Seconds. The instrument can measure up to 3000m using 1 prism with an accuracy of 2mm + 2ppm at an accuracy setting of fine (Topcon Corporation 2006).

The TPS1200 series of total stations are made by Leica Geosystems. The series has a stated angular measurement accuracy ranging from 1 second in the 1201+ instrument to an accuracy of 5 seconds in the 1205+ instrument. The total station can measure to a distance of 3000m in light haze with an accuracy of 1mm + 1ppm in standard mode (Leica Geosystems AG 2006).

Having noted these accuracies it must also be observed that it is difficult, if not impossible, to set up a total station in the field so that it is in perfect adjustment (Survey Computations A Study Book 2005). This means that in the field, errors will be in excess of the stated measurement accuracy presented above. For this reason the accuracy adopted for this project is the accuracy of the Trimble S6, as this represents the lowest distance measurement accuracy presented and is considered by the author to be a more realistic representation of total station error in the field. The accuracy adopted as the S6 measurement accuracy is a 3 second angular accuracy and a distance measurement accuracy of  $\pm 3 \text{ mm} + 2 \text{ ppm}$ .

### 2.6 Plan Requirements

#### 2.6.1 Cadastral Survey Requirements

In Queensland, cadastral surveys are coordinated under the *Survey and Mapping Infrastructure Regulation 2004* which is under the *Survey and Mapping Infrastructure Act 2003* (SMI Act). The objective of the *SMI Act* is to provide for the development, maintenance and improvement of state survey and mapping infrastructure and the maintenance/improvement of cadastral boundaries throughout the state. Other functions of the act include the coordination and integration of surveying and mapping information, improving public access to mapping information and defining and describing administrative boundaries.

Under sections 6(1) & 7(1) of the SMI Act 2003, the Department of Natural Resources and Water have published written standards for cadastral surveying. These standards, 'The Cadastral Survey Requirements 4.0', contain benchmarks relating to the accuracy of cadastral surveys.

Under section 3.4.2 Measurement Accuracy in Cadastral Survey Requirements 4.0, the accuracy required for a cadastral survey is stated as follows.

"The angular misclosure in a surround or the angular deviation from the adopted meridian must not exceed the lesser of:

- 2.5 times ten seconds of arc multiplied by the square root of the number of angles; or
- 2 minutes.

The linear misclosure in a surround must not exceed—

- 10 mm plus 1 part in 5 000 of the total distance traversed; or
- 20 mm plus 1 part in 2 500, if the survey is in rough or broken terrain; or
- 20 mm plus 1 part in 2 000, if another surveyor's work is included in the surround
- 20 mm plus 1 part in 1 000, if a survey effected before 1890 is included in the surround.

All surveyed lines (e.g. boundary lines, connections) must have a vector accuracy of 10mm + 50 ppm."

If plan dimensions are to be displayed correctly to meet the cadastral survey requirements in Queensland, information must be displayed to an accuracy in the order of 10mm plus 1 part in 5 000 of the total distance traversed. Angular errors must not exceed 2 minutes. It should also be noted that all lines must have a vector accuracy of 10mm + 50ppm.

## 2.7 Knowledge Gaps in Previous Research

Previous research has been presented on visualisation methods for representing map distortions (Mulcahy & Clarke 2001). The calculation of distortions in map projections, particularly those using the Transverse Mercator, has also been presented previously (Stem 1989). Previous work on defining local ground-based map projections is also available (Burkholder 1993).

However, there is a gap in previous literature about the errors that occur in local ground-based coordinate systems, when compared to plane bearings and distances that are required for plans. No previous research was found that compared the accuracy of plane measurements in a local ground-based coordinate system to their associated grid values. Previous studies have been conducted into methods of distortion visualisation, including those by Mulcahy & Clarke (2001) and Brainerd
& Pang (1998), but these methods have not been used to model distortions in local map projections.

There is also a gap in the knowledge of the limitations of using local ground-based coordinate systems. This gap relates to distortions that occur within the projection and how these relate to measurement accuracy and the dimensions required to be shown on plans.

This project proposes to fill these gaps by testing and measuring both the distance and angular distortion that occurs in lines formed in local ground-based coordinate systems. The project will cover distance distortions caused by map projection and differences in projection height. These distortions will be calculated and presented visually, relating their magnitude to measurement accuracies and the required accuracy for cadastral surveys in Queensland.

# 2.8 Conclusion

This review has identified the map projections commonly used for local groundbased coordinate systems and the distortions that occur within map projections. The review has also looked at methods that have previously been used to display map distortions and their relevance to this project.

This review demonstrates that there is a distinct lack of previous testing and testing procedures to define the angular and distance errors in local ground-based coordinate systems. Previous methods used to test map projection distortion have been presented to provide a background on distortion testing and can be adapted for use in local projections.

A short analysis of the measurement accuracy of RTK GPS, total stations and the dimensions required for cadastral surveys in Queensland has also been conducted. This review has defined the benchmark errors as 10mm + 1ppm for RTK GPS measurement accuracy, 3mm + 2ppm & 3 seconds as the total station measurement accuracy and 10mm + 50ppm & 2 minutes as the QLD cadastral surveying requirements. These errors will be adopted as the acceptable errors within a local ground-based coordinate system.

Having reviewed the existing literature regarding local ground-based coordinate systems and map projection distortion, the author proposes to undertake an analysis of the distortions that occur within local map projections. The next chapter will specifically outline the testing procedures that will be used in this project and the procedures for the representation of the results.

# **Chapter 3: Research Method**

# 3.1 Introduction

Chapter 2 provided a background into the development of local ground-based coordinate systems and techniques used to graphically represent distortion. It highlighted the need for research into local ground-based coordinate systems and provided the ideas that have helped to form the basis for a method of testing and results reporting in this project.

This chapter will provide details of the testing methods and procedures used to model and present the distortion errors in local ground-based coordinate systems. The aim of this chapter is to define the testing methods used, in a way that will make them reproducible for future testing and use in the field.

The testing method used involved the construction of two local ground-based coordinate systems, based on the Transverse Mercator and Tangent Plane map projections. Geometrical figures were then constructed within these systems and the effects of map projection distortions on these figures calculated. The testing covered the errors in a system relating to distances, angles, height changes and changes in longitude.

# 3.2 Site Location

Local ground-based coordinate systems can be set up anywhere. For this project the primary site chosen for the establishment of local ground-based coordinate systems was in the Toowoomba area, on the University of Southern Queensland (USQ) Campus. The projections set up for testing had a central point, located at the GPS base station ANANGA, located on the Engineering Faculty building.

ANANGA's position is:

MGA coordinates: Easting: 394586.985m Northing: 6946490.639m Elevation (AHD) 718.663

Geographic Coordinates: Latitude: 27° 36' 05.21483" S Longitude: 151° 55' 54.57113" E Ellipsoidal Height: 760.629

A number of other Transverse Mercator projection sites were also defined at differing longitudes between 150° and 153° East at a latitude of 27° 36' 05.21483" South to test the effect of longitude change on coordinate system accuracy.

# 3.3 General Method

# 3.3.1 Projection Definition

Before using a local ground-based coordinate system, the projection it uses must first be designed and defined. There are a number of different projections that can be used in local ground-based coordinate systems and the errors within a local ground-based coordinate system will vary depending upon which one is chosen for use.

The projections chosen for this project included a tangent plane projection using a gnomonic projection and a Transverse Mercator projection. Each projection was set up with the point ANANGA at the centre of the projection and a project height of 718.663m AHD was used. The projections were defined entirely within the software package Trimble Geomatics Office (TGO) using its coordinate system editor.

The Transverse Mercator and Tangent Plane projection methods were chosen, because they are considered to be the two most common types of projections used in Australia for local ground-based surveys and are therefore of most relevance for testing.

#### 3.3.2 Software

A number of software packages were used in this project to prepare test data, perform coordinate calculations, format output from programs suitable to enter into other software packages and also to calculate the errors within the packages.

#### a. Trimble Geomatics Office

Product Version: 1.63 Build 10 Copyright © 1999-2003 Trimble Navigation Limited

This software was used to set up the local projections and the centre of the project site. The software provided algorithms for the calculation of grid distances, ground distances, the grid scale factor of a line and arcto-chord corrections. The software also provided CAD functionality which was used to generate lines between points in the test figures.

#### b. Terramodel

Product Version: 10.4, ToolPak version 4.71 Copyright © 1988-2003 Trimble Navigation Limited

This software was used to transform the input coordinates of the test figures into geographic coordinates suitable for input into WINTER. It provided functions to define and transform between custom projections. Terramodel was also used to produce diagrams for this project.

#### c. Microsoft Excel

Product Version: 2003 (11.8211.8202) SP3 Copyright © 1985-2003 Microsoft Corporation This is a spreadsheet package produced by the Microsoft Corporation. It provided algorithms that allowed the generation of test figures by calculating their coordinates. Excel also provided a means to format and adjust the output from software packages such as TGO, so they were suitable for input into other software packages used in this project. Excel was also used to process the results from the testing and produce graphs.

#### d. Surfer 8

Product Version: 8.02 Copyright © 1993-2002 Golden Software, Inc

This is a contour and gridding package that was used to prepare the distortion figures from results obtained from TGO. It provided an easy means to generate contour maps from data stored in spreadsheets and allowed a wide variety of display options of the results.

#### e. AUSLIG Windows Interpolation software

Product Version: Rev. 5.08 (Windows 9x/NT) Copyright © 1992-2002 AUSLIG

Windows Interpolation software (WINTER) was used to calculate the geoid ellipsoid separation values for points set up in the model using geographic coordinates calculated in Terramodel.

#### 3.3.3 Defining a Local Ground-Based Coordinate System

This involved defining the local ground-based coordinate systems within TGO. The coordinate systems defined were centred at the USQ campus with ANANGA, the central point of the projection, given a coordinate of 5 000 east and 15 000 north. The project elevation for the coordinate system was set at 718.663m.

The coordinate systems, based on the two projections, were placed on an MGA meridian, with the coordinate system using a Transverse Mercator projection

already on the correct meridian and the plane projection rotated so as to be on the MGA meridian. The procedures to set up the coordinate systems used in this project are discussed in detail in Section 3.4.

#### 3.3.4 Generation of Test figures

This step involved defining a number of geometrical figures to test the grid-toground distance difference errors in distances and arc-to-chord angular errors in the coordinate systems. Errors due to height change and longitude variation were also tested. All the data was generated in software with no actual field survey being completed for this project.

To build the test figures, points were generated in Excel in the desired configuration. Points were assigned a custom feature code to allow the generation of lines from the points to obtain the desired figure. The points were exported from Excel in a CSV format and imported to TGO, where the process feature codes function was used to generate lines from the feature codes.

#### 3.3.5 Distance Error Figure

This step involved generating a figure that consisted of a number of lines emanating from a central point. The lines were generated at bearings of 10 degree separation, starting from north and continuing to a bearing of 350 degrees. Lines were generated and were incremented 100m in length, from 100m to 20km from the central point.

The centre of the figure was assigned the coordinates of 5 000 east and 15 000 north, to position it at the centre of the local ground-based coordinate systems.



Figure 3.1: Figure used to test distance errors

#### 3.3.6 Angular Error Figure

Because angular errors vary with direction and are dependent on the length of a line, the angle at which a line intersects another line and the position of the lines within the map projection, it was decided that creating moving figures for the testing of angular errors was necessary.

The moving figures created consisted of a line of fixed length, position and bearing and another 'moving line' (refer to Figure 3.2). The moving line was generated so as to have a constant length and move relative to the fixed line at 10 degree differences in bearing. The result of using this moving line is that the angle between the two lines change, so errors can be tested relative to the size of the angle (see Figure 3.2).



Figure 3.2: The Figure used to test the angular errors

#### 3.3.7 Height Error Figure

Figures to test distortions due to height were constructed using Excel and consisted of a number of lines of changing elevation. Three figures were constructed, each consisting of a 2km line that extended 1km east/west either side of the central point in the projection. To construct this, a line was drawn between points at the end of each line that were incremented in height up or down in 0.5m steps.

The figures generated included a figure in which the elevation of the east end of the line was decreased, while the west end's elevation was increased by the same amount. The next figure involved raising both ends of the line at equal increments above the projection height. The final figure involved decreasing, in equal amounts, the level of both ends of the line below the level of the project site.

### 3.3.8 Moving Longitude Error Figure

For this figure, Microsoft Excel was used to construct a simplified test figure, based on the distance testing figure. The figure consisted of a number of lines extending from the central point in east/west and north/south directions at increments of 100m. The lines extended 40km in the east/west direction and 50km in the north/south direction (see Figure 3.3).



Figure 3.3: The Figure used to test longitude change

# 3.4 Projections

#### 3.4.1 Transverse Mercator Projection

The Transverse Mercator projection method involved selecting a site and then choosing the corresponding MGA mapping zone. The MGA projection was then modified to fit the local area by changing the central scale factor of the projection so that it matched the average height of the project site.

The method in Trimble Geomatics Office to set up a local ground-based coordinate system using a Transverse Mercator projection is as follows:

- 1. Open the Coordinate System Manager
- 2. Copy the GDA Zone relating to the site to a site folder
- 3. Rename the GDA Zone to something meaningful
- 4. Edit the Zone File and under the Projection Tab change:
  - a. In positive coordinate direction check North & East
  - b. Set Central Latitude =  $-0^{\circ} 00' 00'' N$
  - c. Set Central Longitude = 153° 00' 00" (centre longitude of chosen MGA zone)
  - d. Set False Easting
  - e. Set False Northing
  - f. Set Central Scale Factor

Calculating the Central Scale Factor:

- 1. Find the MGA point scale factor for the central point of the site
- 2. Calculate the Datum Scale Factor (DSF) using

$$DSF = \frac{R}{(R+h)}$$

Where: R = Geometric Mean Radius of Curvature at the central point h = Ellipsoidal Height of the site 3. Calculate the Combined Scale Factor using

Combined Scale Factor = MGA point scale factor × Datum Scale Factor

4. Calculate Central Scale Factor using

Central Scale Factor = MGA Central Scale Factor / Combined Scale Factor

Once this had been completed and saved in the coordinate system editor, TGO was used to create a new job. The coordinate system created (using the coordinate system editor) was then selected (under coordinate system settings) as the coordinate system in project properties. The local site settings were then modified by entering the local ground-based coordinate system coordinates and the elevation of the central point of the projection. The resulting computed WGS84 latitude and longitude and ellipsoidal height for the central point of the site were then checked against prior calculations of the known central point. These procedures are reproduced in Appendix F for reference.

#### 3.4.2 Plane Projection

The plane projection method used established a plane projection with the centre of the projection being coincident with the central point of the site. A coordinate system was then set up on this projection and rotated so as to be on an MGA meridian. The plane projection used was the standard projection provided in TGO, which projects from a perspective at the centre of the ellipsoid.

