

University of Southern Queensland
Faculty of Engineering & Surveying

Control of Accumulator in a Continuous Processing Line

A dissertation submitted by

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in fulfilment of the requirements of

ENG4112 Research Project

towards the degree of

Bachelor of Engineering (Computer Systems)

Submitted: October, 2008

Abstract

Continuous processing lines are used in various manufacturing industries including aluminium, paper and steel. Common processes include annealing, pickling, metal coating, hot dip galvanizing, and painting. A standard design feature incorporated by continuous processing line manufacturers is the inclusion of an accumulator at the entry and exits points of the processing section to ensure its continuous operation.

Continuous processing of strip and product quality are key performance targets for operating continuous processing lines. Uncontrolled tension transients in the strip will normally result in poor product quality which can be attributed to poor performance from an accumulator control system. Thus, the optimal method of controlling an accumulator in a continuous processing line is a significant and important issue.

This dissertation presents an analysis of methods for controlling an accumulator to minimise potential sources of disturbance. The analysis included modelling the accumulator dynamics and simulation with popular industrial control methods. Initially a base case was established using a proportional + integral + derivative controller (PID), which is the current standard for industrial control. The base case was used as the benchmark prior to simulation and analysis of alternative or less conventional industrial controllers.

A key emphasis of this dissertation is an association to a real world problem. The project results suggest that the classical PID controller is more suited to this application than the alternatives included in the analysis.

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Acknowledgments

I would like to extend my sincere thanks to my project supervisor Dr. Paul Wen for his guidance and assistance during my thesis.

I would also like to thank my wife Marlene and my daughters Jaeda and Malea for their patience and understanding of my commitment to studies and in particular this project.

MICHAEL RICHARD CRAWFORD-ROSS

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October 2008

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Chapter 1

Introduction

This chapter will introduce the role of an accumulator in a continuous processing line. The importance of maintaining and controlling tension in the line is highlighted and discussed. Lastly, the project aims and objectives are discussed along with a general overview of the dissertation.

1.1 Background and Motivation

Continuous processing lines are used in various manufacturing industries including aluminium, paper and steel. A continuous processing line (CPL) is typically split into three distinct sections termed the entry, process and exit sections. The process section is where the properties of the input material which is typically termed strip or web is altered and is usually required to be operating continuously. Processing of the material commonly involves complex procedures where recovery from a stoppage can be lengthy and can also result in defected material. Typical processes include annealing, pickling, metal coating, hot dip galvanizing, and painting.

The inputs and outputs of a CPL are finite lengths of material usually wound as large coils. As the input and output is finite in length, the entry and exit sections are required to stop to changeover coils. Therefore, there needs to be a method to ensure the process keeps operating. The method most commonly used is to incorporate an entry and exit accumulator to act as buffers. A typical layout for a CPL is shown in Figure 1.1. To generate and maintain the desired strip tensions in the line a strip transport system is employed in the CPL comprising bridles and deflector rolls in strategic locations. A bridle is made up of rolls, gearboxes and motors with the number of rolls and motor/gearbox sizes determined by the line tension required at that point in the line.

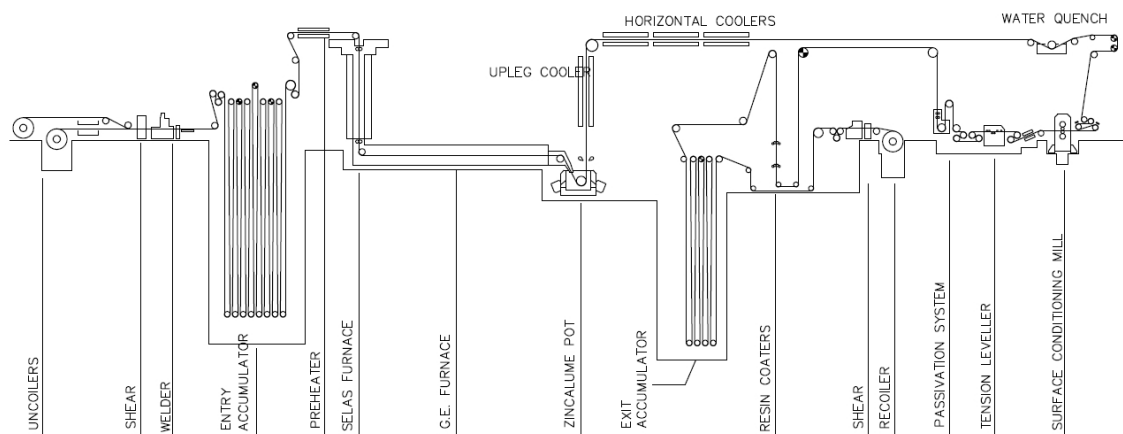


Figure 1.1: Layout of a continuous processing line

Uncontrolled tension transients in the strip will normally result in poor product quality associated with catenary scratches, strip breaks, and bridle slippage. A possible source of disturbance to maintaining desired strip tension in the line can be caused by a poor performing accumulator. For this reason it is crucial for an operating line to have good control of both the entry and exit accumulators.

1.2 Aims and Objectives

The aim of this project is to examine the optimal method of controlling an accumulator in a continuous processing line. The research project has been broken down into the following objectives:

1. To research the current methods used in industry for controlling accumulators
2. To use MATLAB[®] Simulink[®] to model the dynamic system of an accumulator.
3. To simulate the operation of an accumulator with the industry standard controllers and analyse the system response.
4. To critically evaluate the controllers with respect to their performance for controlling the system.
5. To investigate the use of alternate control methodologies.

1.3 Overview of the Dissertation

This dissertation consists of seven chapters. This chapter has introduced the reader to the fundamental components in a continuous processing line. This first chapter has also covered the aims and objectives of the project and highlighted the importance of the research. Chapter 2 provides a literature review of the research topic. In chapter 3 the methodology used to model the dynamics of the accumulator is discussed and the rationale behind the development and implementation strategies is discussed.

Chapters 4 and 5 examine the controller types including design principles and establishes the standard controller currently utilised in industry. Alternative and less conventional controller types are also proposed. Various tuning methods are also discussed and compared. Chapter 6 provides details of the methodology used to

simulate the dynamic system. Base case performance results are used as a benchmark for comparison against the results from alternative control methods. Chapter 6 also includes analysis of the simulation results and critical evaluation of the effectiveness of each control method with regards to performance. A summary of the research and concluding comments including suggested areas for future research work are discussed in chapter 7.

1.4 Summary

In summary, this chapter has provided background information to continuous processing lines and the role of an accumulator in these systems. The framework for the dissertation project was detailed and aims of the research project were discussed.

Chapter 2

Literature Review

2.1 Introduction

The central aim of this project is to examine research into current methods used in industry for controlling accumulators in a CPL. A control system provides an output for a given input and a controller forms one part of that system. There is considerable literature that covers the theory and practice of control systems that can function in a CPL environment (Ogata 2002) and (Nise 2004).

2.2 The Industry Standard Controller

Currently a high proportion of industrial controllers utilise PID (Proportional + Integral + Derivative) control schemes. Research into the control of accumulators conducted by (Pagilla, Garimella, Dreinhoefer & King 2001) indicates that PID control is used extensively in industry. The prevalence of PID control in industry can be attributed in part to its ease of application into most control systems. The controller is robust over a wide range of operating conditions including plant

uncertainty and external disturbances (Shinners 1998). In addition, to its ease of application there are established techniques for tuning the parameters of the PID controller. As detailed in (Ogata 2002), these include working from a set of experimental rules, using an heuristic approach, or arriving at a mathematical solution through incremental analysis.

Although there are a lot of positive aspects in implementing simple PID control, it can be inadequate for good control and disturbance rejection. A disturbance to the control system can be either an internal or external influence that tends to adversely affect the output of the system. One role of the controller is to handle this disturbance and minimise or reject its influence. A possible reason for PID controller inadequacy can be attributed to its fixed structure. The PID controller has three tunable parameters, a proportional term, and integral term and a derivative term. Once the parameters of the PID controller are tuned, that is generally where they remain. In fact (Pagilla, Garimella, Dreinhoefer & King 2003) suggest that this strategy often leads to large tension variations in the accumulator and tension disturbance propagation both upstream and downstream from the accumulator.

2.3 Alternatives to the Industry Standard Controller

With digital computer technology, adaptive control is an alternative method that potentially improves performance on the simple PID controller. This control method uses algorithms to constantly calculate and update the PID controller parameters with optimum values. The PID controller parameters are deemed to be optimum if they enable the controller to meet the desired performance criteria. Digital computers have enabled online tunable PID controller parameters to become a reality. However, the input signal for a control loop is typically analogue in nature. In the digital environment this input signal is digitised through a sample and conversion process. Once the signal is digitised it is then accessible for application in an algorithm to

manipulate the parameters of a PID controller for optimum values prior to acting upon the output of the system. The PID controller in the digital world is shown below:

$$M(z) = \frac{(q_0 + q_1 z^{-1} + q_2 z^{-2})}{(1 - z^{-1})} E(z) \quad (2.1)$$

Which can then be represented as a difference equation:

$$m_k = q_0 e_k + q_1 e_{k-1} + q_2 e_{k-2} + m_{k-1} \quad (2.2)$$

For adaptive control the parameters q_0 , q_1 and q_2 are automatically tuned by an algorithm to provide optimal performance by the controller. Techniques that could be utilised to calculate the parameters include Genetic Algorithms and Fuzzy Control as indicated by (Chung, Lee & Cho 2005).

Another method is using the state variables of the system in the feedback path. Termed state variable feedback this control method requires that all states of the system are accessible and that instrumentation exist for measurement. In some circumstances, it is not possible to measure a certain state variable due to cost, accuracy or availability. In these cases a state observer is needed (Ogata 2002). This control strategy has been used in simulations on a CPL by (Pagilla et al. 2003) with promising results that showed reductions in strip tension variations.

Intelligent control is another field in modern control technology that has a presence in industry. Intelligent control systems include fuzzy control, learning control, expert control, and neural network control. (Driankov, Hellendoorn & Reinfrank 1996) state that fuzzy control has gained popularity. In addition (Nguyen, Prasad, Walker & Walker 2003) confirm that fuzzy and neural network control have been applied successfully throughout industry.

2.4 Summary

In summary, this chapter has identified the PID controller as being the most common control method currently applied in industry. This chapter has outlined reasons for its popularity. In addition some alternative control methods have also been discussed. The review has limited itself to outlining alternative control methods that are currently operating or have been successfully tested in the field. The control methods identified in this review will be used in a simulation study with the dynamic system model of an accumulator. This will be detailed in the next chapter.

Chapter 3

Modelling the Dynamic System

3.1 Introduction

To analytically assess the response of a control method a mathematical model of the controlled plant is required. A mathematical model is a set of equations that represent the dynamics of the plant which are derived from the physical laws governing its behaviour. For a mathematical model or dynamic system model typically Kirchoff's laws are used for electrical systems and Newton's laws are used for mechanical systems.

3.2 Theoretical Basis

A dynamic system model is obtained through the detailed analysis of a system but with approximations to allow the simplification of the equations. System analysis can be achieved analytically by applying the fundamental physical laws of science and engineering or empirically using system identification techniques. (Klee 2007) suggests that dynamic system models are inexact and there is often a trade off

between complexity and usefulness. The application for the dynamic system model will usually determine the level of complexity required. For instance, space travel is an application that would require complex modelling. Extreme levels of accuracy are needed to launch a rocket and have it travel along a defined trajectory to a final destination. Less critical applications may not require the same levels of accuracy and can utilise a model comprising lesser complexity. A model, regardless of the accuracy is the starting point of a simulation study. The results should then be analysed to determine their validity before proceeding.

The carriage dynamics of an accumulator can be modelled as a mechanical system exhibiting mass-spring-damper like characteristics. A mass-spring-damper system is displayed in Figure 3.1 and can be described mathematically by 3.1.

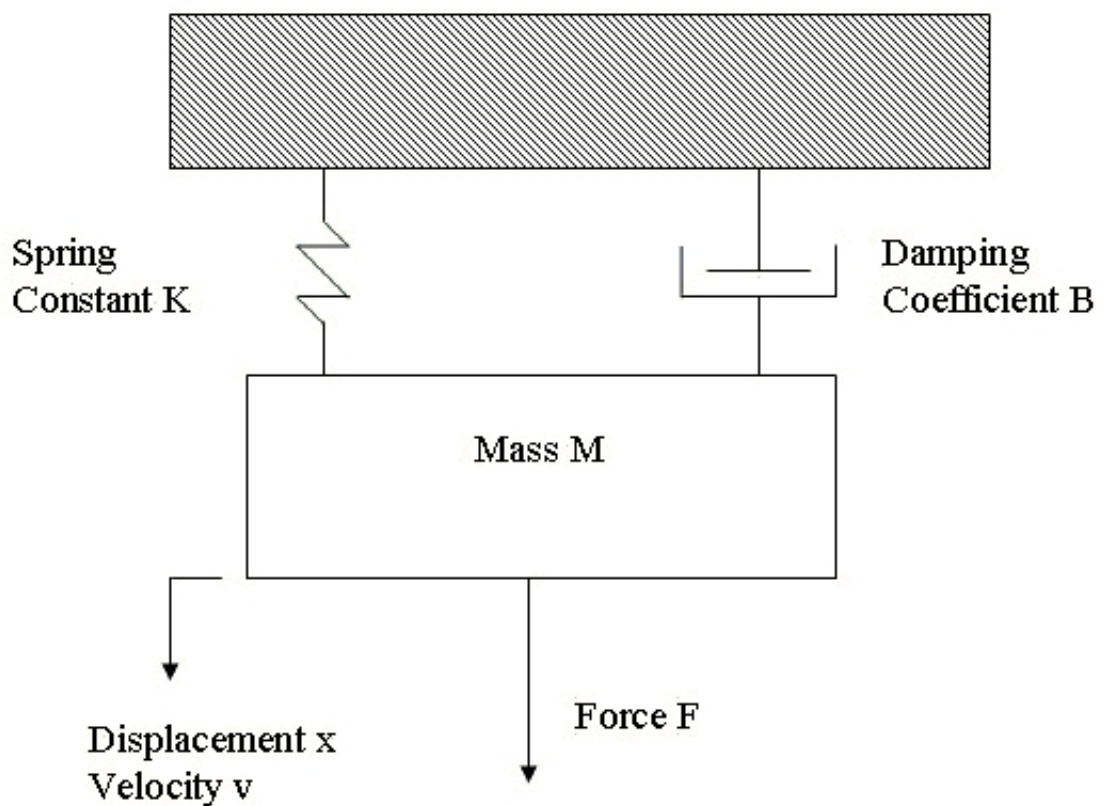


Figure 3.1: A mass-spring-damper system

$$f(t) = M \frac{d^2x(t)}{dt^2} + B \frac{dx(t)}{dt} + Kx(t) \quad (3.1)$$

3.3 Dynamic System Model

The accumulator tower in a CPL is a mechanical system with a complex arrangement of gears, rolls, pulleys and drive system. An accumulator arrangement is displayed in Figure 3.2.