The following procedure was used to establish local ground-based coordinate systems using a plane projection in TGO:

- 1. Open Coordinate System Manager
- 2. Edit Add Coordinate System Plane
- 3. Name the Projection
- 4. Select a Datum either WGS84 or ITRF
- 5. Select Geoid Grid Model
- 6. Select Ausgeoid 98
- 7. In the Projection screen change:

- a. Positive coordinate direction to North and East
- b. Central Latitude to the Latitude of the central point of the site
- c. Central Longitude to the longitude of the central point of the site
- d. Set the height above the ellipsoid to the ellipsoidal height of the site
- e. Set the height above the geoid to the elevation on the site
- f. Set rotation angle (in seconds) to the MGA grid convergence of the central point to bring the coordinate system onto MGA meridian
- g. Set False Northing
- h. Set False Easting
- i. Set the Scale Factor to 1

In step (f), the grid convergence was calculated using the Redfearn spreadsheet available from Geoscience Australia.

Once this projection had been created, TGO was opened and a new job created. The coordinate system of this new job was then edited in 'project properties' and the plane projection, created previously, selected. The local site settings were then edited and the local ground-based system coordinates and the elevation of the central point entered. The resulting computation of WGS84 latitude and longitude and the ellipsoidal height of the central point of the site were then checked against prior calculations. These procedures are reproduced in Appendix G for reference.

# 3.5 Validation and Testing

#### 3.5.1 Distance Testing

The process of testing the distance errors within a projection involved using TGO to calculate the grid and ground distances along the lines of the distance error testing figure. To do this, a custom report was then created which consisted of the To Point, From Point, Grid Scale Factor, Grid Distance, Ground Distance and Ellipsoidal Distance. The report was then formatted into a spreadsheet format using Excel.

The spreadsheet was then used to calculate the difference between grid and ground distances along all the lines in the figure and this difference was assumed to be the error due to projection. This error was then compared to measurement and plan

accuracies until the grid/ground distance difference exceeded the required accuracies. At this point it was concluded that this distance was the maximum that could be travelled from the central point of the figure in each local ground-based coordinate system.

#### 3.5.2 Angle Testing

The process of testing the angular errors involved using TGO to calculate the arcto-chord correction for each of the lines in the distance testing figure. Again, to do this, a custom report was set up in Excel to output the arc-to-chord correction for each line. The Transverse Mercator projection results were also verified using a modified spreadsheet based on the GRIDCALC spreadsheet available from Geoscience Australia.

TGO was used to calculate the arc-to-chord correction for each of the lines in the distance testing figure. The corrections for these lines relate to the bearing of the lines and not the angle between the lines. To test the error in the angle, the angular testing figure was constructed using lines from of the distance testing figure. The lines were held at a fixed length and the arc-to-chord correction for the moving line was added or subtracted to the arc-to-chord correction of the fixed line, depending on its location in the projection. The results of this process are the angle between the two lines and the error in this angle, due to projection.

The GRIDCALC spreadsheet was modified to make computations on the local Transverse Mercator projection used at the site. The spreadsheet was then modified to allow multiple lines to be computed at once. This involved using the local coordinates for the start and end of each of the lines, which were converted to MGA coordinates for computations. The arc-to-chord correction for each of the lines was then extracted from the spreadsheet and compared to the results from TGO, to provide a check on the computations.

#### 3.5.3 Height Change Testing

The affect of the height on distances represented in the system was calculated by importing the height change testing figure into TGO. A custom report was then run

and output the height of the line, the grid distance and the ground distance. The difference between the grid and ground distances of the line was then calculated and taken to be the error, due to height change in the line. The error in each of the lines in the figure was then compared to the acceptable measurement accuracies and a distance above or below the projection grid determined, when the line error exceeded the measurement accuracy.

Each of the three height testing figures were tested using the procedure mentioned above and the results stated as a distance above or below the projection level that can be travelled.

#### 3.5.4 Longitude Variation Testing

The longitude variation testing involved setting up a number of projections at the same latitude and differing longitudes. The latitude chosen was the same as the Toowoomba test site, with the longitudes being varied every 30 minutes from 150 degrees to a longitude of 153 degrees. Another site was also set up at 152° 45' to help clarify the results. These longitudes were chosen, as they represent the western side of MGA zone 56. This is because one of the local ground-based coordinate systems used in this project used a Transverse Mercator projection, based on MGA zone 56.

When testing local ground-based coordinates systems based on a Transverse Mercator projection, it was necessary to set up a new site for every longitude. This involved the re-computation of the central scale factor for each projection at each site. For the plane projection, however, no re-computation of the scale factor was necessary, but the MGA gird convergence for the rotation angle needed to be computed and applied as a rotation at each site. For both projections, the geoid/ellipsoid separation value for the central point of projection required to be computed using WINTER, to maintain a constant elevation for each of the projections.

The testing of the projection sites involved using TGO to produce a report stating the grid and ground distances for each of the lines in the test figure. The difference between the grid and ground distances were then assumed to be projection error and were compared to the measurement accuracies to come up with a distance from the central point in a north/south and east/west direction that can be travelled before measurement accuracy is exceeded.

#### 3.5.5 Error Modelling

The modelling used varied depending upon the type of testing completed and the results from this testing. A combination of contour maps (isoline method), graphs and tables were used to present and model the errors in local ground-based coordinate systems.

The modelling of errors in the distance testing figure was achieved using the Surfer 8 software package. The software was used to create a contour map of the difference between the projection error of a line and a number of measurement accuracies. To complete the modelling, the endpoint of each of the lines in the distance testing figure was assigned coordinates in the local ground-based coordinate system. A height value was then assigned as the difference between measurement accuracy and grid-to-ground difference error in the line. The resulting grid was then contoured and coloured to graphically display the error in the local ground-based to display the maximum distance from the central point at particular bearings that can be travelled before measurement accuracy is exceeded.

Because of the number of variables involved, angular errors could not be modelled in such a fashion as the distance errors. It was therefore concluded that the most appropriate way to represent these errors was through the use of a number of graphs. The graphs were constructed with respect to a particular fixed line (fixed distance and bearing) and a moving line of a fixed length. The combined arc-tochord correction for the angle between the two lines was then plotted on the y axis and the bearing of the moving line was plotted along the x axis.

Horizontal distance errors due to elevation change were presented using tables. This allows easy quantitative representation and analysis of the errors. This is considered the simplest way to represent these errors so as to provide the most information to the user.

For the longitude change testing, the distance from the central point that can be travelled in an east/west or north/south direction before measurement accuracies and plan requirements were exceeded was calculated. These distances were then averaged in the north/south and east/west directions and used to produce a graph depicting longitude verses the distance travelled before the measurement accuracies are exceeded. These graphs were produced for local ground-based coordinate systems using both the Transverse Mercator and plane projections.

#### 3.5.6 Accuracy Assessment

The accuracy of the data has been limited by the accuracy possible to be output from the TGO software. The format of the output data consisted of a custom report from the software. The accuracies in this report are unable to be changed in the software and limited the level of accuracy possible for the determination of distances and angles in this project. The accuracies that were achieved from TGO included distances measured to the millimetre and angles measured to 5 decimal places of a second. As such the angular accuracy output from TGO was adequate for testing purposes, however a better distance accuracy would have resulted in more precise results.

The method of creating lines within TGO also affected the accuracy of the data used in this project. The method of creating the lines using field codes resulted in the lines having a direction running from the outside point into the centre of the testing figures. The direction of this line has resulted in the calculation of the arc-to-chord correction for the line, being from the outer end of the line towards the centre of the figure. This problem was able to be fixed in the Transverse Mercator projection with the use of spreadsheets to check computations. However the reverse arc-tochord corrections for the plane projection were not able to be computed on mass, due to the absence of suitable spreadsheets to perform checks. After carrying out a number of checks and due to the small nature of the difference between the forward and reverse arc-to-chord corrections of a line, it was determined that the use of the reverse arc-to-chord correction for the plane projection would be of a suitable accuracy for the purpose of this project.

# 3.6 Conclusion

This chapter described the process and techniques used for testing in this project. It defined the selection of the primary test site for the project, as centred at the USQ GPS base station ANANGA. The test figures used for the testing in this project were also outlined and described in this chapter. The procedures for setting up local ground-based coordinate systems in TGO, based on Transverse Mercator and Plane projections, were also outlined. The methods used for calculating errors and determining their magnitude relative to GPS and Total station measurement accuracies were also discussed in this chapter. The process and methods for presenting these errors using the isoline method, graphs and tables were also outlined.

Since not all consumers have access to the same software packages that have been used in this project, and each package operates in a difference way, the use of the procedures mentioned above may not be explicitly applicable to all users. However, they will provide useful information to users about the setup of a local groundbased coordinate system.

# **Chapter 4: Results**

# 4.1 Introduction

This chapter presents the results of the testing of the accuracy of a number of local ground-based coordinate systems based on Transverse Mercator and Plane projections. It presents a number of graphs, tables and maps representing the errors caused by the process of projection as well as the effect that the location of the test site has on accuracy.

The aim is that this chapter will give the reader a better understanding of the limitations of local ground-based coordinate systems so that they are able make more informed choices regarding their use and possible applications.

The presented results cover the four areas of testing, including distance and bearing errors due to projection, as well as the errors caused by height variation away from the project height and the effect of longitude on site accuracy. The results have been divided into two sections to provide clarity: Transverse Mercator; and Plane Projection.

# 4.2 Explanation of Results Shown

The results presented in this chapter make reference to a number of measurement values and accuracies. These refer to the adopted measurement accuracies and required plan dimensions that were discussed in Chapter Two. To briefly restate these accuracies, they are:

- RTK GPS Measurement accuracy: 10mm + 1ppm
- Total Station Measurement accuracy: 3mm + 2ppm and 3 seconds angular
- QLD Cadastral Surveying Requirements: 10mm + 50ppm and 2 minutes angular

### 4.3 Transverse Mercator

The results shown below are for local ground-based coordinate systems, based on the Transverse Mercator projection. The analysis of the results has been divided into sections depending on the tests that have been completed.

#### 4.3.1 Distance Testing

The results for the distance testing figure placed at the centre of the site are shown below. The results are presented as diagrams. More comprehensive data relating to the maximum distance that can be accomplished from the figure's central point before the measurement accuracies are exceeded is presented in Appendix B.



Figure 4.1: Grid/Ground distance difference from the central point of the site

Figure 4.1 depicts the difference between the grid and ground distances in the distance error testing figure. The errors are for lines emanating from the central point of the site and displayed using the isoline method. All dimensions are shown in metres.



Figure 4.2: Area where projection distortion does not exceed RTK GPS measurement accuracy

Figure 4.2 depicts, in green, the area where grid/ground distance difference does not exceed RTK GPS measurement accuracy. The site is centred at 0, 0 and the distance testing figure extends from the centre of the figure to a distance of 20km. All dimensions are shown in metres.

#### 4.3.2 Angle Testing

Below are a number of graphs presenting the error in the angle between two lines that emanate from the centre of the projection. The angular error was calculated using the angle testing figure described in Chapter Three. The error presented represents the combined arc-to-chord correction of the two lines in the testing figure and was found by adding their arc-to-chord corrections.



Figure 4.3: Angular Error in the angle testing figure, using a fixed line of 2000m at 30 degrees and a 3500m moving line

In Figure 4.3 the fixed line was held at a bearing of 30 degrees and a distance of 2000m and the moving line was held at a distance of 3 500m.



Figure 4.4: Angular Error in the angle testing figure, using a fixed line of 8000m at 110 degrees and a 5000m moving line



Figure 4.5: Angular Error in the angle testing figure, using a fixed line of 10 000m at 0 degrees and a 10 000m moving line

In Figure 4.4 the fixed line was held at a bearing of 110 degrees and a distance of 8000m and the moving line was held at a distance of 5000m. In Figure 4.5 the fixed line was held at a bearing of 0 degrees and a distance of 10 000m and the moving line was held at a distance of 10 000m.

#### 4.3.3 Height Change Testing

Presented below is the distance that can be travelled away from the project height of a Transverse Mercator projection before the measurement accuracies are exceeded. Testing was only carried out to a distance of 50m above or below the height of the project coordinate system.

Measurement Accuracy	Height from project site
10mm + 1ppm	41.5m Above
	37.5m Below
3mm + 2ppm	22.5m Above
	18.5m Below
10mm + 50ppm	Not exceeded within 50m
	of project height

Table 4.1: Transverse Mercator Height Change Testing Results

### 4.3.4 Site Longitude Testing

Below is shown the maximum distance that can be travelled from the project's central point before the measurement accuracies are exceeded for a number of projects set up at differing longitudes. The distance from the central point presented is an average of the distance in both directions. The cadastral survey requirements are not presented in Figure 4.6, because they were not exceeded within the 40km distance from the central point tested.



Figure 4.6: Effects of Longitude on site accuracy in an east/west direction

The results for the north/south testing limits of the projection are not included, because the Transverse Mercator projection error did not exceed the measurement accuracies within 50km of the centre of the site, which was the limit for testing.

# 4.4 Plane Projection

The results shown below are for local ground-based coordinate systems based on a plane projection and rotated onto an MGA meridian. The analysis of the results has been divided into sections depending on the tests completed.

#### 4.4.1 Distance Testing

The results for the distance testing figure that was placed at the centre of the site are shown below. For a more complete set of results consult Appendix C. Appendix C contains a table of the distances from the central point at varying bearings, before measurement accuracies are exceeded.



Figure 4.7: Grid/Ground distance difference from the central point of the site

Figure 4.7 shows an isoline map of grid/ground distance difference in the distance testing figure. All errors shown are for lines emanating from the central point of the site. All dimensions shown are in metres.



accuracy

Figure 4.8 shows, in green, the area where the grid/ground distance difference does not exceed RTK GPS measurement accuracy for lines emanating from the central point of projection. All dimensions are shown in metres.

#### 4.4.2 Angle Testing

Below are a number of graphs presenting the error in the angle between two lines that emanate from the centre of the projection. The angular error was calculated using the angle testing figure described in Chapter Three. The error presented represents the combined arc-to-chord correction of the two lines in the testing figure and was found by adding their arc-to-chord corrections.



Figure 4.9: Angular Error in the angle testing figure, using a fixed line of 2000m at 30 degrees and a 3500m moving line



Figure 4.10: Angular Error in the angle testing figure, using a fixed line of 8000m at 110 degrees and a 5000m moving line



Figure 4.11: Angular Error in the angle testing figure, using a fixed line of 10 000m at 0 degrees and a 10 00m moving line

# 4.4.3 Height Change Testing

Presented below is the distance that can be travelled away from the project height of a plane projection before the measurement accuracies are exceeded. Testing was only carried out to a distance of 50m above or below the height of the project coordinate system.

Measurement Accuracy	Height from project site
10mm + 1ppm	39.5m Above
	39.5m Below
3mm + 2ppm	20.5m Above
	20.5m Below
10mm + 50ppm	Not exceeded within 50m
	of project height

Table 4.2: Plane Projection Height Change Testing Results

#### 4.4.4 Site Longitude Testing

Below is shown the maximum distance that can be travelled from the project's central point before the measurement accuracies are exceeded for a number of projects set up at differing longitudes. The distance from the central point presented is an average of the distance in both directions. It should be noted that the Queensland cadastral survey requirements are not presented, because they were not exceeded within the 40km east/west and 50km north/south tested in this project.