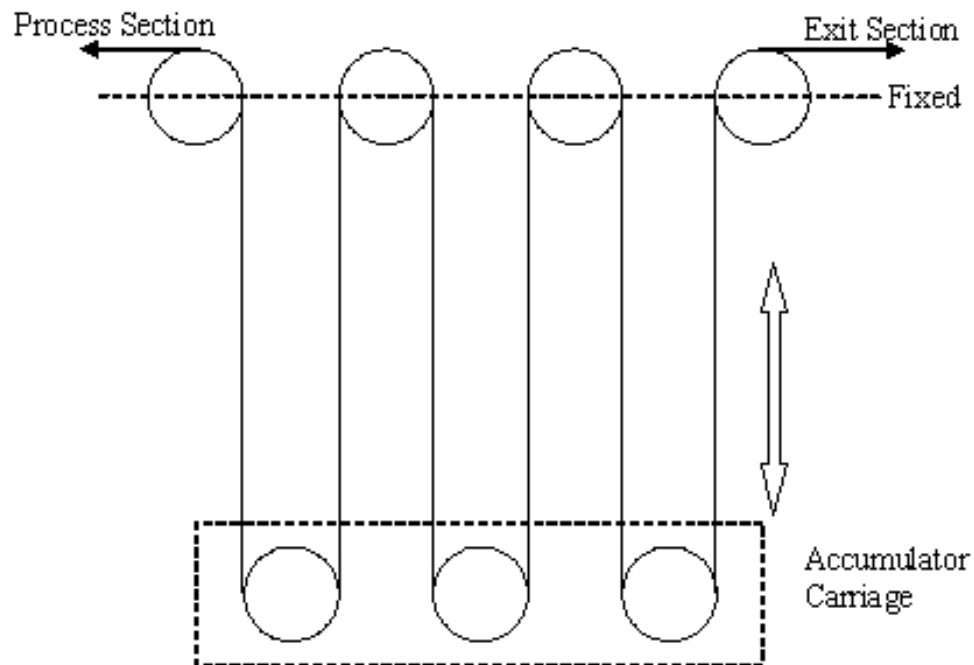


Figure 3.2: Sketch of an accumulator

Developing the dynamic model of the accumulator mechanical system from first principles is a complicated task and worthy of a project dedicated solely to this topic. This being the case along with the timeframe of the project, a model of the plant has been generated based on the dynamics of the accumulator carriage and its web spans as provided by (Pagilla, Garimella, Dreinhofer & King 2000).

The accumulator carriage dynamics are:

$$M_c \frac{d^2 x_c(t)}{dt^2} = F_h(t) - F_d(t) - M_c g - N t_c(t) \quad (3.2)$$

The average tension dynamics are:

$$\frac{dt_c}{dt} = \frac{AE}{x_c(t)} \frac{1}{N} (v_e(t) - v_p(t)) + \frac{AE}{x_c(t)} x_c(t) - \frac{1}{x_c(t)} t_c(t) x_c(t) \quad (3.3)$$

Where:

M_c	carriage mass;	t_c	average sum of tensions in the web spans
x_c	carriage position;	v_p	process section velocity;
F_h	controlled force;	v_e	exit section velocity;
F_d	disturbance force;	A	area of cross section of the strip;
g	acceleration due to gravity;	E	modulus of elasticity of the strip;
N	number of spans;		

3.4 Implementation

The software MATLAB[®] Simulink[®] is an industry and academic standard and has been successfully implemented to model and simulate the dynamic system of the

accumulator. Displayed in Figure 3.3 is a block diagram of the model developed in Simulink®.

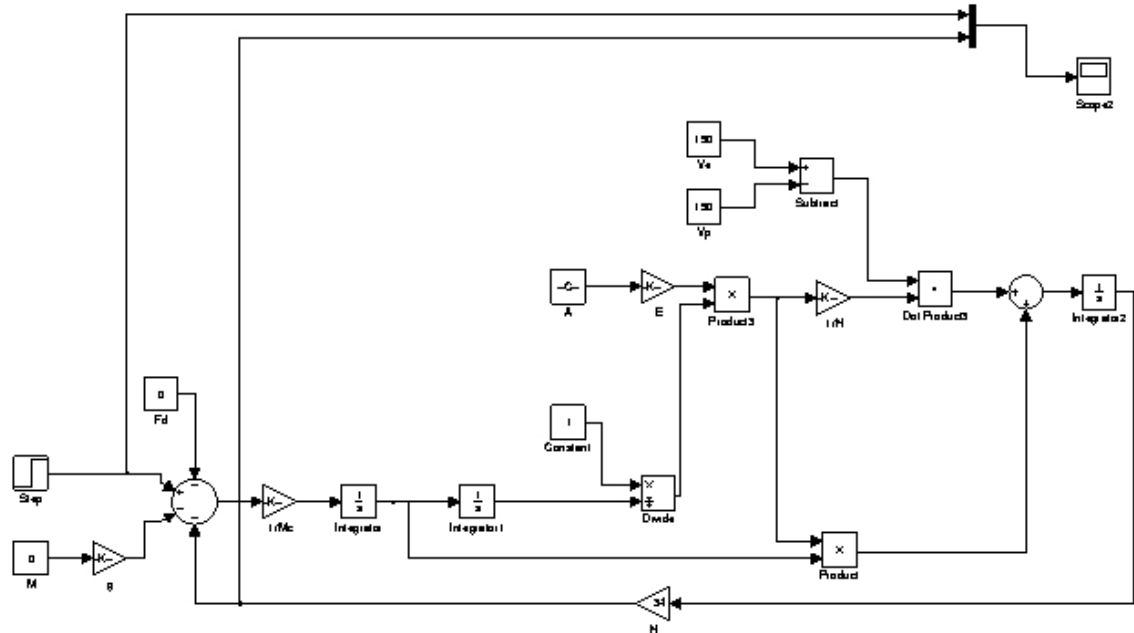


Figure 3.3: Simulink model of the dynamic system for the accumulator

Figure 3.3 has been developed with a step input connected to the position of the controlled force. This has been done so that the unit step response of the mechanical system can be assessed. Examining the response provides insight into the mechanical system's physical parameters and will be used when designing the controller. The unit step response for the accumulator dynamic system is shown in Figure 3.4. It clearly exhibits the characteristics of an undamped response. The system oscillates at approximately 23 rads/sec with an amplitude of 2 when subjected to the unit step input. The oscillatory nature of the system is attributed to the elasticity of the strip contained within the accumulator.

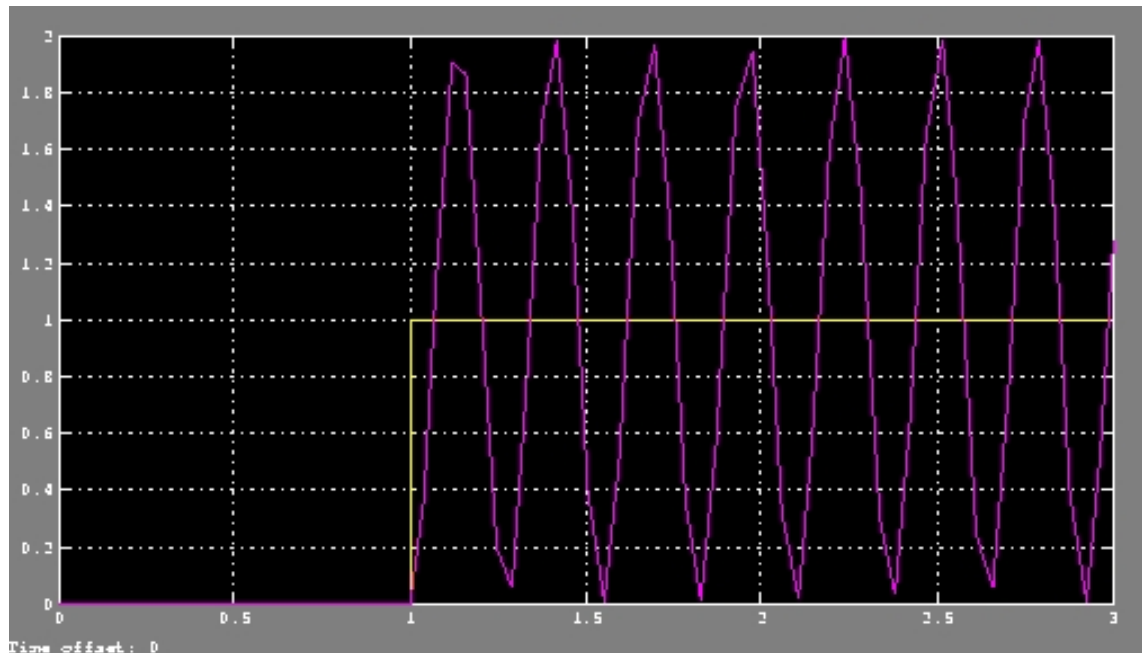


Figure 3.4: Unit step response for the accumulator dynamic system

As mentioned previously the accumulator dynamics are modelled based on a linear second order system. The two quantities that describe the characteristics of a second order system transient response are the damping ratio and the natural frequency. The general second order transfer function is shown below.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (3.4)$$

Where:

- ζ damping ratio;
- ω_n natural frequency;

3.5 Summary

In summary, this chapter described a dynamic system model for an accumulator in a CPL. The accumulator model is used in the current simulation study to determine performance results for the control methods identified in this literature review. This chapter also detailed the deficient performance characteristics of the accumulator dynamics when simulated. To yield the desired performance, the deficiencies are required to be designed out of the system. The next chapter will describe the design of a controller to enhance the system.