Figure 4.12: Effects of Longitude on site accuracy in an east/west direction



Figure 4.13: Effects of Longitude on site accuracy in a north/south direction

# 4.5 Conclusion

This chapter has presented the results from the testing carried out in this project. The limitations of local ground-based coordinate systems were presented, regarding grid-to-ground distance errors and angular errors due to projection, as well as the effect of height change above and below the level of the projection and the effect of longitude on site accuracy. The intention of the chapter was to inform the reader of the magnitude of the errors within these systems and the best application for the systems, based on these errors. In doing this, the distance and angular accuracy and the effect that height change and longitude has on the accuracy of the site has also been presented.

The results presented in this chapter can be summarised as follows: The accuracy of local ground-based coordinate systems varies depending on the projection method used. The effect that the projection method has on distance errors influences the distance that can be travelled from the central point of the site before RTK GPS measurement accuracy is exceeded, by distortion error in the line. These distances have been presented graphically and range from approximately 19km from the central point in the plane projection, to a distance of approximately 3km east/west

in the Transverse Mercator projection. Angular errors have also been presented and differ depending on the projection method. The maximum combined angular error in each projection method ranges from approximately 0.1 of a second in the plane projection to approximately 5.3 seconds in the Transverse Mercator projection.

External site influences, including longitude and height above or below the project site height, have also been presented as having varying affects on the accuracy of local ground-based coordinates systems, depending on the projection method used. Errors due to height above or below the projection level before RTK GPS measurement accuracy is exceeded, range from a uniform distance of 39.5m above or below in the Plane projection to unequal distances of 41.5m above & 37.5m below in a Transverse Mercator projection. Site accuracy due to longitude can also be seen to vary in the Transverse Mercator projection. The distance before RTK GPS measurement accuracy is exceeded from the central point the of site ranges from approximately 17.4km to 1.8km. However, in a plane projection this distance remains constant at approximately 19km.

The figures and tables presented in this chapter are discussed in the following chapter. These discussions focus on the information presented in sections 4.3 and 4.4 and explain the implications of the results presented in this chapter.

# **Chapter 5: Analysis and Discussion**

# 5.1 Introduction

The purpose of this chapter is to provide an in-depth analysis of the results presented in the previous chapter. The discussion will focus on the results presented in Sections 4.3 and 4.4, with the focus of discussion on irregularities identified in the results, and an explanation of the results focusing on four main areas:

- The effects of distance from the central point of the map projection
- Angular distortions
- Effects of height change on distance accuracy
- Effects of longitude on site distance accuracy

The aim of this chapter is to explain and interpret the results from the testing conducted in this project. From this interpretation, it is expected that the reader will gain an understanding of the magnitude of errors in the systems tested and the limitations when using such systems, with respect to measurement accuracies and plan requirements.

The analysis in this chapter has been divided into the following two main sections: the Transverse Mercator projection & the Plane projection. A comparison between projection methods in relation to their distance and angular accuracy will be presented and recommendations as to their suitable uses made. These recommendations will then be demonstrated through the use of a practical validation.

# 5.2 Transverse Mercator Projection

#### 5.2.1 The effects of distance from the central point

Errors in the distances in Figures 4.1 & 4.2 can be seen to increase with distance from the central point of the site in local ground-based coordinate systems based on Transverse Mercator projections. The errors presented in Figure 4.1 can also be seen to be dependent on the bearing of the line in the projection. As expected, because the scale error in Transverse Mercator projections increases away from the central meridian of the site in an east/west direction, the Transverse Mercator projection performs poorly in the east/west direction. This is shown with the distance from the central point of the projection at the Toowoomba test site before RTK GPS measurement accuracy is exceeded being 3km. However, in a north/south direction, the RTK GPS measurement accuracy of the system was not exceeded within the 20km tested. The results of testing using the longitude test figure indicate that the accuracy is still not exceeded at a distance of 50km from the central point (see Section 5.2.4). This means that it is possible to travel long distances strictly in a north/south direction using Transverse Mercator projections as long as the shape of the site limits the east/west extent severely.

The Transverse Mercator projection can also be seen to produce ground distances on the eastern side of the projection that are longer than grid distances. Ground distances on the western side of the projection, however, are shorter than grid distances. This is represented in Figure 4.1, with negative grid-ground distance differences on the eastern side and positive on the western side of the site. The rationale behind this is that the level project site and Transverse Mercator projection grid are coincident at the site's central meridian. On the eastern side of the central meridian, the level surface diverges away from the projection grid in an upwards direction, while on the western side, the level surface diverges in a downwards direction. Because of the convergence of plumb lines, distances measured below the projection surface, as on the western side of site, will be shorter than lines measured above the projection surface, as on the eastern side.

An interesting effect of the Transverse Mercator projection was observed regarding the accuracy of distances in the projection. It was found that it is possible to go further from the central point of the site towards the central meridian of the projection, than it is to go in a direction away from the central meridian. This effect can be seen to occur at the Toowoomba site, with the distances presented in Appendix B being longer on the eastern side of the site than the western side. It is proposed that the source of this difference is because the difference between the level surface and the Transverse Mercator grid is diverging at a greater rate on the western side than the eastern side of the grid. This same effect is discussed in Section 5.2.4, under site longitude change.

#### 5.2.2 The Angular Distortions

The effect that a Transverse Mercator projection has on the angular error is presented in Figures 4.3, 4.4 and 4.5. It can be seen that these errors vary depending on the length of the lines used and the initial starting bearing at which the fixed line is held. These errors can be seen to follow a sinusoidal pattern, regardless of the position of the fixed line. It can be seen that the angular error between the two lines is generally at a minimum when the angle between the lines is 180 degrees. The slight deviation from 180 degrees before the error is at a minimum can be put down to the fact that the lines in Figures 4.2 and 4.4 have different lengths and therefore produce slightly different magnitudes of arc-to-chord corrections. Therefore, their corrections will not be equal at 180 degrees. In Figure 4.5, where the lines are the same length, there is no such problem. The combined arc-to-chord correction is represented in Figure 5.1 with the positive correction for the northern line being the same as the negative correction for the southern line. Therefore, the situation that occurs in Figure 4.5 where there is a zero correction at 180 degrees.



Figure 5.1: Sign of arc-to-chord corrections in a Transverse Mercator projection

In Figures 4.3, 4.4 and 4.5, the combined arc-to-chord correction at the bearings of 90 degrees and 270 degrees is equal to the arc-to-chord correction for the single fixed line. This is because the arc-to-chord correction for the moving lines at these bearings is zero, as it is perpendicular to the central meridian. This can be seen in Figure 5.1, with there being no arc-to-chord correction for the line perpendicular to the central meridian.

From the results, it can be seen that the variation of the angular errors between test figures ranges from a maximum of 5.3 seconds in Figure 4.5 down to a maximum of 1.4 seconds in Figure 4.3. It is unlikely that distances of 10 000m will be measured in a local ground-based coordinate system, based on a Transverse

Mercator projection, in all directions, because of the distance errors discussed in Section 5.2.1. It is therefore considered that the errors presented in Figure 4.3 are a more realistic representation of the possible errors that will be encountered. Another reason for adopting the errors presented in Figure 4.3 is that the distances used are approximately that of the maximum ranges of the total stations, presented in Section 2.5.2. This will therefore allow direct comparison with their measurement accuracies, without the need to consider multiple setups.

When comparing the angular accuracy, the two standards that have been defined are the total station angular measurement accuracy and the cadastral surveying requirements in Queensland. The total station angular measurement error is 3 seconds and the cadastral survey requirements are that the angular errors must not exceed 2 minutes. In all the testing that has been conducted, no errors have exceeded 2 minutes, so this requirement does not hinder the use of local groundbased coordinate systems. However, the angular measurement accuracy of a total station of 3 seconds, that was adopted, will have limiting effects on the use of local ground-based coordinate systems. If lines of 10km, measured in the projection were used, then the angular measurement accuracy would be exceeded. However, if a maximum distance from the central point of 3km is adopted, the angular measurement error does not exceed the measurement accuracy of a total station.

#### 5.2.3 The effects of Height Change on Accuracy

The maximum distance that can be travelled above or below the project site before a number of measurement accuracies are exceeded is presented in Table 4.1. More detailed results are also presented in Appendix D. Results from the testing indicate that distances measured at a ground surface above the level of the local grid will be longer than that portrayed in the local grid. Distances measured at a ground surface below the level of the local grid also display a difference to distances measured in the local grid, with the ground distances being shorter. This is due to the convergence of plumblines, which means that horizontal distances are dependent on elevation. These results correspond with the evidence presented in Section 2.3.3.

However, this does not explain the results in this project, where there is a difference between the height before measurement accuracy is exceeded in a projection above
the level of the projection and below (refer to Table 4.1). The results indicate that it is possible to travel further in an upwards direction than in a downwards direction. Initially, it was thought that the project site was the cause of the problem. However, a check was performed by setting up another project site at the central meridian of the zone and similar results were obtained. A closer inspection of the results has revealed that there is only a 0.002m difference between the error in the length of the lines. Because of the computation process, this has caused the difference to occur in heights that can be travelled. It is proposed that this 0.002m difference can be accounted for by the distortions that occur in the projection and the rounding errors that have occurred in the process of exporting the data from TGO.

## 5.2.4 The Effects of Longitude on Site Accuracy

In Figure 4.6, the distance that can be travelled from the projection's central point does not display a linear relationship in regard to longitude. This can be explained by the fact that as one goes towards the edge of the projection zone, the angle between the projection grid and the level surface changes. This can be seen in Figure 5.2 as the difference between the angles  $\phi_1 \& \phi_2$ , where the local grid intersects the level surface.



Figure 5.2: The Effect of Longitude Change

To reduce the distances on the level surface to distances on the local grid, the level surface (ground) distance is projected onto the local grid. The length of the line, when projected, will therefore be affected by the angle between the local grid and the level surface. This can be better understood by imagining an infinitely small section of the level surface, so as it can be represented as a straight line as shown in Figure 5.3.



Figure 5.3: Close-up of a local grid intersecting the level surface

The length of line X on the level surface can be portrayed on the Local Grid through the use of the cosine relationship. This means that the relationship between the local grid distance and the level surface distance is related to a trigonometric identity and is therefore related to the angle as well. This trigonometric relationship can explain the non-linear relationship between the distance travelled before accuracy limits are exceeded and the longitude of the site shown in Figure 4.6.

A results figure has not been included for the distance that can be travelled in a north/south direction from the central point in the Transverse Mercator projection, as the measurement accuracies were not exceeded within the 50km limit of the testing. The accuracy of the projection as the distance increased actually became proportionally better than the measurement accuracies chosen and resulted in divergence between the two values. Theoretically, if this were to keep continuing, the projection would be well under the measurement accuracy for distances upwards of hundreds of kilometres in a purely north/south direction. However, it must be noted that this would not result in a practical site design, as the east/west width restriction would need to be very close to the north/south direction, resulting in an unusable, narrow site.

## 5.3 Plane Projection

#### 5.3.1 The effects of Distance from the Central Point

The grid-to-ground distance difference errors in Figures 4.7 and 4.8 can be seen to increase with distance from the central point of the site in local ground-based coordinate systems using a plane projection. Unlike the Transverse Mercator projection, the extent of these errors appears to be symmetrical around the central point of the projection, with direction not affecting the errors significantly. At the Toowoomba test site, it is possible to go approximately 19km from the central point of the projection in any direction before the RTK GPS measurement accuracy is exceeded (see Appendix C for more detailed information). It is also worth noting that total station measurement accuracy and the QLD cadastral surveying requirements were not exceeded within the 20km distance from the central point of the projection that was tested. However, in the testing carried out for the effect of longitude on site performance (see Section 5.3.4), it was found that the total station measurement accuracy was exceeded at an average distance of 22 750m in a north/south direction and 22 800m in an east/west direction, from the central point of the site.

Because a plane projection uses a plane that is brought into contact with a point on the ellipsoid and the ellipsoid diverges away from this plane in all directions evenly, the symmetrical error seen in these results can be seen to reflect this even divergence. This means that for local ground-based coordinate systems, using a plane projection the shape of the accurate area of the site will approximate a circle around the central point, no matter where the system is located. The negative values in Figure 4.7 also reflect this divergence, indicating that the ground distance is greater than the grid distance.

#### 5.3.2 The Angular Distortions

The effect that a plane projection has on the accuracy of angles in local groundbased coordinate systems is presented in Figures 4.9, 4.10 and 4.11. It can be seen that the errors in the angles are dependent on the length and direction of the line in the projection. The direction at which the fixed line was held can be seen to have little effect on the error in the angles between the lines. This is because, in a plane projection, the arc-to-chord errors from the central point of the projection are so minute that they have little effect on the total error between the lines. This can be seen in the sinusoidal effect that the error appears to display. This effect is caused by the fact that the arc-to-chord corrections for each of the cardinal directions in the projection are zero. The errors increase from the cardinal directions to a point midway between the cardinal directions, from where they decrease to reach zero at the next cardinal direction.

The slight offset from a perfect zero error in the cardinal directions can be seen in Figures 4.9, 4.10 and 4.11. These can be explained through two effects that the software has had on processing. The first effect is the slight errors that TGO has produced in calculating the arc-to-chord corrections for the lines in the plane projection (see Section 3.5.6). This error can be seen in Figure 4.11 as the slight offset from zero at the start of the line. The second effect is that Excel has fitted the curved line in the figures to provide a best fit to the data. This has resulted in a slightly skewed presentation of the lines, which is evident when they cross the axis.

The total angular error in the plane projection can also be considered minor, with none of the errors presented in Figures 4.9, 4.10 and 4.11 exceeding an error of 1 second, even over a 10km line. This can be explained by the fact that the lines in the testing figure emanate from the central point of the projection, which limits the distortions in the directions in the lines. However, if the lines did not emanate from the central point of the projection, which limits the manate from the central point of the projection, these errors are likely to increase.

When comparing the angular accuracy to the two standards; the total station angular measurement accuracy and the Queensland cadastral surveying requirements, it can be seen that the plane projection's combined arc-to-chord error does not exceed the 3 seconds or 2 minutes defined. This means that for the use of local ground-based coordinate systems it can be considered that the angular accuracy does not greatly hinder the use of plane projection based systems.

#### 5.3.3 The effects of Height Change on Accuracy

The maximum distance that can be travelled above or below the project site before the selected measurement accuracies are exceeded is presented in Table 4.2 and more detailed information is available in Appendix E. Results again indicate that distances measured at a ground surface above the level of the local grid will be longer than portrayed in the local grid. Further, distances measured below the level of the local grid will be shorter than those measured in the local grid. This is due to the convergence of plumblines between which the horizontal distances are measured.

Unlike the Transverse Mercator projection, the distance that can be achieved above and below the level of the local grid was consistent and reflects the constant divergence of plumblines that occur above and below the level of the test site.

#### 5.3.4 The Effects of Longitude on Site Accuracy

As displayed in Figures 4.12 and 4.13, the distance that can be travelled from the central point in plane projection is not affected by the longitude of the site. This is because, unlike the Transverse Mercator projection, a separate projection surface is set up for each site and the same projection is not scaled to fit each site (see Figure 5.4). Each of the plane projections diverge from the level surface from the centre of each site at a constant rate. This results in the distance that can be travelled from the centre of each site being approximately equal.



Figure 5.4: Setting up Plane Projections at differing Longitudes

In Figures 4.12 and 4.13 there can be seen to be small variations in the length that can be travelled from the centre of the site, due to longitude change. The magnitude of these errors is minimal and can be explained by the method used to set up the sites. The sites have been defined at a constant elevation. However, the height of the site above the ellipsoid and the height above the level surface (geoid) change relative to each other. This change between the level surface elevation and the ellipsoidal height affects the computation of grid distances relative to the reference ellipsoid and has resulted in the perceived small changes, due to longitude. The irregularity of the geoid, compared to the ellipsoid, also explains the random nature of the errors between sites.

## 5.4 Comparison between Projection Methods

It was found that the distance that can be travelled from the projection's central point, the angular errors and the shape of the accurate area within local ground-based coordinate systems changed depending on the projection method used. The effect that longitude has on the accuracy of local ground-based coordinate systems and the effect of height changes are also dependent on the projection method used.

Local ground-based coordinate systems based on the Transverse Mercator projection record a shorter distance that can be travelled from the central point of the site in an east/west direction than that which can be achieved using local ground-based coordinate systems based on a plane projection. This difference is quite significant, with the distance that can be travelled from the centre of the Toowoomba test site being 3km in the Transverse Mercator projection, as opposed to 19km in the plane projection before RTK GPS measurement accuracy is exceeded. The Transverse Mercator projection, however, far outperforms the plane projection in a strictly north/south direction. In the testing, the Transverse Mercator projection does not exceed any of the measurement accuracies within 50km of the centre of the site. However, the plane projection exceeds the RTK GPS measurement accuracy at a distance of approximately 19km. The difference in distribution of these errors between projection methods has resulted in a difference in the shape of the accurate areas for both projections. It can be seen in Figures 4.2 and 4.8 that the accurate area of a Transverse Mercator projection is long and narrow in a north/south direction, whereas the accurate area of a plane projection forms a circular area around the central point of the site.

The angular errors in both of the projections are of such a minor magnitude that neither exceeded the measurement accuracy of a total station before the distance of the line required had already exceeded the distance measurement accuracies. The plane projection recorded, on average, a lower angular error, with the maximum error being approximately 0.1 of a second, as opposed to a maximum error in the Transverse Mercator projection of just over 5 seconds. These angular errors, however, have little significance in restricting the use of local ground-based coordinate systems when compared to the distance error restrictions.

The effect that longitude has on the accuracy of local ground-based coordinate systems also depends on the projection method used. The distance that can be travelled east/west in a Transverse Mercator projection, before the RTK GPS measurement accuracy is exceeded, ranges from approximately 1.8km to 17.4km. In a plane projection, however, the distance that can be travelled remained independent of longitude with a constant distance east/west of approximately 19km maintained throughout all testing.

The height that can be travelled above or below the level of the projection also changed depending on the projection method. In a Transverse Mercator projection it is possible to go 41.5m or 37.5m above or below the projection level. However, in a plane projection it is possible to go 39.5m above or below the level of the projection. This is a slight variation between methods and is not considered to be of major significance when differentiating between the projection methods.

From this analysis it can be seen that a local ground-based coordinate system based on a plane projection offers a better-shaped, accurate area and performs better distance-wise than the Transverse Mercator projection in an east/west direction. This considered, and the fact that local ground-based coordinate systems using a plane projection are longitude-independent, makes them a better choice for an all round projection than the Transverse Mercator projection.

However, it must also be concluded that the Transverse Mercator projection does outperform the plane projection in certain circumstances and would therefore be a better choice for some applications. The major advantage of the Transverse Mercator projection is that its accuracy is far superior to the plane projection in a purely north/south direction, making it suitable for mapping narrow north/south sites such as pipelines or easements.

# 5.5 Practical Validation

To demonstrate the application and limitations of local ground-based coordinate systems, two local ground-based coordinate systems, one based on the plane projection and the other based on a Transverse Mercator projection, have been set up at the Toowoomba test site. A fictitious lot was then constructed and imported into both of these systems within the accurate areas defined in sections 5.2 and 5.3. The ground distances and the associated grid distances are displayed in Figure 5.5.



Figure 5.5: Dimensions of the Lot at the Toowoomba Test Site

As it can be seen, both the Transverse Mercator and the plane projection represent all the ground distances and bearings of the lot at the project site accurately, with none of the adopted measurement accuracies being exceeded.

The same local ground-based coordinate systems were then extended to a site at Withcott, approximately 11.8km's away and 544m different in elevation. The same figure used in the testing at the Toowoomba site was imported into each of the local ground-based coordinate systems and centred at the Withcott site. Each of the boundaries was then remeasured in the same local ground-based coordinate systems that are based at the Toowoomba test site. The results are shown in Figure 5.6.



Figure 5.6: Dimensions of the Lot at the Withcott Test Site

It can be seen that there is a large deterioration in the accuracy with which the local ground-based coordinate systems represent the boundaries of the lot. The grid distances presented in both systems exceed the allowable RTK GPS measurement accuracy in all boundaries and as such should not be used to represent ground distances on plans.

This deterioration is caused by the movement of the subject site outside the accurate area defined in sections 5.2 and 5.3. Even though the site appears to be within the 19km accurate area defined for a plane projection, the large variation in project height must also be considered. Using this example as an illustration, the affect of misusing local ground-based coordinate systems can lead to grossly wrong distances being shown on survey plans and their use should be confined to the parameters defined.

## 5.6 Conclusion

This chapter has presented an in-depth analysis and discussion of the results achieved in this project. The discussion has focused on the differences between the two map projection techniques: the Transverse Mercator projection and the plane projection. Comparisons have also been made between the two projections and recommendations and limitations of their use have been noted. The validity of these recommendations in relation to distance from the central point of the projection and height change has also been discussed, with the presentation of a practical validation in the Toowoomba and Withcott area.

The distribution of distance and angular errors in the projections have been discussed and explained. The results due to site location, including height above or below the level of the projection and the affect of longitude on the accuracy of the site, have also been explored and explained. It has been discussed why longitude has no affect on plane projections, but has a non-linear affect on accuracy in Transverse Mercator projections. The irregularities in height change errors and the combined angular errors have also been outlined and explanations presented.

It has been concluded in this chapter that a local ground-based coordinate system based on a plane projection offers a more suitable all-round coordinate system than the Transverse Mercator projection. This is because of the superior east/west coverage of the plane projection and the more consistent shape of the accurate mapping area. The plane projection was also chosen because its accuracy is independent of longitude, with no major change in accuracy between sites placed at differing longitudes.

It has also been concluded, however, that the Transverse Mercator projection is far superior than the plane projection is a north/south direction and would be more suitable than the plane projection to map a narrow north/south site such as an easement or a pipeline.

# **Chapter 6: Conclusion and Recommendations**

## 6.1 Introduction

In this chapter the results and recommendations of this project will be summarised and presented. The significance of this research will be revisited, as well as the practical application and uses of this project. The limitations of the research will be presented and the aim of the project addressed.

The aim of this chapter is to provide a summary of the results obtained and recommendations made in this project. It is expected that upon reading this chapter, the reader will get an overview of the work involved in this project and the results that have been obtained from this project.

This will be achieved through the presentation of the key findings of the research and the practical implications of these findings. The significance of this research will then be outlined and recommendations for future work into this area presented.

# 6.2 Key Findings

This project has found that the accuracy of local ground-based coordinate systems is dependent on the projection method used and a number of site specific factors, including the height above and below the projection level and the longitude of the site.

#### 6.2.1 Differences in Projection Methods

It has been found that it is possible to move approximately 3km east/west and approximately 50km north/south away from the central point of the projection site in a Transverse Mercator projection, before the errors induced into the plan distances solely from the process of map projection exceed RTK GPS measurement accuracy. This distance, however, is different when the plane projection is used, with the user being able to move away from the central point of the site approximately 19km in any direction before RTK GPS measurement accuracy is exceeded by projection error. The distance difference, before map projection

distortion exceeds RTK GPS measurement accuracy between the Transverse Mercator and the Tangent Plane projection, has resulted in markedly differently shaped accurate areas around the central point of the site. The shape of the area varies from a narrow north/south section in a Transverse Mercator projection to the more practical circular area around the central point of projection that occurs in the plane projection.

#### 6.2.2 Site Dependent Factors

The height that a project site is above or below the projection level (representing the average project height) has been found to influence the accuracy of local ground-based coordinate systems. It has been concluded that distances of 41.5m above and 37.5m below the level of the projection in a Transverse Mercator projection can be achieved before RTK GPS measurement accuracy is exceeded by distance errors in the surveyed line. In a plane projection, however, it is possible to achieve a distance of 39.5m above or below projection level before RTK GPS measurement accuracy is exceeded by distance errors in the surveyed before RTK GPS measurement accuracy is exceeded by distance a distance of 39.5m above or below projection level before RTK GPS measurement accuracy is exceeded by distance errors in the surveyed line.

Longitude was also found to have an affect on local ground-based coordinate systems defined using a Transverse Mercator projection, with no significant effect on the accuracy of plane projection based systems found. In the Transverse Mercator system, the effect of longitude was found to be non-linear. The system performed best at the central longitude of the zone of 153°, with the distance that could be achieved east or west of the central point being approximately 17.4km, before RTK GPS measurement accuracy is exceeded by the grid-to-ground distance error in the surveyed line. Because the Transverse Mercator projection is centred at the central meridian and is scaled to fit other sites, the accuracy of the Transverse Mercator projection is influenced by the changing nature of the angle between the level surface and the Transverse Mercator grid. This means that the distance that can be achieved east or west of the central point of the site decreases with longitude away from the central meridian, because of the divergence of the level surface and Transverse Mercator grid. This can be seen in the testing by the decrease of the distance east or west of the central point before the RTK GPS measurement accuracy exceeds the distance error in the surveyed lines, from approximately 17.4km at the central meridian to approximately 1.8km at the edge of the MGA zone at 150° longitude.

# 6.3 Practical Implications and Recommendations

The practical implications of the research conducted in this project can be summarised as follows:

- **Distance from the centre of the site**: Do not travel further than 3km east/west in a Transverse Mercator based system and 19km in a plane based coordinate system from the central point of the site. Otherwise RTK GPS measurement accuracy will be exceeded by projection errors.
- **Heights**: Do not go above the level of the adopted map projection plane by more than 41.5m in a Transverse Mercator system and 39.5m in a plane projection based system. Further, do not go below the level of the projection by more than 37.5m in a Transverse Mercator system or 39.5m in a plane projection system or RTK GPS measurement accuracy will be exceeded by distance error in the lines due to the convergence of plumb lines.
- Always check the longitude of the site when using a local groundbased coordinate system based on a Transverse Mercator projection and make adjustments for the accuracy possible. It may be better to use a plane projection based system if near the edge of the zone.

# 6.4 Significance of Research

Little previous research has been conducted into the limitations of local groundbased coordinate systems and with the increased adoption of new technologies such as RTK GPS leading to the increased use of local ground-based coordinate systems, it is important to quantify the associated errors and to provide validity to, and guidelines for, the process. This project has filled this gap in previous knowledge by providing procedures for establishing local ground-based coordinate systems and quantifying their accuracy with respect to a number of specified measurement and legal requirements.

# 6.5 Limitations of the Study

The scope of this project was such that only two projection methods were tested: the Transverse Mercator and the Tangent Plane projections. The testing also focused on the testing of lines that emanated from the central point or passed through the central point of the projection and no testing was done on the accuracy of lines that occur at different positions around the site.

Other limitations of this research include the lack of in-depth study into the effect that height change has on the accuracy of local ground-based coordinate systems, with only one line being used to determine these limitations. This, however, is still sufficient for the purpose of this project to determine limitations of the systems, as it is representative of the errors in lines that occur due to the convergence of plumb lines.

Longitude change testing has only been completed on one side of a UTM zone and testing has only been conducted in one zone. It is expected that the errors obtained will be mirrored on the opposite side of the zone and in other mapping zones. However, this has not been confirmed in this project. Another limitation of the longitude testing is that the testing was only conducted to find out the distance errors of lines in the cardinal directions. Comprehensive testing into the angular errors and errors in lines at directions other than the cardinal directions has not been carried out, as it is beyond the scope of this project.

# 6.6 Recommendations for Future Work

It is recommended that further testing be carried out on other projection types. Only Transverse Mercator and the tangent plane projection methods have been tested, but other projection methods are also used to define local map projections. One common projection used in local ground-based coordinate systems, which was not included in testing and would benefit from further research, is the Lambert conic conformal projection.

Further research into the effect that the position of a line in relation to the centre of a local site has, is also recommended. This is because all the testing completed in

this project focused on errors produced from lines running from the centre of the site and no lines were tested that did not run through the centre point. It is expected, however, that the line's position within the context of the site will affect both its angular and distance accuracy and should be the subject of further research.

Research into the affect that height change has on the accuracy of local groundbased coordinate systems is also recommended. This is because, for the purposes of this project, a single 2km line was used to determine the limitations due to height change and the use of different length lines and lines at differing bearings will affect these results.

An in-depth exploration of the effect that longitude has on the accuracy of local ground-based coordinate systems is also suggested. The author expects that the errors presented in this project will be mirrored on the opposite side of the zone and in other mapping zones. However, further research to confirm this is recommended. Expanding the scope of the longitude testing, to take into account lines at differing bearings to those in the cardinal directions and including angular errors in the analysis, is also recommended.

## 6.7 Close

This project has achieved its aim "to develop a procedure, determine the limitations, and validate the use of local ground-based coordinate systems with respect to dimensions shown on plans when conducting RTK GPS surveys". The aim was accomplished through the development of procedures to define local ground-based coordinate systems and the development of procedures to test the accuracy of these systems. This accuracy was then compared to RTK GPS and total station measurement accuracies and the current cadastral survey requirements in Queensland to quantify the limitations of such systems. The process was then validated through the use of a practical validation.

It was found that the accuracy of local ground-based coordinate systems is dependent on the projection method used. It was found that in a local ground-based coordinate system, defined using a plane projection, it is possible to go approximately 19km from the central point of the site in all directions. This makes it a better system for all-round use than a system defined using the Transverse Mercator projection, in which it is possible to go approximately 3km east or west of the central point before RTK GPS measurement accuracy is exceeded by distance errors due to projection. However, it was also concluded that the Transverse Mercator projection offers far superior accuracy in a strictly North/South direction than the Plane projection.

# **Appendix A: Project Specification**

University of Southern Queensland

## FACULTY OF ENGINEERING AND SURVEYING

#### ENG4111/4112 Research Project PROJECT SPECIFICATION

FOR: Jeff Pickford

- TOPIC: How to Establish Map Projections to facilitate the use of GPS on Ground-Based Surveys
- SUPERVISOR: Peter Gibbings

SPONSORSHIP: University of Southern Queensland

PROJECT AIM: To develop a procedure, determine the limitations, and validate the use of local ground-based coordinate systems with respect to dimensions shown on plans when conducting RTK GPS surveys.