Chapter 4

The Industry Standard Controller

4.1 Introduction

The most widely utilised controller in industrial process control is the three term controller or PID (Proportional + Integral + Derivative). Considering the prevalence of the PID controller in industrial process control this controller forms the benchmark response for comparative analysis with alternative controllers. To obtain the benchmark response, a controller had to be designed for simulation with the accumulator model. Control systems are typically designed by applying established engineering techniques and procedures to achieve required performance specifications. The PID controller design process will be discussed in the following sections.

4.2 Background

A block diagram of the PID controller is displayed in Figure 4.1.

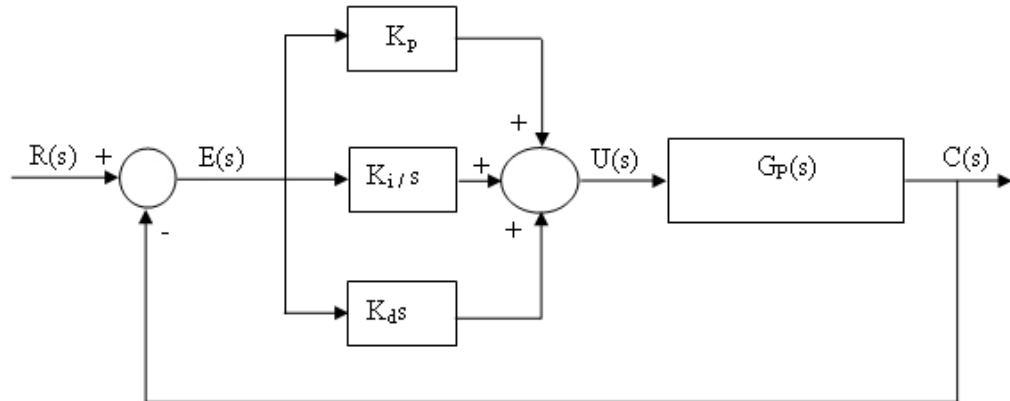


Figure 4.1: Block diagram of a PID controller

The PID controller is a form of compensating controller that exhibits robust performance over a wide range of operating conditions including external disturbances (Shinners 1998), (Dorf & Bishop 2008). The PID controller can be implemented by applying all three terms but quite often not all are required. A control system is designed to perform at required specifications and quite often this can be achieved by applying only a partial combination of the three terms. A PID controller consists of three separate terms acting upon the error of the system (K_p , K_i and K_d) as represented in Figure 4.1. The controller can be implemented as a straight proportional only controller by setting the integral and derivative terms to zero. Thus, this leaves only the proportional term active but this is the least likely application of the controller. Other scenarios are to combine the proportional and derivative terms to form a PD controller or the proportional and integral terms to form a PI controller. Hence, as the three terms of the controller are summed together,

any term (K_p , K_i or K_d) set to zero is rendered inactive. The combination of a PD controller establishes a lead type compensator and the combination of a PI controller establishes a lag type compensator and lastly combining all three terms establishes a lead / lag type compensator. These types of controllers are cascade type compensators. Table 4.1 displays the controllers with associated functionality (Nise 2004). Cascade compensation implies that the controller is in series with the forward-path transfer function as detailed in Figure 4.2.

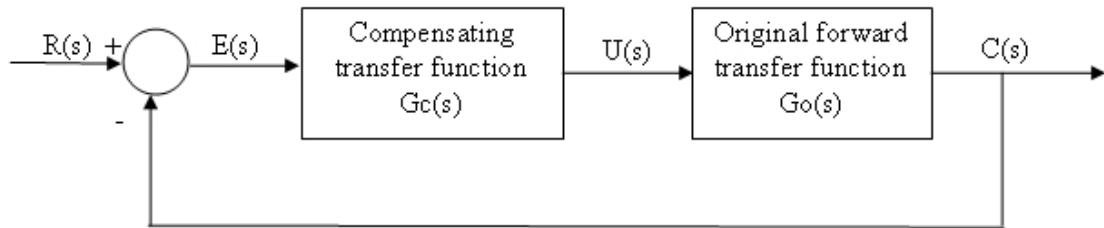


Figure 4.2: Cascade compensation

The transfer function of the PID controller $G_c(s)$ is

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s \quad (4.1)$$

If the integral term K_i is set to zero a PD controller is formed and the transfer function is

$$G_c(s) = K_p + K_d s \quad (4.2)$$

Likewise if the derivative term K_d is set to zero a PI controller is formed and the transfer function is

$$G_c(s) = K_p + \frac{K_i}{s} \quad (4.3)$$

Table 4.1: Types of controllers

Controller	Transfer Function	Function
PI	$K \frac{s+z_c}{s}$	Improve steady state error
PD	$K(s+z_c)$	Improve transient response
PID	$K \frac{(s+z_{lag})(s+z_{lead})}{s}$	Improve steady state error and transient response

The PID controller consists of three terms working on the error of the system and is represented in the time domain by the equation:

$$m(t) = K \left[e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_d \frac{de(t)}{dt} \right] \quad (4.4)$$

Where:

- $m(t)$ output of the controller;
- $e(t)$ input error signal;
- K gain constant;
- T_i the integral time in seconds;
- T_d the derivative time in seconds;

4.3 Designing the Controller

When designing systems there should exist performance specifications so the designer has the required knowledge prior to commencing the design process. Required specifications include transient response and steady state accuracy (error) (Kuo & Golnaraghi 2003). Transient response is the time response of a system to go from initial state to final state. Steady state accuracy is the difference between the input and the output of a system as time approaches infinity. Each specification is tested against a prescribed test input. Typical test input waveforms include a step, ramp and parabola. The specific parameters are typically unique to each application and dictate to the designer the minimum system response that must be designed. The specifications also provide the designer with the information necessary so that the final designed system type can be determined. The type for a second order system is typically determined according to the transient response specification. The specifications for the accumulator control system are contained in Table 4.2.

Table 4.2: Performance specifications for the accumulator system

Specification	Value
Steady state error	Zero for a step input
Transient response	Critically damped for a step input

Often the system to be controlled is not the desired type so a compensating controller type is inserted into the control system to compensate for deficient performance. The compensating controller achieves this by altering the system structure with the addition of poles and zeros based on the type of controller applied to the system. It is this process that the control system design engineer must undertake to obtain the desired performance.

The design of systems can be carried out in either the time domain or the frequency domain. Design in the time domain is performed using root locus techniques. Nichols chart, Bode plot and polar plot are tools used when designing in the frequency domain. Root locus techniques tend to be more intuitive than their frequency domain counterparts. The root locus technique is suited for cascade compensation design and was employed during the design process for the accumulator control system.

The following design procedure detailed in Figure 4.3 was utilised when designing of the PID controller (Nise 2004).