#### PROGRAMME: (Issue A, 6th March 2008)

1. Analysis of the errors in a GPS system through a literature review

2. Develop procedures for defining local based map projections suitable for a number of uses

3. Develop methods for defining errors associated with local coordinate systems, so the accuracy of plan dimensions can be determined. This includes looking at errors that occur in map projections in relation to height differences, distances and bearings, with results being output in tables or charts.

4. Review of the errors associated with the use of local coordinate systems and their affect on output dimensions

5. Validate the procedures in step two through the use of a test site and examining the results

6. Discussion the results & possible implications with respect to: Legal issues & the need for checks and redundancies

7. Recommend the most appropriate procedure for a particular type of survey

8. Submit an academic dissertation of an acceptable standard

AGREED A Himpod

Student: Jeff Pickford Date: 06 / 03 / 2008

Posith

Supervisor: Peter Gibbings Date: 06 / 03 / 2008

# Appendix B: Transverse Mercator Projection Accuracy Limits

Bearing	GPS (m)	Cadastral (m)	Total Station (m)
0	-	-	-
10	8400	-	8800
20	5500	-	5100
30	4500	-	4100
40	4000	-	3300
50	3700	-	2900
60	3500	-	2800
70	3400	-	2700
80	3300	-	2700
90	3300	-	2600
100	3300	-	2700
110	3400	-	2700
120	3500	-	2800
130	3700	-	2900
140	4000	-	3300
150	4600	-	4100
160	5600	-	5100
170	8400	-	8900
180	-	-	-
190	9900	-	11200
200	6300	-	5900
210	4800	-	4200
220	4200	-	3300
230	3500	-	2800
240	3300	-	2600
250	3100	-	2500
260	3100	-	2300
270	3000	-	2200
280	3100	-	2300
290	3100	-	2500
300	3300	-	2600
310	3600	-	2800
320	4200	-	3300
330	4800	-	4200
340	6400	-	6100
350	10200	-	11300

- Denotes the accuracy was not exceeded within testing limits

# **Appendix C: Plane Projection Accuracy Limits**

Bearing	GPS (m)	Cadastral (m)	Total Station (m)
0	19000	-	-
10	19000	-	-
20	19100	-	-
30	19100	-	-
40	19100	-	-
50	19200	-	-
60	19300	-	-
70	19300	-	-
80	19400	-	-
90	19400	-	-
100	19400	-	-
110	19400	-	-
120	19400	-	-
130	19400	-	-
140	19400	-	-
150	19400	-	-
160	19400	-	-
170	19400	-	-
180	19400	-	-
190	19400	-	-
200	19400	-	-
210	19400	-	-
220	19400	-	-
230	19400	-	-
240	19400	-	-
250	19400	-	-
260	19400	-	-
270	19400	-	-
280	19300	-	-
290	19300	-	-
300	19300	-	-
310	19200	-	-
320	19200	-	-
330	19100	-	-
340	19100	-	-
350	19000	-	-

- Denotes the accuracy was not exceeded within testing limits

# Appendix D: Transverse Mercator Projection Height Change Results

Elevation of the project site was 718.663m

Elevation	Height	Grid	Distance	Ground	Distance	Grid-Ground
AHD	Difference (m)	(m)		(m)		Distance (m)
668.663	50		2000		1999.984	0.016
669.163	49.5		2000		1999.984	0.016
669.663	49		2000		1999.984	0.016
670.163	48.5		2000		1999.984	0.016
670.663	48		2000		1999.984	0.016
671.163	47.5		2000		1999.984	0.016
671.663	47		2000		1999.985	0.015
672.163	46.5		2000		1999.985	0.015
672.663	46		2000		1999.985	0.015
673.163	45.5		2000		1999.985	0.015
673.663	45		2000		1999.985	0.015
674.163	44.5		2000		1999.985	0.015
674.663	44		2000		1999.986	0.014
675.163	43.5		2000		1999.986	0.014
675.663	43		2000		1999.986	0.014
676.163	42.5		2000		1999.986	0.014
676.663	42		2000		1999.986	0.014
677.163	41.5		2000		1999.986	0.014
677.663	41		2000		1999.987	0.013
678.163	40.5		2000		1999.987	0.013
678.663	40		2000		1999.987	0.013
679.163	39.5		2000		1999.987	0.013
679.663	39		2000		1999.987	0.013
680.163	38.5		2000		1999.987	0.013
680.663	38		2000		1999.987	0.013
681.163	37.5		2000		1999.988	0.012
681.663	37		2000		1999.988	0.012
682.163	36.5		2000		1999.988	0.012
682.663	36		2000		1999.988	0.012
683.163	35.5		2000		1999.988	0.012
683.663	35		2000		1999.988	0.012
684.163	34.5		2000		1999.989	0.011
684.663	34		2000		1999.989	0.011
685.163	33.5		2000		1999.989	0.011
685.663	33		2000		1999.989	0.011
686.163	32.5		2000		1999.989	0.011
686.663	32		2000		1999.989	0.011
687.163	31.5		2000		1999.989	0.011
687.663	31		2000		1999.99	0.01
688.163	30.5		2000		1999.99	0.01
688.663	30		2000		1999.99	0.01
689.163	29.5		2000		1999.99	0.01
689.663	29		2000		1999.99	0.01
690.163	28.5		2000		1999.99	0.01

690.663	28	2000	1999.991	0.009
691.163	27.5	2000	1999.991	0.009
691.663	27	2000	1999.991	0.009
692.163	26.5	2000	1999.991	0.009
692.663	26	2000	1999.991	0.009
693.163	25.5	2000	1999.991	0.009
693.663	25	2000	1999.992	0.008
694.163	24.5	2000	1999.992	0.008
694.663	24	2000	1999.992	0.008
695.163	23.5	2000	1999.992	0.008
695.663	23	2000	1999.992	0.008
696.163	22.5	2000	1999.992	0.008
696.663	22	2000	1999.992	0.008
697.163	21.5	2000	1999.993	0.007
697.663	21	2000	1999.993	0.007
698.163	20.5	2000	1999.993	0.007
698.663	20	2000	1999.993	0.007
699.163	19.5	2000	1999.993	0.007
699.663	19	2000	1999.993	0.007
700.163	18.5	2000	1999.994	0.006
700.663	18	2000	1999.994	0.006
701.163	17.5	2000	1999.994	0.006
701.663	17	2000	1999.994	0.006
702.163	16.5	2000	1999.994	0.006
702.663	16	2000	1999.994	0.006
703.163	15.5	2000	1999.995	0.005
703.663	15	2000	1999.995	0.005
704.163	14.5	2000	1999.995	0.005
704.663	14	2000	1999.995	0.005
705.163	13.5	2000	1999.995	0.005
705.663	13	2000	1999.995	0.005
706.163	12.5	2000	1999.995	0.005
706.663	12	2000	1999.996	0.004
707.163	11.5	2000	1999.996	0.004
707.663	11	2000	1999.996	0.004
708.163	10.5	2000	1999.996	0.004
708.663	10	2000	1999.996	0.004
709.163	9.5	2000	1999.996	0.004
709.663	9	2000	1999.997	0.003
710.163	8.5	2000	1999.997	0.003
710.663	8	2000	1999.997	0.003
711.163	7.5	2000	1999.997	0.003
711.663	7	2000	1999.997	0.003
712.163	6.5	2000	1999.997	0.003
712.663	6	2000	1999.997	0.003
713.163	5.5	2000	1999.998	0.002
713.663	5	2000	1999.998	0.002
714.163	4.5	2000	1999.998	0.002
714.663	4	2000	1999.998	0.002
715.163	3.5	2000	1999.998	0.002
715.663	3	2000	1999.998	0.002
716.163	2.5	2000	1999.999	0.001
716.663	2	2000	1999.999	0.001
717.163	1.5	2000	1999.999	0.001

718.163     0.5     2000     1999.999     0.001       718.663     0     2000     2000     0       719.663     -1     2000     2000     0       720.663     -2     2000     2000     0       720.663     -2     2000     2000     0       721.163     -3.5     2000     2000     0       721.63     -3.5     2000     2000     0       722.163     -3.5     2000     2000.001     -0.001       723.63     -4.5     2000     2000.001     -0.001       724.63     -5.5     2000     2000.001     -0.001       724.63     -6.5     2000     2000.001     -0.001       724.63     -6.5     2000     2000.002     -0.002       726.63     -7     2000     2000.002     -0.002       726.63     -9     2000     200.002     -0.002       727.63     -9     2000     200.003     -0.003       728.63     -10	717.663	1	2000	1999.999	0.001
718.663     0     2000     1999.999     0.001       719.663     -0.5     2000     2000     0       720.163     -1.5     2000     2000     0       720.163     -2     2000     2000     0       721.163     -2.5     2000     2000     0       721.163     -3.5     2000     2000     0       722.63     -4     2000     2000     0     0       722.63     -4     2000     2000.001     -0.001       723.63     -5     2000     2000.001     -0.001       723.63     -5     2000     2000.001     -0.001       724.63     -6.5     2000     2000.002     -0.002       726.63     -7     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       728.663     -11     2000     2000.003     -0.003       729.663	718.163	0.5	2000	1999.999	0.001
719.163     -0.5     2000     2000     0       719.163     -1     2000     2000     0       720.163     -1.5     2000     2000     0       721.63     -2.5     2000     2000     0       721.63     -3.5     2000     2000     0       721.63     -3.5     2000     2000     0       722.63     -4     2000     2000     0       723.63     -5     2000     2000.001     -0.001       723.63     -5     2000     2000.001     -0.001       724.63     -6.5     2000     2000.001     -0.001       725.63     -7     2000     2000.002     -0.002       726.63     -8     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       727.663     -9.5     2000     2000.002     -0.002       728.63     -10     2000     2000.003     -0.003       728.63     -11	718.663	0	2000	1999.999	0.001
719.663     -1     2000     2000     0       720.63     -1.5     2000     2000     0       720.663     -2     2000     2000     0       721.163     -2.5     2000     2000     0       721.163     -3.5     2000     2000     0       722.163     -3.5     2000     2000.001     -0.001       723.163     -4.5     2000     2000.001     -0.001       723.663     -5     2000     2000.001     -0.001       724.63     -6     2000     2000.001     -0.001       725.63     -7     2000     2000.002     -0.002       726.63     -8     2000     2000.002     -0.002       727.63     -8.5     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       728.663     -11     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       730.163	719.163	-0.5	2000	2000	0
720.163     -1.5     2000     2000     0       720.163     -2     2000     2000     0       721.163     -2.5     2000     2000     0       722.663     -3     2000     2000     0       722.663     -4     2000     2000.001     -0.001       723.663     -5     2000     2000.001     -0.001       724.663     -6     2000     2000.001     -0.001       723.663     -5     2000     2000.001     -0.001       724.663     -6     2000     2000.002     -0.002       725.663     -7     2000     2000.002     -0.002       726.663     -8     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       728.63     -10     2000     2000.002     -0.002       728.63     -11     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.63	719.663	-1	2000	2000	0
720.663     -2     2000     2000     0       721.163     -2.5     2000     2000     0       721.63     -3.5     2000     2000     0       722.663     -4     2000     2000.001     -0.001       723.63     -5     2000     2000.001     -0.001       723.63     -5     2000     2000.001     -0.001       724.63     -6     2000     2000.001     -0.001       724.63     -6     2000     2000.001     -0.001       725.63     -7     2000     2000.002     -0.002       726.63     -8     2000     2000.002     -0.002       727.63     -9.5     2000     2000.002     -0.002       728.63     -9     2000     2000.002     -0.002       728.63     -10     2000     2000.003     -0.003       728.63     -10     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       729.663	720.163	-1.5	2000	2000	0
721.163     -2.5     2000     2000     0       721.663     -3     2000     2000     0       722.163     -3.5     2000     2000     0       722.163     -4.5     2000     2000.001     -0.001       723.163     -4.5     2000     2000.001     -0.001       724.163     -5.5     2000     2000.001     -0.001       724.63     -6.5     2000     2000.001     -0.001       725.663     -7     2000     2000.002     -0.002       726.663     -8     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       728.163     -9.5     2000     2000.002     -0.002       728.663     -10     2000     2000.003     -0.003       729.163     -1.5     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003	720.663	-2	2000	2000	0
721.663     -3     2000     2000     0       722.163     -3.5     2000     2000     0       722.663     -4     2000     2000.001     -0.001       723.163     -4.5     2000     2000.001     -0.001       723.663     -5     2000     2000.001     -0.001       724.663     -6     2000     2000.001     -0.001       725.663     -7     2000     2000.002     -0.002       726.663     -8     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       728.663     -11     2000     2000.003     -0.003       729.663     -12     2000     2000.003     -0.003       731.163     -12.5     2000     2000.003     -0.003       731.163     -13     2000     2000.003     -0.003	721.163	-2.5	2000	2000	0
722.163     -3.5     2000     2000     0       722.663     -4     2000     2000.001     -0.001       723.163     -4.5     2000     2000.001     -0.001       723.163     -5     2000     2000.001     -0.001       724.163     -5.5     2000     2000.001     -0.001       724.663     -6     2000     2000.001     -0.001       725.163     -5.5     2000     2000.002     -0.002       726.63     -7     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       728.663     -11     2000     2000.002     -0.003       729.663     -11     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       739.663     -12     2000     2000.003     -0.003       731.163     -14.5     2000     2000.003     -0.004 <td>721.663</td> <td>-3</td> <td>2000</td> <td>2000</td> <td>0</td>	721.663	-3	2000	2000	0
722.663     4     2000     200.001     -0.001       723.163     -4.5     2000     2000.001     -0.001       723.663     -5     2000     2000.001     -0.001       724.163     -5.5     2000     2000.001     -0.001       724.663     -6     2000     2000.001     -0.001       725.663     -7     2000     2000.002     -0.002       726.663     -8     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       728.663     -10     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       729.163     -10.5     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.003	722.163	-3.5	2000	2000	0
723.163 $4.5$ 20002000.001 $-0.001$ 723.663 $-5$ 20002000.001 $-0.001$ 724.163 $-5.5$ 20002000.001 $-0.001$ 725.163 $-6.5$ 20002000.001 $-0.001$ 725.663 $-7$ 20002000.002 $-0.002$ 726.163 $-7.5$ 20002000.002 $-0.002$ 726.663 $-8$ 20002000.002 $-0.002$ 727.163 $-8.5$ 20002000.002 $-0.002$ 727.663 $-9$ 20002000.002 $-0.002$ 728.163 $-10$ 20002000.002 $-0.002$ 728.663 $-10$ 20002000.002 $-0.002$ 729.163 $-10.5$ 20002000.003 $-0.003$ 729.663 $-11$ 20002000.003 $-0.003$ 730.663 $-11.5$ 20002000.003 $-0.003$ 730.663 $-12$ 20002000.003 $-0.003$ 731.663 $-13.5$ 20002000.003 $-0.004$ 732.663 $-14$ 20002000.004 $-0.004$ 734.663 $-15.5$ 20002000.004 $-0.004$ 734.663 $-16$ 20002000.005 $-0.005$ 736.63 $-19$ 20002000.005 $-0.005$ 736.63 $-19$ 20002000.005 $-0.005$ 736.63 $-16$ 20002000.005 $-0.005$ 736.63 $-16$ 20002000.005 $-0.005$ 736.63 $-19$ 20002000.	722.663	-4	2000	2000.001	-0.001
723.663     .5     2000     2000.001     .0.001       724.163     .5.5     2000     2000.001     .0.001       724.663     .6.5     2000     2000.001     .0.001       725.163     .6.5     2000     2000.002     .0.002       726.663     .7.5     2000     2000.002     .0.002       726.663     .8.5     2000     2000.002     .0.002       727.663     .9     2000     2000.002     .0.002       728.663     .10     2000     2000.002     .0.002       728.663     .10.5     2000     2000.002     .0.002       728.663     .10.5     2000     2000.003     .0.003       729.663     .11     2000     2000.003     .0.003       730.663     .12     2000     2000.003     .0.003       731.63     .13     2000     2000.003     .0.003       731.63     .14.5     2000     2000.004     .0.004       733.63     .14.5     2000     2000.004     .0.	723,163	-4.5	2000	2000.001	-0.001
724.163     -5.5     2000     2000.001     -0.001       724.663     -6     2000     2000.001     -0.001       725.163     -6.5     2000     2000.002     -0.002       725.663     -7     2000     2000.002     -0.002       726.663     -8     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       729.663     -11     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.003       731.63     -14     2000     2000.003     -0.003       731.63     -14.5     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004	723.663	-5	2000	2000.001	-0.001
724.663     -6     2000     2000.001     -0.001       725.163     -6.5     2000     2000.002     -0.002       726.163     -7.5     2000     2000.002     -0.002       726.163     -7.5     2000     2000.002     -0.002       726.163     -8     2000     2000.002     -0.002       727.163     -8.5     2000     2000.002     -0.002       728.663     -9     2000     2000.002     -0.002       728.163     -9.5     2000     2000.002     -0.002       728.163     -10.5     2000     2000.003     -0.003       729.163     -11.5     2000     2000.003     -0.003       730.663     -11     2000     2000.003     -0.003       731.163     -12.5     2000     2000.003     -0.003       731.163     -15     2000     2000.003     -0.004       732.163     -13.5     2000     2000.004     -0.004       733.163     -14.5     2000     2000.004 <td< td=""><td>724.163</td><td>-5.5</td><td>2000</td><td>2000.001</td><td>-0.001</td></td<>	724.163	-5.5	2000	2000.001	-0.001
725.163     -6.5     2000     2000.001     -0.001       725.663     -7     2000     2000.002     -0.002       726.163     -7.5     2000     2000.002     -0.002       726.663     -8     2000     2000.002     -0.002       727.163     -8.5     2000     2000.002     -0.002       728.663     -9     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       728.663     -10     2000     2000.003     -0.003       729.163     -10.5     2000     2000.003     -0.003       730.663     -11     2000     2000.003     -0.003       731.663     -11.5     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.004       732.663     -14     2000     2000.004     -0.004       732.663     -14     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004<	724.663	-6	2000	2000.001	-0.001
725.663     -7     2000     2000.002     -0.002       726.663     -8     2000     2000.002     -0.002       727.163     -8.5     2000     2000.002     -0.002       727.163     -8.5     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       728.663     -10     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       730.663     -13     2000     2000.003     -0.003       731.663     -13     2000     2000.004     -0.004       732.663     -14     2000     2000.004     -0.004       733.163     -14.5     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       734.663     -16     2000     2000.005     -0.005 <td>725.163</td> <td>-6.5</td> <td>2000</td> <td>2000.001</td> <td>-0.001</td>	725.163	-6.5	2000	2000.001	-0.001
726.163     -7.5     2000     2000.002     -0.002       726.663     -8     2000     2000.002     -0.002       727.163     -8.5     2000     2000.002     -0.002       727.163     -9.5     2000     2000.002     -0.002       728.163     -9.5     2000     2000.002     -0.002       728.663     -10     2000     2000.003     -0.003       729.163     -10.5     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.163     -12.5     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.004       732.663     -14     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       733.663     -15     2000     2000.005     -0.005       736.63     -16     2000     2000.005     -0.00	725.663	-7	2000	2000.002	-0.002
726.663    B     2000     2000.002     -0.002       727.163     -8.5     2000     2000.002     -0.002       727.663     -9     2000     2000.002     -0.002       728.163     -9.5     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       729.663     -10.5     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.003       732.663     -14     2000     2000.004     -0.004       732.663     -14     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       733.663     -16     2000     2000.005     -0.005       735.663     -17     2000     2000.005     -0.005 </td <td>726,163</td> <td>-7.5</td> <td>2000</td> <td>2000.002</td> <td>-0.002</td>	726,163	-7.5	2000	2000.002	-0.002
101010     101010     101010     101010       727.163     -8.5     2000     2000.002     -0.002       728.163     -9.5     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       728.663     -10     2000     2000.003     -0.003       729.163     -10.5     2000     2000.003     -0.003       730.663     -11     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.003       732.663     -14     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       733.663     -16     2000     2000.004     -0.004       734.163     -15.5     2000     2000.005     -0.005       735.663     -17     2000     2000.005     -0.005  <	726 663	-8	2000	2000.002	-0.002
11100     100     1000     1000     1000       727.663     -9     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       729.663     -10.5     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.663     -12     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.003       731.663     -13     2000     2000.004     -0.004       732.663     -14     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       734.663     -16     2000     2000.004     -0.004       735.663     -17     2000     2000.005     -0.005       736.63     -16     2000     2000.005     -0.005       736.63     -17     2000     2000.005     -0.005 <td>727 163</td> <td>-8.5</td> <td>2000</td> <td>2000.002</td> <td>-0.002</td>	727 163	-8.5	2000	2000.002	-0.002
T28.163     -9.5     2000     2000.002     -0.002       728.663     -10     2000     2000.002     -0.002       729.163     -10.5     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       730.163     -11.5     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.63     -12.5     2000     2000.003     -0.003       731.63     -13.5     2000     2000.003     -0.003       732.663     -14     2000     2000.004     -0.004       732.663     -14     2000     2000.004     -0.004       733.163     -14.5     2000     2000.004     -0.004       734.663     -15     2000     2000.004     -0.004       735.663     -17     2000     2000.005     -0.005       736.663     -16.5     2000     2000.005     -0.005       736.663     -17     2000     2000.005	727 663	-9	2000	2000.002	-0.002
728.663     -10     2000     2000.002     -0.002       729.163     -10.5     2000     2000.003     -0.003       729.663     -11     2000     2000.003     -0.003       730.163     -11.5     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.163     -12.5     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.003       731.663     -13.5     2000     2000.004     -0.004       732.663     -14     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       733.663     -16     2000     2000.004     -0.004       734.663     -16     2000     2000.005     -0.005       735.663     -17     2000     2000.005     -0.005       736.663     -18     2000     2000.005     -0.005       736.663     -19     2000     2000.005     -0.	728 163	-9.5	2000	2000.002	-0.002
120.003     100     1000     10000     10000     100000     100000     100000     100000     100000     100000     100000     100000     100000     100000     100000     100000     100000     10000000     1000000000     100000000000000     1000000000000000000000000000000000000	728 663	-10	2000	2000.002	-0.002
120.103     1005     1000     100005     10005     10005 <t< td=""><td>729 163</td><td>-10.5</td><td>2000</td><td>2000.002</td><td>-0.003</td></t<>	729 163	-10.5	2000	2000.002	-0.003
11.1     1000     10000     10000     10000       730.163     -11.5     2000     2000.003     -0.003       730.663     -12     2000     2000.003     -0.003       731.163     -12.5     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.003       732.163     -13.5     2000     2000.004     -0.004       733.163     -14.5     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       734.663     -16     2000     2000.004     -0.004       735.163     -16.5     2000     2000.005     -0.005       736.63     -17     2000     2000.005     -0.005       736.63     -18     2000     2000.005     -0.005       736.63     -18     2000     2000.005     -0.005       737.663     -19     2000     2000.005     -0.005       738.663     -20     2000     2000.006     -0.006 <td>729.663</td> <td>-11</td> <td>2000</td> <td>2000.003</td> <td>-0.003</td>	729.663	-11	2000	2000.003	-0.003
100.103     1105     1000     10000     10000     10000       730.663     -12     2000     2000.003     -0.003       731.163     -12.5     2000     2000.003     -0.003       731.663     -13     2000     2000.003     -0.003       732.163     -13.5     2000     2000.004     -0.004       733.663     -14     2000     2000.004     -0.004       733.663     -14.5     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       734.663     -16     2000     2000.004     -0.004       735.663     -16.5     2000     2000.005     -0.005       735.663     -17     2000     2000.005     -0.005       736.663     -18     2000     2000.005     -0.005       737.163     -18.5     2000     2000.005     -0.005       738.663     -20     2000     2000.005     -0.005       738.663     -20     2000     2000.00	730 163	-11.5	2000	2000.003	-0.003
100.000     12     2000     2000.003     0.003       731.163     -12.5     2000     2000.003     -0.003       731.63     -13.5     2000     2000.004     -0.004       732.63     -14     2000     2000.004     -0.004       733.163     -14.5     2000     2000.004     -0.004       733.663     -14     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       734.663     -16     2000     2000.004     -0.004       735.663     -16.5     2000     2000.005     -0.005       735.663     -17     2000     2000.005     -0.005       736.663     -18     2000     2000.005     -0.005       737.663     -19     2000     2000.005     -0.005       738.63     -19.5     2000     2000.005     -0.005       738.63     -20     2000     2000.005     -0.005       738.63     -20.5     2000     2000.006     -0.006	730.663	-12	2000	2000.003	-0.003
131     2000     2000,003     0.003       731.663     -13     2000     2000,003     -0.003       732.163     -13.5     2000     2000,004     -0.004       732.663     -14     2000     2000,004     -0.004       733.163     -14.5     2000     2000,004     -0.004       733.663     -15     2000     2000,004     -0.004       734.163     -15.5     2000     2000,004     -0.004       734.663     -16     2000     2000,004     -0.004       735.163     -16.5     2000     2000,005     -0.005       735.663     -17     2000     2000,005     -0.005       736.663     -18     2000     2000,005     -0.005       737.663     -19     2000     2000,005     -0.005       738.163     -19.5     2000     2000,005     -0.005       738.63     -20     2000     2000,006     -0.006       739.163     -21.5     2000     2000,006     -0.006	731 163	-12.5	2000	2000.003	-0.003
732.163 $-13.5$ $2000$ $2000.004$ $-0.004$ $732.663$ $-14$ $2000$ $2000.004$ $-0.004$ $733.163$ $-14.5$ $2000$ $2000.004$ $-0.004$ $733.663$ $-15$ $2000$ $2000.004$ $-0.004$ $734.63$ $-15.5$ $2000$ $2000.004$ $-0.004$ $734.63$ $-16.5$ $2000$ $2000.004$ $-0.004$ $735.163$ $-16.5$ $2000$ $2000.005$ $-0.005$ $735.663$ $-17$ $2000$ $2000.005$ $-0.005$ $736.63$ $-17$ $2000$ $2000.005$ $-0.005$ $736.63$ $-18$ $2000$ $2000.005$ $-0.005$ $737.63$ $-19$ $2000$ $2000.005$ $-0.005$ $737.63$ $-19.5$ $2000$ $2000.005$ $-0.005$ $738.63$ $-20$ $2000$ $2000.006$ $-0.006$ $739.163$ $-21.5$ $2000$ $2000.006$ $-0.006$ $740.163$ $-21.5$ $2000$ $2000.006$ $-0.006$ $741.63$ $-22.5$ $2000$ $2000.006$ $-0.006$ $741.63$ $-23.5$ $2000$ $2000.007$ $-0.007$ $742.663$ $-24$ $2000$ $2000.007$ $-0.007$ $743.663$ $-25$ $2000$ $2000.007$ $-0.007$ $743.663$ $-24.5$ $2000$ $2000.007$ $-0.007$ $743.663$ $-25$ $2000$ $2000.007$ $-0.007$ $743.663$ $-25$ $2000$ $2000.007$ $-0.007$	731 663	-13	2000	2000.003	-0.003
102.103 $1000$ $2000$ $2000.004$ $0.004$ $732.663$ $-14$ $2000$ $2000.004$ $-0.004$ $733.163$ $-14.5$ $2000$ $2000.004$ $-0.004$ $733.663$ $-15$ $2000$ $2000.004$ $-0.004$ $734.163$ $-15.5$ $2000$ $2000.004$ $-0.004$ $734.663$ $-16$ $2000$ $2000.004$ $-0.004$ $735.163$ $-16.5$ $2000$ $2000.005$ $-0.005$ $735.663$ $-17$ $2000$ $2000.005$ $-0.005$ $736.63$ $-17.5$ $2000$ $2000.005$ $-0.005$ $736.63$ $-18$ $2000$ $2000.005$ $-0.005$ $737.63$ $-18.5$ $2000$ $2000.005$ $-0.005$ $737.63$ $-19$ $2000$ $2000.005$ $-0.005$ $738.63$ $-20$ $2000$ $2000.006$ $-0.006$ $739.63$ $-21.5$ $2000$ $2000.006$ $-0.006$ $740.63$ $-22.5$ $2000$ $2000.006$ $-0.006$ $741.63$ $-22.5$ $2000$ $2000.006$ $-0.006$ $741.63$ $-23.5$ $2000$ $2000.007$ $-0.007$ $742.63$ $-24.5$ $2000$ $2000.007$ $-0.007$ $743.63$ $-24.5$ $2000$ $2000.007$ $-0.007$ $743.663$ $-25$ $2000$ $2000.007$ $-0.007$ $743.663$ $-25$ $2000$ $2000.007$ $-0.007$ $743.663$ $-25$ $2000$ $2000.007$ $-0.007$ </td <td>732 163</td> <td>-13.5</td> <td>2000</td> <td>2000.004</td> <td>-0.004</td>	732 163	-13.5	2000	2000.004	-0.004
101.000     101.000     1000.001     0.001       733.163     -14.5     2000     2000.004     -0.004       733.663     -15     2000     2000.004     -0.004       734.163     -15.5     2000     2000.004     -0.004       734.663     -16     2000     2000.004     -0.004       735.163     -16.5     2000     2000.005     -0.005       735.663     -17     2000     2000.005     -0.005       736.663     -17.5     2000     2000.005     -0.005       736.663     -18     2000     2000.005     -0.005       736.663     -18.5     2000     2000.005     -0.005       737.663     -19     2000     2000.005     -0.005       738.663     -20     2000     2000.005     -0.005       738.663     -20     2000     2000.006     -0.006       739.163     -20.5     2000     2000.006     -0.006       740.663     -22     2000     2000.006     -0.006 </td <td>732 663</td> <td>-14</td> <td>2000</td> <td>2000.004</td> <td>-0.004</td>	732 663	-14	2000	2000.004	-0.004
100.100     100     100     100     1000 <th< td=""><td>733 163</td><td>-14.5</td><td>2000</td><td>2000.004</td><td>-0.004</td></th<>	733 163	-14.5	2000	2000.004	-0.004
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	733 663	-15	2000	2000.001	-0.004
734.663     -16     2000     2000.004     -0.004       735.163     -16.5     2000     2000.005     -0.005       735.663     -17     2000     2000.005     -0.005       736.63     -17     2000     2000.005     -0.005       736.63     -17     2000     2000.005     -0.005       736.63     -18     2000     2000.005     -0.005       737.63     -18.5     2000     2000.005     -0.005       737.63     -19     2000     2000.005     -0.005       738.63     -19.5     2000     2000.005     -0.005       738.63     -20     2000     2000.005     -0.005       738.63     -20     2000     2000.006     -0.006       739.63     -21     2000     2000.006     -0.006       740.63     -21.5     2000     2000.006     -0.006       741.63     -23.5     2000     2000.007     -0.007       742.63     -24     2000     2000.007     -0.007	734,163	-15.5	2000	2000.004	-0.004
735.163     -16.5     2000     2000.005     -0.005       735.163     -16.5     2000     2000.005     -0.005       735.663     -17     2000     2000.005     -0.005       736.163     -17.5     2000     2000.005     -0.005       736.663     -18     2000     2000.005     -0.005       737.163     -18.5     2000     2000.005     -0.005       737.663     -19     2000     2000.005     -0.005       738.163     -19.5     2000     2000.005     -0.005       738.663     -20     2000     2000.005     -0.005       738.663     -20     2000     2000.006     -0.006       739.163     -20.5     2000     2000.006     -0.006       739.663     -21     2000     2000.006     -0.006       740.163     -22.5     2000     2000.006     -0.006       741.163     -22.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007	734 663	-16	2000	2000.004	-0.004
735.663     -17     2000     2000.005     -0.005       735.663     -17.5     2000     2000.005     -0.005       736.163     -17.5     2000     2000.005     -0.005       736.663     -18     2000     2000.005     -0.005       737.163     -18.5     2000     2000.005     -0.005       737.663     -19     2000     2000.005     -0.005       738.663     -19.5     2000     2000.005     -0.005       738.663     -20     2000     2000.006     -0.006       739.163     -20.5     2000     2000.006     -0.006       739.663     -21     2000     2000.006     -0.006       740.163     -22.5     2000     2000.006     -0.006       741.163     -22.5     2000     2000.007     -0.007       742.663     -23     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007	735 163	-16.5	2000	2000.005	-0.005
736.163     -17.5     2000     2000.005     -0.005       736.163     -17.5     2000     2000.005     -0.005       736.663     -18     2000     2000.005     -0.005       737.163     -18.5     2000     2000.005     -0.005       737.663     -19     2000     2000.005     -0.005       738.163     -19.5     2000     2000.005     -0.005       738.663     -20     2000     2000.006     -0.006       739.163     -20.5     2000     2000.006     -0.006       739.663     -21     2000     2000.006     -0.006       740.663     -22.5     2000     2000.006     -0.006       741.163     -22.5     2000     2000.006     -0.006       742.163     -23.5     2000     2000.007     -0.007       742.663     -24.5     2000     2000.007     -0.007       743.663     -24.5     2000     2000.007     -0.007       743.663     -24.5     2000     2000.007	735.663	-17	2000	2000.005	-0.005
100.100     11.0     1000     1000     1000       736.663     -18     2000     2000.005     -0.005       737.163     -18.5     2000     2000.005     -0.005       737.663     -19     2000     2000.005     -0.005       738.163     -19.5     2000     2000.005     -0.005       738.663     -20     2000     2000.006     -0.006       739.163     -20.5     2000     2000.006     -0.006       739.663     -21     2000     2000.006     -0.006       740.663     -22     2000     2000.006     -0.006       741.63     -22.5     2000     2000.006     -0.006       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	736 163	-17.5	2000	2000.005	-0.005
737.163     -18.5     2000     2000.005     -0.005       737.163     -19     2000     2000.005     -0.005       737.663     -19     2000     2000.005     -0.005       738.163     -19.5     2000     2000.005     -0.005       738.663     -20     2000     2000.006     -0.006       739.163     -20.5     2000     2000.006     -0.006       739.663     -21     2000     2000.006     -0.006       740.163     -21.5     2000     2000.006     -0.006       741.663     -22.5     2000     2000.006     -0.007       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	736.663	-18	2000	2000.005	-0.005
737.663     -19     2000     2000.005     -0.005       738.163     -19.5     2000     2000.005     -0.005       738.663     -20     2000     2000.006     -0.006       739.163     -20.5     2000     2000.006     -0.006       739.663     -21     2000     2000.006     -0.006       740.163     -21.5     2000     2000.006     -0.006       740.663     -22     2000     2000.006     -0.006       741.663     -22.5     2000     2000.007     -0.007       742.163     -23.5     2000     2000.007     -0.007       743.663     -24.5     2000     2000.007     -0.007       743.663     -24.5     2000     2000.007     -0.007	737,163	-18.5	2000	2000.005	-0.005
738.163     -19.5     2000     2000.005     -0.005       738.663     -20     2000     2000.006     -0.006       739.163     -20.5     2000     2000.006     -0.006       739.663     -21     2000     2000.006     -0.006       740.163     -21.5     2000     2000.006     -0.006       740.663     -22     2000     2000.006     -0.006       741.163     -22.5     2000     2000.006     -0.006       741.663     -23     2000     2000.007     -0.007       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	737.663	-19	2000	2000.005	-0.005
738.663     -20     2000     2000.006     -0.006       738.663     -20.5     2000     2000.006     -0.006       739.163     -20.5     2000     2000.006     -0.006       739.663     -21     2000     2000.006     -0.006       740.163     -21.5     2000     2000.006     -0.006       740.663     -22     2000     2000.006     -0.006       741.663     -22.5     2000     2000.007     -0.007       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	738,163	-19.5	2000	2000.005	-0.005
739.163     -20.5     2000     2000.006     -0.006       739.663     -21     2000     2000.006     -0.006       740.163     -21.5     2000     2000.006     -0.006       740.663     -22     2000     2000.006     -0.006       741.63     -22.5     2000     2000.006     -0.006       741.663     -23     2000     2000.007     -0.007       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	738.663	-20	2000	2000.006	-0.006
739.663     -21     2000     2000.006     -0.006       740.163     -21.5     2000     2000.006     -0.006       740.663     -22     2000     2000.006     -0.006       741.163     -22.5     2000     2000.006     -0.006       741.663     -23     2000     2000.007     -0.007       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	739,163	-20.5	2000	2000.006	-0.006
740.163     -21.5     2000     2000.006     -0.006       740.663     -22     2000     2000.006     -0.006       741.163     -22.5     2000     2000.006     -0.006       741.663     -23     2000     2000.007     -0.007       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	739.663	-21	2000	2000.006	-0.006
740.663     -22     2000     2000.006     -0.006       741.163     -22.5     2000     2000.006     -0.006       741.663     -23     2000     2000.007     -0.007       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	740.163	-21.5	2000	2000.006	-0.006
741.163     -22.5     2000     2000.006     -0.006       741.663     -23     2000     2000.007     -0.007       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	740.663	-22	2000	2000.006	-0.006
741.663     -23     2000     2000.007     -0.007       742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	741.163	-22.5	2000	2000.006	-0.006
742.163     -23.5     2000     2000.007     -0.007       742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	741.663	-23	2000	2000.007	-0.007
742.663     -24     2000     2000.007     -0.007       743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	742.163	-23.5	2000	2000.007	-0.007
743.163     -24.5     2000     2000.007     -0.007       743.663     -25     2000     2000.007     -0.007	742.663	-24	2000	2000.007	-0.007
743.663     -25     2000     2000.007     -0.007	743.163	-24.5	2000	2000.007	-0.007
	743.663	-25	2000	2000.007	-0.007
744.163   -25.5   2000   2000.007   -0.007	744.163	-25.5	2000	2000.007	-0.007