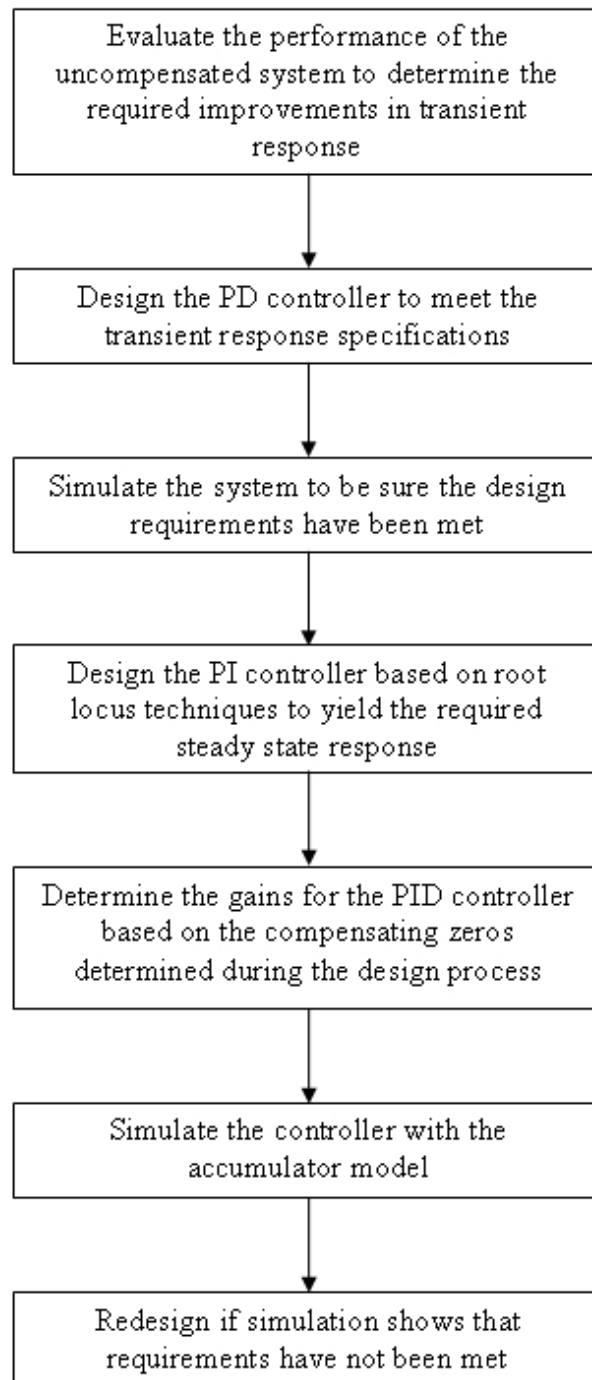


Figure 4.3: Flow diagram of the design process

Step 1. The open loop response of the dynamic system model was shown in Figure 3.4 and revealed the system to be undamped. The undamped system can be represented as a transfer function by the following equation.

$$G_p(s) = K \frac{529}{s^2 + 529} \quad (4.5)$$

The open loop system can be visualised using the root locus which is a time domain design tool. Displayed in Figure 4.4 is the root locus for the open loop system which has two symmetrical poles located on the imaginary axis. Two poles symmetrically positioned on this axis indicate that the system will exhibit an oscillatory or undamped response.

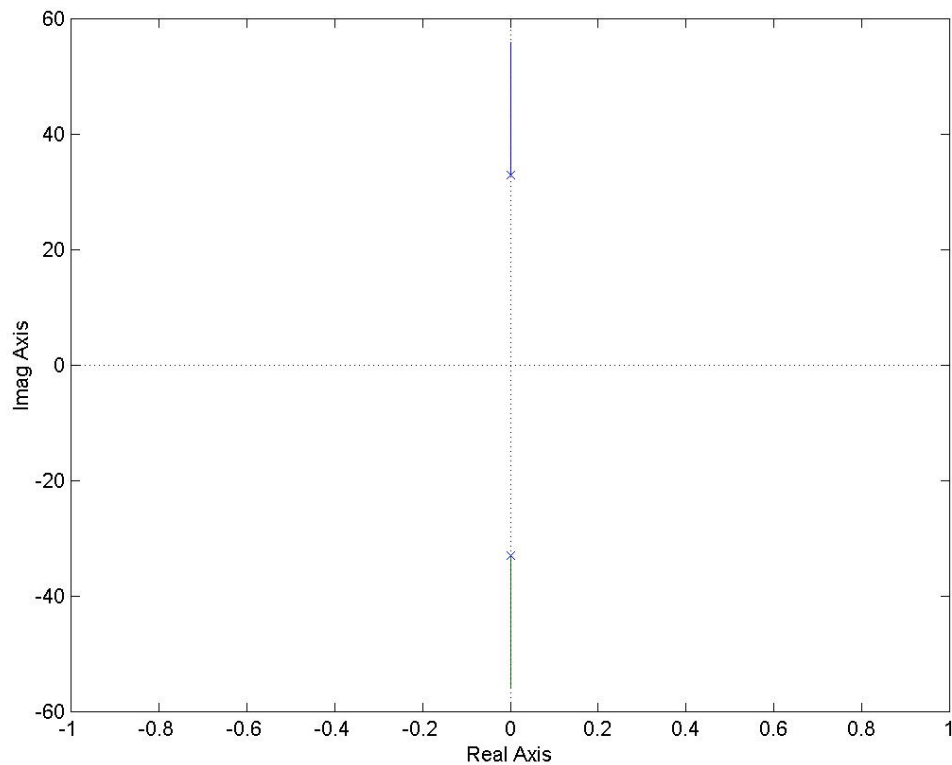


Figure 4.4: Root locus plot of the open loop system

Step 2. A damping factor needed to be introduced to the system to eliminate the oscillatory response of the dynamic system. The specifications state that the system is to be designed for a critically damped response. To achieve a critically damped second order system the transfer function must have a damping ratio of 1. The transfer function of a standard second order system is shown in the equation below.

$$G_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4.6)$$

(Ogata 2002) suggests that the PD controller can be approximated by the following equation.

$$G_c(s) = K_p(1 + T_d s) \approx (1 + T_d s) \quad (4.7)$$

Combining the approximate controller and the standard second order system gives the following transfer function.

$$G(s) = \frac{\omega_n^2 + T_d \omega_n^2 s}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4.8)$$

The resulting closed loop transfer function of the standard second order system with the PD controller is given by.

$$T(s) = \frac{\omega_n^2 + T_d \omega_n^2 s}{s^2 + (2\zeta\omega_n + T_d \omega_n^2) s + \omega_n^2} \quad (4.9)$$

Currently the system damping ratio ζ is zero which leads to the following approximation.

$$T_d \omega_n^2 \approx 2\zeta\omega_n \quad (4.10)$$

From this

$$T_d = \frac{2\zeta}{\omega_n} = \frac{2}{23} = 0.087 \quad (4.11)$$

Now that T_d has been found this value is then placed back into the controller equation to find the zero location and subsequently the gains of the controller Kp and Kd . Rearranging the equation to form the standard representation for a transfer function zero gives $(s + 11.5)$ which equates to introducing a zero on the real axis at -11.5. The gains of the PD controller are set to $Kp = 11.5$ and $Kd = 1$. The root locus plot of the PD compensated system is displayed in Figure 4.5. The PD compensator zero is signified on the root locus plot by the circle located on the real axis at -11.5.