744.663	-26	2000	2000.008	-0.008
745.163	-26.5	2000	2000.008	-0.008
745.663	-27	2000	2000.008	-0.008
746.163	-27.5	2000	2000.008	-0.008
746.663	-28	2000	2000.008	-0.008
747.163	-28.5	2000	2000.008	-0.008
747.663	-29	2000	2000.008	-0.008
748.163	-29.5	2000	2000.009	-0.009
748.663	-30	2000	2000.009	-0.009
749.163	-30.5	2000	2000.009	-0.009
749.663	-31	2000	2000.009	-0.009
750.163	-31.5	2000	2000.009	-0.009
750.663	-32	2000	2000.009	-0.009
751.163	-32.5	2000	2000.01	-0.01
751.663	-33	2000	2000.01	-0.01
752.163	-33.5	2000	2000.01	-0.01
752.663	-34	2000	2000.01	-0.01
753.163	-34.5	2000	2000.01	-0.01
753.663	-35	2000	2000.01	-0.01
754.163	-35.5	2000	2000.01	-0.01
754.663	-36	2000	2000.011	-0.011
755.163	-36.5	2000	2000.011	-0.011
755.663	-37	2000	2000.011	-0.011
756.163	-37.5	2000	2000.011	-0.011
756.663	-38	2000	2000.011	-0.011
757.163	-38.5	2000	2000.011	-0.011
757.663	-39	2000	2000.012	-0.012
758.163	-39.5	2000	2000.012	-0.012
758.663	-40	2000	2000.012	-0.012
759.163	-40.5	2000	2000.012	-0.012
759.663	-41	2000	2000.012	-0.012
760.163	-41.5	2000	2000.012	-0.012
760.663	-42	2000	2000.013	-0.013
761.163	-42.5	2000	2000.013	-0.013
761.663	-43	2000	2000.013	-0.013
762.163	-43.5	2000	2000.013	-0.013
762.663	-44	2000	2000.013	-0.013
763.163	-44.5	2000	2000.013	-0.013
763.663	-45	2000	2000.013	-0.013
764.163	-45.5	2000	2000.014	-0.014
764.663	-46	2000	2000.014	-0.014
765.163	-46.5	2000	2000.014	-0.014
765.663	-47	2000	2000.014	-0.014
766.163	-47.5	2000	2000.014	-0.014
766.663	-48	2000	2000.014	-0.014
767.163	-48.5	2000	2000.015	-0.015
767.663	-49	2000	2000.015	-0.015
768.163	-49.5	2000	2000.015	-0.015
768.663	-50	2000	2000.015	-0.015

# Appendix E: Plane Projection Height Change Results

Elevation of the project site was 718.663m

Elevation	Height	Grid	Distance	Ground	Distance	Grid-Ground
(m)	Difference (m)	(m)		(m)		Distance (m)
668.663	-50		2000		1999.984	0.016
669.163	-49.5		2000		1999.984	0.016
669.663	-49		2000		1999.985	0.015
670.163	-48.5		2000		1999.985	0.015
670.663	-48		2000		1999.985	0.015
671.163	-47.5		2000		1999.985	0.015
671.663	-47		2000		1999.985	0.015
672.163	-46.5		2000		1999.985	0.015
672.663	-46		2000		1999.986	0.014
673.163	-45.5		2000		1999.986	0.014
673.663	-45		2000		1999.986	0.014
674.163	-44.5		2000		1999.986	0.014
674.663	-44		2000		1999.986	0.014
675.163	-43.5		2000		1999.986	0.014
675.663	-43		2000		1999.987	0.013
676.163	-42.5		2000		1999.987	0.013
676.663	-42		2000		1999.987	0.013
677.163	-41.5		2000		1999.987	0.013
677.663	-41		2000		1999.987	0.013
678.163	-40.5		2000		1999.987	0.013
678.663	-40		2000		1999.987	0.013
679.163	-39.5		2000		1999.988	0.012
679.663	-39		2000		1999.988	0.012
680.163	-38.5		2000		1999.988	0.012
680.663	-38		2000		1999.988	0.012
681.163	-37.5		2000		1999.988	0.012
681.663	-37		2000		1999.988	0.012
682.163	-36.5		2000		1999.989	0.011
682.663	-36		2000		1999.989	0.011
683.163	-35.5		2000		1999.989	0.011
683.663	-35		2000		1999.989	0.011
684.163	-34.5		2000		1999.989	0.011
684.663	-34		2000		1999.989	0.011
685.163	-33.5		2000		1999.99	0.01
685.663	-33		2000		1999.99	0.01
686.163	-32.5		2000		1999.99	0.01
686.663	-32		2000		1999.99	0.01
687.163	-31.5		2000		1999.99	0.01
687.663	-31		2000		1999.99	0.01
688.163	-30.5		2000		1999.99	0.01
688.663	-30		2000		1999.991	0.009
689.163	-29.5		2000		1999.991	0.009
689.663	-29		2000		1999.991	0.009
690.163	-28.5		2000		1999.991	0.009

690.663	-28	2000	1999.991	0.009
691.163	-27.5	2000	1999.991	0.009
691.663	-27	2000	1999.992	0.008
692.163	-26.5	2000	1999.992	0.008
692.663	-26	2000	1999.992	0.008
693.163	-25.5	2000	1999.992	0.008
693.663	-25	2000	1999.992	0.008
694.163	-24.5	2000	1999.992	0.008
694.663	-24	2000	1999.992	0.008
695.163	-23.5	2000	1999.993	0.007
695.663	-23	2000	1999.993	0.007
696,163	-22.5	2000	1999,993	0.007
696,663	-22	2000	1999,993	0.007
697,163	-21.5	2000	1999,993	0.007
697,663	-21	2000	1999.993	0.007
698,163	-20.5	2000	1999.994	0.006
698 663	-20	2000	1999 994	0.006
699 163	-19.5	2000	1999 994	0.006
699 663	-19	2000	1999 994	0.006
700 163	-18.5	2000	1999 994	0.006
700.663	-18	2000	1999 994	0.006
701 163	-17.5	2000	1999.995	0.005
701.663	-17	2000	1999.995	0.005
702 163	-16.5	2000	1000.000	0.005
702.103	-16	2000	1999.995	0.005
702.003	-15.5	2000	1999.995	0.005
703.663	-13.5	2000	1999.995	0.005
704 163	-14.5	2000	1999.995	0.005
704.103	-14.5	2000	1999.995	0.003
705 163	13.5	2000	1999.990	0.004
705.103	-13.3	2000	1999.990	0.004
705.003	-13	2000	1999.990	0.004
706.663	-12.5	2000	1999.990	0.004
700.003	-12	2000	1999.990	0.004
707.103	-11.5	2000	1999.990	0.004
707.003	-11	2000	1999.997	0.003
708.103	-10.5	2000	1999.997	0.003
700.003	-10	2000	1999.997	0.003
709.103	-9.0	2000	1999.997	0.003
710 163	-9	2000	1999.997	0.003
710.103	-0.5	2000	1000.007	0.003
710.003	-0	2000	1999.997	0.003
711.103	-7.5	2000	1999.998	0.002
712.162	-1	2000	1999.990	0.002
712.103	-0.0	2000	1999.996	0.002
712.003	-0	2000	1999.996	0.002
712.000	-5.5	2000	1999.998	0.002
714 460	-5	2000	1999.998	0.002
714.000	-4.5	2000	1999.999	0.001
745.400	-4	2000	1999.999	0.001
715.103	-3.5	2000	1999.999	0.001
715.003	-3	2000	1999.999	0.001
710.103	-2.5	2000	1999.999	0.001
/16.663	-2	2000	1999.999	0.001
/1/.163	-1.5	2000	2000	0

717.663	-1	2000	2000	0
718.163	-0.5	2000	2000	0
718.663	0	2000	2000	0
719.163	0.5	2000	2000	0
719.663	1	2000	2000	0
720.163	1.5	2000	2000	0
720.663	2	2000	2000.001	-0.001
721.163	2.5	2000	2000.001	-0.001
721.663	3	2000	2000.001	-0.001
722.163	3.5	2000	2000.001	-0.001
722.663	4	2000	2000.001	-0.001
723.163	4.5	2000	2000.001	-0.001
723.663	5	2000	2000.002	-0.002
724.163	5.5	2000	2000.002	-0.002
724.663	6	2000	2000.002	-0.002
725.163	6.5	2000	2000.002	-0.002
725.663	7	2000	2000.002	-0.002
726.163	7.5	2000	2000.002	-0.002
726.663	8	2000	2000.003	-0.003
727.163	8.5	2000	2000.003	-0.003
727.663	9	2000	2000.003	-0.003
728.163	9.5	2000	2000.003	-0.003
728.663	10	2000	2000.003	-0.003
729.163	10.5	2000	2000.003	-0.003
729.663	11	2000	2000.003	-0.003
730.163	11.5	2000	2000.004	-0.004
730.663	12	2000	2000.004	-0.004
731.163	12.5	2000	2000.004	-0.004
731.663	13	2000	2000.004	-0.004
732.163	13.5	2000	2000.004	-0.004
732.663	14	2000	2000.004	-0.004
733.163	14.5	2000	2000.005	-0.005
733.663	15	2000	2000.005	-0.005
734.163	15.5	2000	2000.005	-0.005
734.663	16	2000	2000.005	-0.005
735.163	16.5	2000	2000.005	-0.005
735.663	17	2000	2000.005	-0.005
736.163	17.5	2000	2000.005	-0.005
736.663	18	2000	2000.006	-0.006
737.163	18.5	2000	2000.006	-0.006
737.663	19	2000	2000.006	-0.006
738.163	19.5	2000	2000.006	-0.006
738.663	20	2000	2000.006	-0.006
739.163	20.5	2000	2000.006	-0.006
739.663	21	2000	2000.007	-0.007
740.163	21.5	2000	2000.007	-0.007
740.663	22	2000	2000.007	-0.007
741.163	22.5	2000	2000.007	-0.007
741.663	23	2000	2000.007	-0.007
742.163	23.5	2000	2000.007	-0.007
742.663	24	2000	2000.008	-0.008
743.163	24.5	2000	2000.008	-0.008
743.663	25	2000	2000.008	-0.008
744.163	25.5	2000	2000.008	-0.008

744.663	26	2000	2000.008	-0.008
745.163	26.5	2000	2000.008	-0.008
745.663	27	2000	2000.008	-0.008
746.163	27.5	2000	2000.009	-0.009
746.663	28	2000	2000.009	-0.009
747.163	28.5	2000	2000.009	-0.009
747.663	29	2000	2000.009	-0.009
748.163	29.5	2000	2000.009	-0.009
748.663	30	2000	2000.009	-0.009
749.163	30.5	2000	2000.01	-0.01
749.663	31	2000	2000.01	-0.01
750.163	31.5	2000	2000.01	-0.01
750.663	32	2000	2000.01	-0.01
751.163	32.5	2000	2000.01	-0.01
751.663	33	2000	2000.01	-0.01
752.163	33.5	2000	2000.011	-0.011
752.663	34	2000	2000.011	-0.011
753.163	34.5	2000	2000.011	-0.011
753.663	35	2000	2000.011	-0.011
754.163	35.5	2000	2000.011	-0.011
754.663	36	2000	2000.011	-0.011
755.163	36.5	2000	2000.011	-0.011
755.663	37	2000	2000.012	-0.012
756.163	37.5	2000	2000.012	-0.012
756.663	38	2000	2000.012	-0.012
757.163	38.5	2000	2000.012	-0.012
757.663	39	2000	2000.012	-0.012
758.163	39.5	2000	2000.012	-0.012
758.663	40	2000	2000.013	-0.013
759.163	40.5	2000	2000.013	-0.013
759.663	41	2000	2000.013	-0.013
760.163	41.5	2000	2000.013	-0.013
760.663	42	2000	2000.013	-0.013
761.163	42.5	2000	2000.013	-0.013
761.663	43	2000	2000.013	-0.013
762.163	43.5	2000	2000.014	-0.014
762.663	44	2000	2000.014	-0.014
763.163	44.5	2000	2000.014	-0.014
763.663	45	2000	2000.014	-0.014
764.163	45.5	2000	2000.014	-0.014
764.663	46	2000	2000.014	-0.014
765.163	46.5	2000	2000.015	-0.015
765.663	47	2000	2000.015	-0.015
766.163	47.5	2000	2000.015	-0.015
766.663	48	2000	2000.015	-0.015
767.163	48.5	2000	2000.015	-0.015
767.663	49	2000	2000.015	-0.015
768.163	49.5	2000	2000.016	-0.016
768.663	50	2000	2000.016	-0.016

# Appendix F: Procedure to Establish a Local Ground-Based Coordinate System using a Transverse Mercator Projection

The method below is to establish a local ground-based coordinate system in TGO using a Transverse Mercator Projection.

- 1. Open the Coordinate System Manager in TGO
- 2. Copy the GDA Zone relating to the site to a site folder
- 3. Rename the GDA Zone to something meaningful
- 4. Edit the Zone File and under the Projection Tab change
  - a. In positive coordinate direction check North & East
  - b. Set Central Latitude =  $-0^{\circ} 00' 00'' N$
  - c. Set Central Longitude = 153° 00' 00" (centre longitude of chosen MGA zone)
  - d. Set False Easting
  - e. Set False Northing
  - f. Set Central Scale Factor (calculated below)

#### Calculating the Central Scale Factor:

- 1. Find the MGA point scale factor for the central point of the site
- 2. Calculate the Datum Scale Factor (DSF) using

$$DSF = \frac{R}{(R+h)}$$

Where: R = Geometric Mean Radius of Curvature at the central point h = Ellipsoidal Height of the site

3. Calculate the Combined Scale Factor using

Combined Scale Factor = MGA point scale factor  $\times$  Datum Scale Factor

4. Calculate the Central Scale Factor using

Central Scale Factor = MGA central Scale Factor (0.9996) / Combined Scale Factor

Once this is completed save the coordinate system in the coordinate system editor. Open TGO and create a new job. In project properties under the coordinate system settings select the coordinate system created using the coordinate system editor. Modify the local site settings by entering the local ground-based coordinate system coordinates and the elevation of the central point of the site. Check the resulting computed WGS84 latitude and longitude and ellipsoidal height for the central point of the site with your own calculations.

# Appendix G: Procedure to Establish a Local Ground-Based Coordinate System using a Plane Projection

The method below is to establish a local ground-based coordinate system in TGO using a Plane Projection.

- 1. Open Coordinate System Manager
- 2. Edit Add Coordinate System Plane
- 3. Name the Projection
- 4. Select a Datum either WGS84 or ITRF
- 5. Select Geoid Grid Model
- 6. Select Ausgeoid 98
- 7. In the Projection screen change:
  - a. Positive coordinate direction to North and East
  - b. Central Latitude to the Latitude of the central point of the site
  - c. Central Longitude to the longitude of the central point of the site
  - d. Set the height above the ellipsoid to the ellipsoidal height of the site
  - e. Set the height above the geoid to the elevation on the site
  - f. Set rotation angle (in seconds) to the MGA grid convergence of the central point to bring the coordinate system onto MGA meridian
  - g. Set False Northing
  - h. Set False Easting
  - i. Set the Scale Factor to 1

In step (f) the grid convergence was calculated using the redfearn spreadsheet available from Geoscience Australia.

Once this projection is created, create a new job in TGO. In the project properties select the coordinate system created above. Edit the local site settings and input the local ground-based system coordinates and the elevation of the central point. Check the resulting computation of WGS84 latitude and longitude and the ellipsoidal height of the central point with your own calculations.

# **References:**

Allan, A Hollwey, J & Maynes, J 1968, *Practical Field Surveying and Computations*, William Heinemann Ltd, London

Brainerd J & Pang A 1998, Floating ring: a new tool for visualizing distortion in map projections, *Computer Graphics International 1998 Proceedings*, 22-26 June 1998, p. 466-480, Viewed 18 May 2008, IEE Xplore, Digital Object Identifier 10.1109/CGI.1998.694300

Burkholder, EF 1991, Computation of Horizontal/Level Distances, *Journal of Surveying Engineering*, Vol. 117, No. 3, p. 104-117

Burkholder, EF 1993, Design of a Local Coordinate System for Surveying,Engineering, and LIS/GIS, *Surveying and Land Information Systems*, Vol. 53, No. 1, pp. 29-40

Department of Natural Resources and Mines 2005, *Cadastral Survey Requirements*, Version 4.0, Standards and Guidelines, Department of Natural Resources and Water, Brisbane, Queensland, viewed 15 May 2008, <http://www.nrw.qld.gov.au/about/policy/documents/2093/pdfs/cadastralsurveyv4. pdf>

Dion, T 2002, *Land Development for Civil Engineers*, 2nd edn, Wiley & Sons Inc, New York, New York

Estopinal, S 1992, *A Guide to Understanding Land Surveys*, 2<sup>nd</sup> edn, John Wiley and Sons, New York, New York

Inter-Governmental Committee on Surveying and Mapping 2008, Geocentric Datum of Australia Technical Manual, ver. 2.3, Inter-Governmental Committee on Surveying and Mapping, Canberra

<http://www.icsm.gov.au/gda/gdatm/index.html>

Iliffe J 2002, *Datums and Map Projections for remote sensing, GIS and Surveying*, Whittles Publishing, Latheronwheel, Caithness, Scotland

Leica Geosystems AG 2006, *Leica TPS1200+ Series Technical Data*, Leica Geosystems AG, Heerbrugg, Switzerland, Viewed 6 August 2008, <a href="http://www.leica-geosystems.com/common/shared/downloads/inc/downloader.asp?id=2655">http://www.leica-geosystems.com/common/shared/downloads/inc/downloader.asp?id=2655</a>

Leica Geosystems AG 2007, *Leica GPS1200 Series Technical Data*, Leica Geosystems AG, Heerbrugg, Switzerland, Viewed 12 May 2008, <a href="http://www.leica-geosystems.com/common/shared/downloads/inc/downloader.asp?id=2670">http://www.leica-geosystems.com/common/shared/downloads/inc/downloader.asp?id=2670</a>

Lemmon, T & Gerdan, G 1999, The Influence of the Number of Satellites on the Accuracy of RTK GPS Positions, *The Australian Surveyor*, Vol. 44, No. 1, pp. 64-70

Maling, D.H 1992, *Coordinate Systems and Map Projections*, 2nd end, Pergamon Press, Oxford, England

Mulcahy, K & Clarke, K 2001, Symbolisation of Map Projection Distortion: A Review, *Cartography and Geographic Information Science*, Vol. 28, No. 3, pp.167-181

Natural Resources Canada 2005, *Glossary of Remote Sensing Terms*, Natural Resources Canada, Ottawa, Ontario, Canada, viewed 20 May 2008, <a href="http://www.ccrs.nrcan.gc.ca/glossary/index\_e.php?id=311">http://www.ccrs.nrcan.gc.ca/glossary/index\_e.php?id=311</a>

Petrotechnical Open Standards Consortium (POSC) 1997, POSC Epicentre, Version 2.2, Usage Guide, Energistics, Houston, Texas, USA, Viewed 10 May 2008, <a href="http://www.posc.org/Epicentre.2\_2/DataModel/ExamplesofUsage/eu\_toc.html">http://www.posc.org/Epicentre.2\_2/DataModel/ExamplesofUsage/eu\_toc.html</a>

Stem, J 1989, *State Plane Coordinate System of 1983*, NOAA Manual NOS NGS 5, National Geodetic Survey, Silver Spring, Maryland, viewed 28 April 2008, <http://www.ngs.noaa.gov/PUBS\_LIB/ManualNOSNGS5.pdf> Survey and Mapping Infrastructure Act 2003 (Queensland)

Survey and Mapping Infrastructure Regulation 2004 (Queensland)

*SVY 1104 N/A Survey Computations A Study Book*, 2005, University of Southern Queensland (USQ), Toowoomba, Australia.

*SVY 3107 N/A Geodetic Surveying B Study Book*, 2007, University of Southern Queensland (USQ), Toowoomba, Australia.

Trimble Navigation Limited 2006, *Trimble 5800 GPS System*, Datasheet, Trimble Navigation Limited, Dayton, Ohio, Viewed 12 May 2008, <a href="http://trl.trimble.com/docushare/dsweb/Get/Document-32185/022543-016C\_5800GPS\_DS\_0706\_lr.pdf">http://trl.trimble.com/docushare/dsweb/Get/Document-32185/022543-016C\_5800GPS\_DS\_0706\_lr.pdf</a>

Trimble Navigation Limited 2007a, *Trimble S6 Total Station*, Datasheet, Trimble Navigation Limited, Drayton, Ohio, Viewed 6 August 2008, <a href="http://trl.trimble.com/docushare/dsweb/Get/Document-208580/022543-0984">http://trl.trimble.com/docushare/dsweb/Get/Document-208580/022543-0984">http://trl.trimble.com/docushare/dsweb/Get/Document-208580/022543-0984</a>

Trimble Navigation Limited 2007b, *Trimble R8 GNSS System*, Datasheet, Trimble Navigation Limited, Dayton, Ohio, Viewed 12 May 2008, <a href="http://trl.trimble.com/docushare/dsweb/Get/Document-140079/022543-079F\_TrimbleR8GNSS\_DS\_0507\_lr.pdf">http://trl.trimble.com/docushare/dsweb/Get/Document-140079/022543-079F\_TrimbleR8GNSS\_DS\_0507\_lr.pdf</a>

Topcon Corporation 2006, *GPT9000A Series GTS 900A Series Robotic Total Station System*, Brochure, ABC Lasers, Arcadia Ridge, Queensland, Viewed 8 August 2008, <http://www.abclasers.com.au/brochures/GPT9000SeriesBroch.pdf>

Topcon Positioning Systems Inc 2008, *GR-3 Triple Constellation Receiver*, Topcon Positioning Systems Inc, Livermore, California, viewed 12 May 2008 <http://www.topconpositioning.com/about-us/contact-info/> Robinson, AH, Morrison, JL, Muehrcke, PC, Kimerling AJ & Guptill SC 1995, *Elements of Cartography*, 6th edn, J. Wiley & Sons Inc, New York, New York

Wisconsin State Cartographers Office 2004, *Wisconsin Coordinate Systems*, Handbook, Wisconsin State Cartographers Office, Madison, Wisconsin, USA, Viewed 29<sup>th</sup> April 2008, <http://www.sco.wisc.edu/pubs/wiscoord/wiscoord.php>

Wolf, PR & Brinker, RC 1994, *Elementary Surveying*, 9th edn, HarperCollins, New York, New York

Wolf, PR & Ghilani, CD 2002, *Elementary Surveying an Introduction to Geomatics*, 10th edn, Prentice Hall, Upper Saddle River, New Jersey, USA.