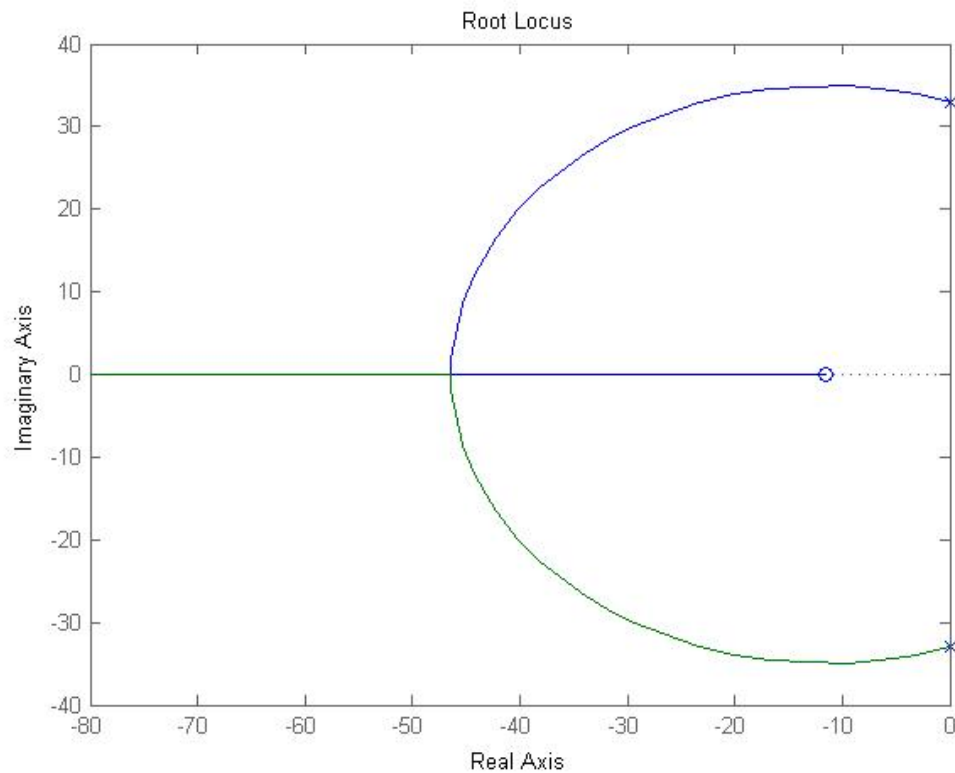


Figure 4.5: Root locus plot of the PD compensated system

Step 3. The PD compensated system is now simulated to analyse the response of the system against the performance specifications. The system response to a step input is displayed in Figure 4.6.

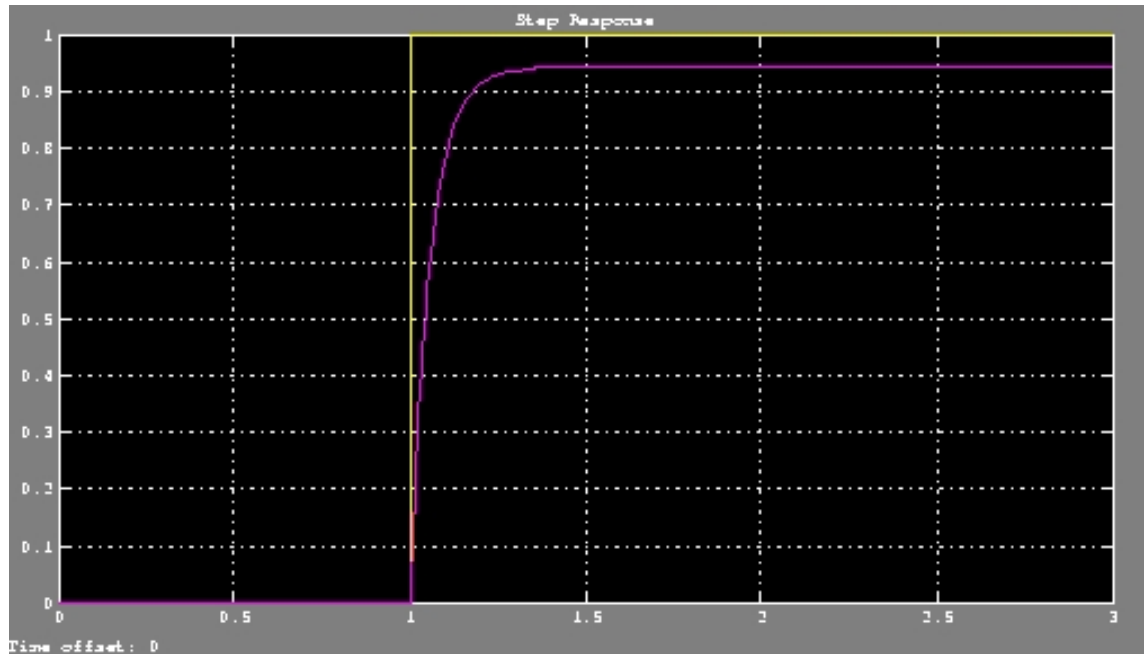


Figure 4.6: Unit step response of the PD compensated system

The PD compensated system with the introduction of the zero, eliminates the oscillatory characteristics of the open loop system to now exhibit a response that is of the critically damped type. The system still requires further design process as there is a steady state error present.

Step 4. The next step is to design the PI controller. Initially the zero for the integral is placed close to the origin to minimise adversely affecting the existing design. The zero is positioned at -1.

Step 5. The controller gains are determined based on the current design. $K_p = 11.5$, $K_i = 1$ and $K_d = 1$.

Step 6. The system is simulated with the controller parameters set according to the design to test for the required response.

Step 7. The system response still exhibits a steady state error with the initial integral gain parameter of 1. The design then went through an iterative process from step 4 to step 7 until the desired response was achieved. The desired response was achieved with an integral zero placed at -11 on the real axis of the root locus. The controller gains for the design are $K_p = 11.5$, $K_i = 11$ and $K_d = 1$. The root locus plot of the closed loop PID controller with the introduction of the zero and the pole is shown in Figure 4.7. The system response to a step input is displayed in Figure 4.8.

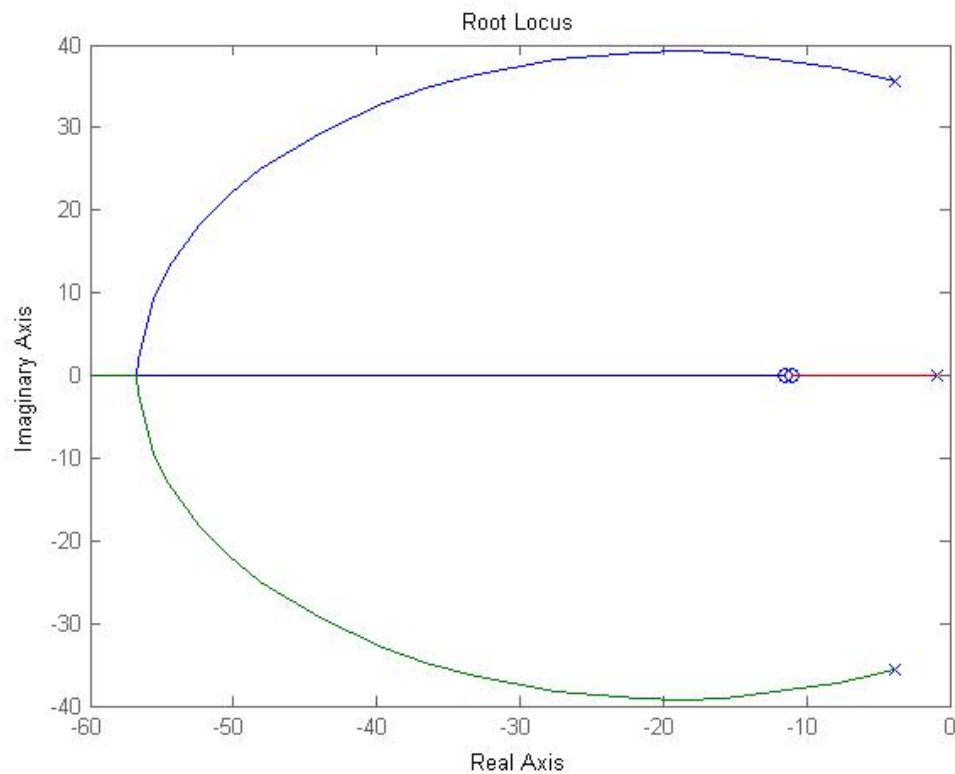


Figure 4.7: Root locus plot of the PID compensated system

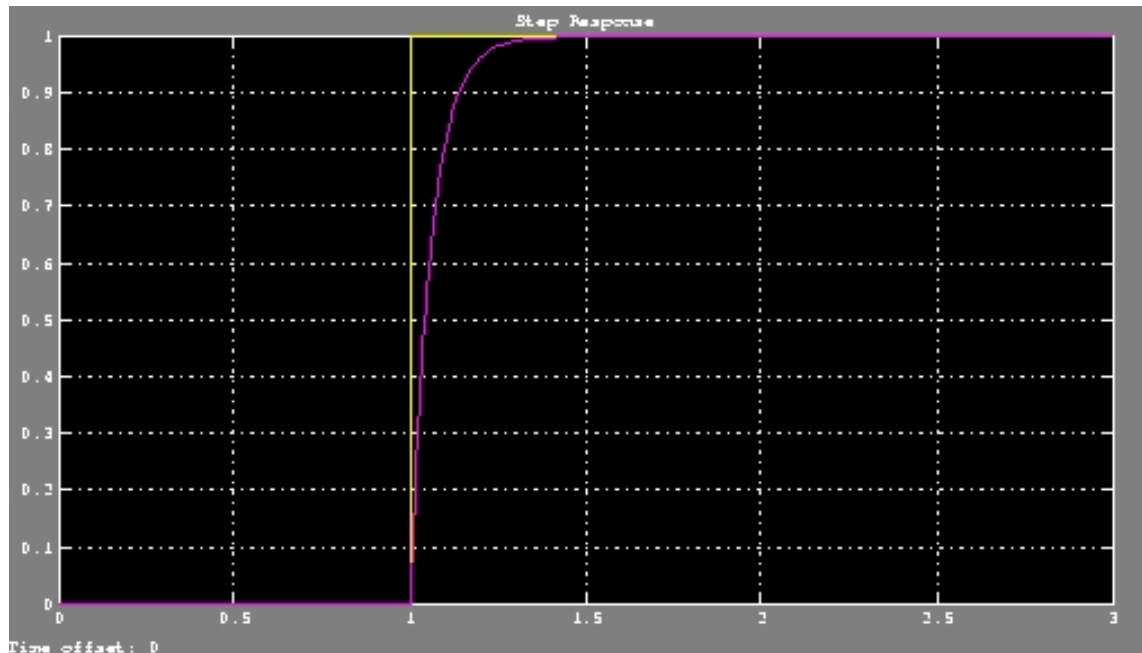


Figure 4.8: Unit step response of the PID compensated system

4.4 Tuning

In the section above the PID controller was designed using time domain design techniques to calculate the controller parameters. In most real life situations the plant mathematical model is not known or is too complex so theoretical methods cannot be utilised. In these situations alternative approaches are required. The PID controller has been adopted as the industrial standard and there exists experimental approaches that can be applied to calculate the parameters or tune the controller as it is termed in industry. A well established and simply implemented method commonly applied in industry is the Ziegler-Nichols PID tuning method. There are two methods to the Ziegler-Nichols tuning rules, the step-response method and the ultimate-sensitivity method.

The first method introduces a unit step input and the response is analysed. For this method to work the plant response should resemble an S-shape. The S-shape curve is

characterised by two constants. The first constant is the delay time (L) and the second is the time constant (T). The PID controller parameters are then calculated as detailed in Table 4.3.

Table 4.3: Ziegler-Nichols tuning rules based on step response

Controller Type	K	T_i	T_d
P	T/L	∞	0
PI	0.9T/L	L/0.3	0
PID	1.2T/L	2L	0.5L

The second method starts with the integral gain set to ∞ and the derivative gain set to zero so that pure proportional control is achieved. The proportional gain value is then increased until the system becomes unstable and this is then documented as the critical gain K_{cr} . The controller parameters are then calculated as detailed in the table 4.4.

Table 4.4: Ziegler-Nichols tuning rules based on ultimate-sensitivity

Controller Type	K	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$K_{cr}/1.2$	0
PID	$0.6K_{cr}$	$0.5K_{cr}$	$0.125K_{cr}$

Although the tuning methods presented in this section are established and well proven they are not always applicable. In this instance with a naturally unstable system it is not possible to identify K_{cr} due to the system already oscillating at a

natural frequency. Thus, it is necessary to be familiar with numerous techniques to determine the optimal parameters of a controller.

4.5 Summary

In this chapter a PID controller has been designed to suit required system performance specifications. Established time domain design techniques were employed for the design process. The controller was also simulated with the accumulator model to assess the system response to a step input. The simulation results obtained will form the base case for comparison against alternative control methods.

Chapter 5

Alternatives to the Industry Standard Controller

5.1 Introduction

There exist numerous alternatives to the industry standard PID controller. An alternative control method that is applied successfully in industry is state variable feedback control. This control method is usually associated with a group of controllers that form, modern control systems. Modern control system design was largely due to advanced control requirements prior to World War II. The requirement was driven by the need for accurate tracking systems, particularly missiles tracking aircraft. State variable feedback has been successfully tested by (Pagilla et al. 2003) during a study on an aluminium CPL.

The majority of alternative controllers, excluding state variable feedback lie in the domain of what is termed advanced control. Advanced control methods include the following.

Adaptive Control can be defined as an intelligent feedback control system that has the ability to adjust its characteristics in a changing environment. The controller in this scheme is supposed to identify changes in the

controlled plant and make adjustments so the output performs according to specified criteria.

Model Predictive Control utilises a known model of the controlled plant to predict the future output based on historical data of the process. Complete accuracy of the model is not required. Any mismatch between the model and plant can be overcome by online identification techniques. Regardless of the level of accuracy required for the model, design of this system is complex and requires deep knowledge of the controlled plant.

Intelligent Control includes expert control which is a knowledge based decision control algorithm. This intelligent control method is suited for fault diagnosis and scheduling type problems. Another intelligent control method is neural network control, which is based on artificial neural networks. Neural network controllers also require a known model of plant to function. Fuzzy control is a method structured according to the mathematical principles of fuzzy set theory. Operating knowledge of the process is represented by mathematical terms and implemented as fuzzy control.

The alternative controller detailed in the following sections is the fuzzy controller. Fuzzy control has been used extensively in commercial markets and has been comprehensively researched. Another contributing factor for examining this controller is the acceptance of this technology into standard programming blocks of industrial control hardware. This indicates that there is a role for the fuzzy controller in some form within an industrial environment. This dissertation attempts to apply the fuzzy control method to control an accumulator in an industrial process line.

5.2 Background

The concept of fuzzy control was first presented in the 1960's (Nguyen et al. 2003). Fuzzy control methodology attempts to mimic human thinking and react using intelligence of the operating process. The mathematical principle behind fuzzy control is set theory. Traditional set theory is boolean in nature, where this is represented as either true or false. According to traditional set theory an object is a member of a set and represented as a logic 1, or it is not, which is represented as a logic 0. Unlike traditional set theory, fuzzy set theory is based on a vagueness rather than precision (Kovacic & Bogdan 2006). This implies that there is a degree of membership that exists within the bounds of 0 to 1. This degree of membership notion allows the vagueness of fuzzy control to be applied.

Fuzzy controllers can be applied to solve the same control problems as more conventional controllers. This can be achieved using true fuzzy control or by emulating a traditional controller like PID. Fuzzy control comprises a fuzzy inference engine, membership functions and fuzzy rules and is displayed in Figure 5.1 (Kovacic & Bogdan 2006). A fuzzy inference engine formulates the mapping of the input to the output for the fuzzy control system. The fuzzy inference process involves membership functions and associated fuzzy rules. Typical inference engine models are the Mamdani method and the Tagaki-Sugeno method. E. H. Mamdani is renowned for developing an experimental fuzzy control system for a steam engine and boiler in 1974. His design synthesized a set of linguistic control rules obtained from experienced operators (Nguyen et al. 2003). The Tagaki-Sugeno method differs from the Mamdani method in that the defuzzification process is not required. This method can be applied when there is greater knowledge of the plant dynamics.

Membership functions are applied for fuzzification and defuzzification of signals. Essentially this equates to the processing of the input signals into the inference engine

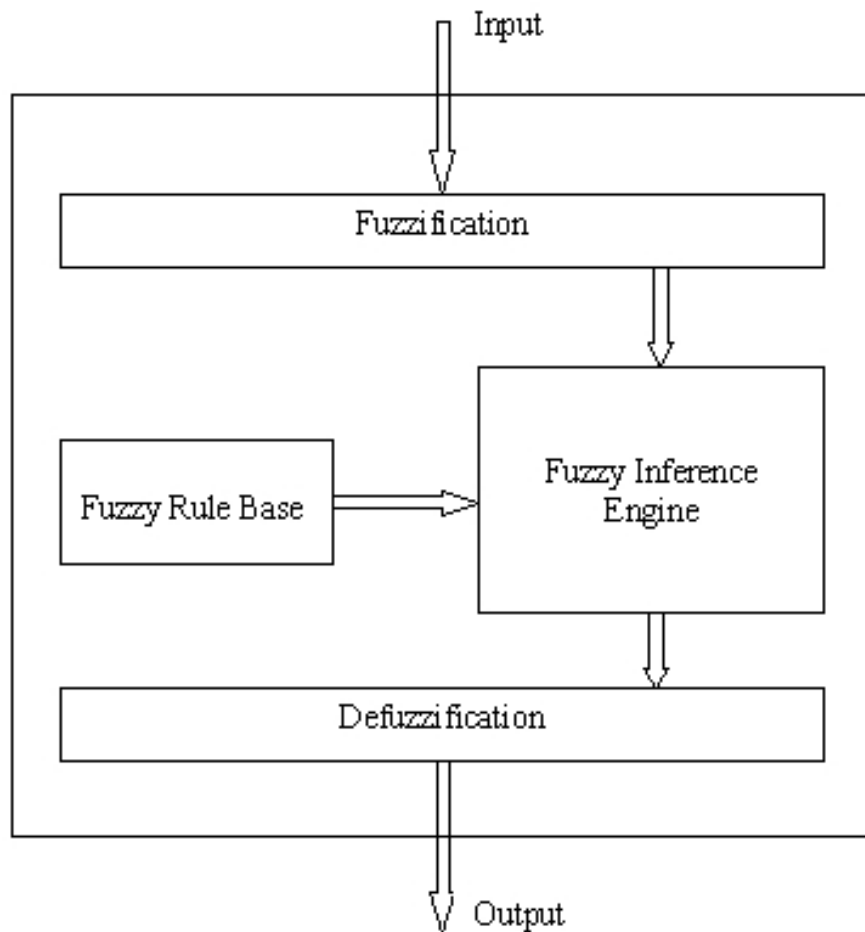


Figure 5.1: The structure of a fuzzy logic controller

and the outputs from the inference engine. These statements imply that a membership function converts a physical numerical value into a linguistic value or vice versa. This is achieved by associating the value with a membership degree that lies between 0 and 1. In contrast, ordinary set theory associates with only a 0 or a 1, there is no middle ground or vagueness. Ordinary set theory can also be termed a crisp set, for example A is a member of X or A is not a member of X .

A membership function is physically represented on an axis by a shape. The type of shape is typically selected according to the particular application involved. Common shapes mostly applied as membership functions in fuzzy control theory are triangular,

trapezoidal, Gaussian, and sigmoidal Z and S functions (Nguyen et al. 2003). These shapes are depicted in the following figures.

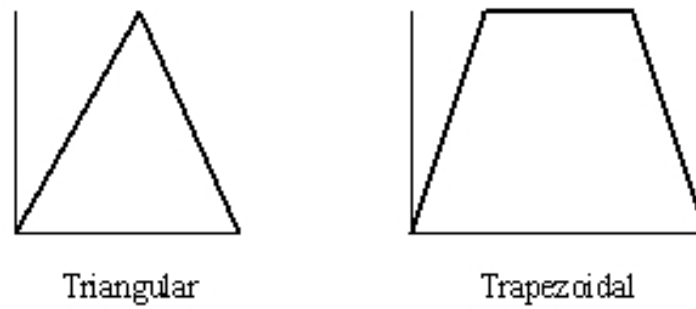


Figure 5.2: Triangular and trapezoidal membership function shapes

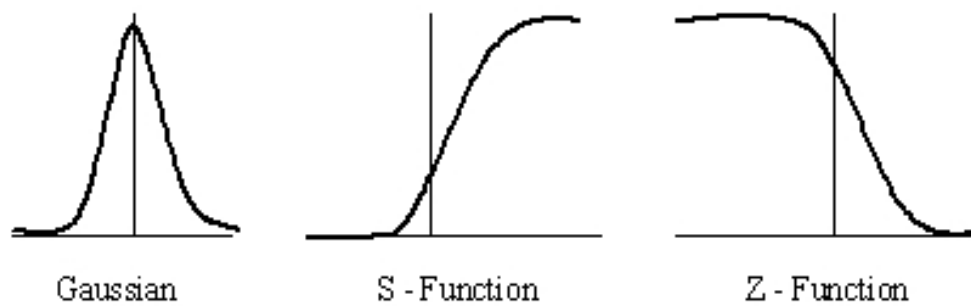


Figure 5.3: Gaussian, Sigmoidal S and Z membership function shapes

A fuzzy rules base is a combination of linguistic rules consisting of natural expressions that contain IF-THEN statements. The rules are interpreted by the fuzzy inference engine and are required to conform to the expected format of the mechanism employed by the engine. Typically in fuzzy control this mechanism is derived from one of following inference models developed by pioneers in the history of fuzzy control.

- Mamdani model
- Larson model
- Takagi-Sugeno model

5.3 Designing the Controller

A fuzzy controller is implemented in a control loop in similar fashion as a conventional controller. Generally, if a fuzzy control system is designed to replace a conventional single input - single output controller (SISO) a derivative of the control error is applied as a second input (Kovacic & Bogdan 2006). This is displayed in Figure 5.4.

The first step in the design process is the choice of fuzzy inference engine to solve the problem. In this case, the Mamdani method is chosen. This method is a simpler approach as it does not require a model of the plant for development.

The next step of the design process is to conceptualise the rules table. The inputs to the fuzzy controller are the tension error and its derivative or the rate of change. In the case of the accumulator system, the error is classed as being one of five possible states. The second input is the rate of change related to the error and this was determined to be in one of three possible states. These are displayed in Table 5.1 along with the output states. The output is mapped to the controlled force and has seven possible states.

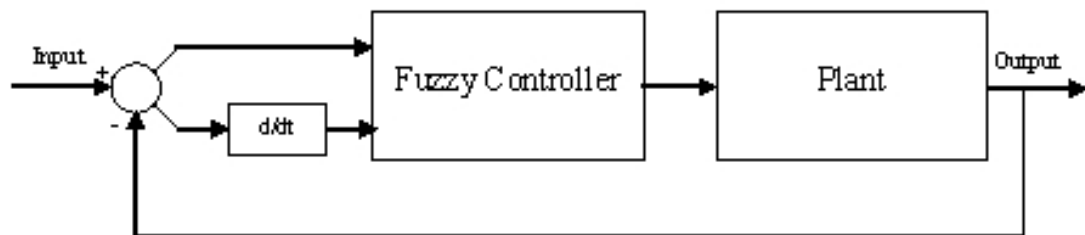


Figure 5.4: Block diagram of a fuzzy control loop

Table 5.1: Rules table for the fuzzy controller

Rate of Change	Error				
	NL	NS	Z	PS	PL
N	NL	NM	NS	PS	PL
Z	NL	NM	Z	PM	PL
P	NL	NS	PS	PM	PL

Where:

N Negative; S Small;
 Z Zero; M Medium;
 P Positive; L Large;

Following on, is the definition of the membership functions. These are essentially the blocks that process the inputs and outputs of the fuzzy control system. All membership functions for the system were chosen to comprise the triangular variety. This type is one of the more popular choices when designing systems (Kovacic & Bogdan 2006). The configured blocks are displayed in Figures 5.5, 5.6 and 5.7.

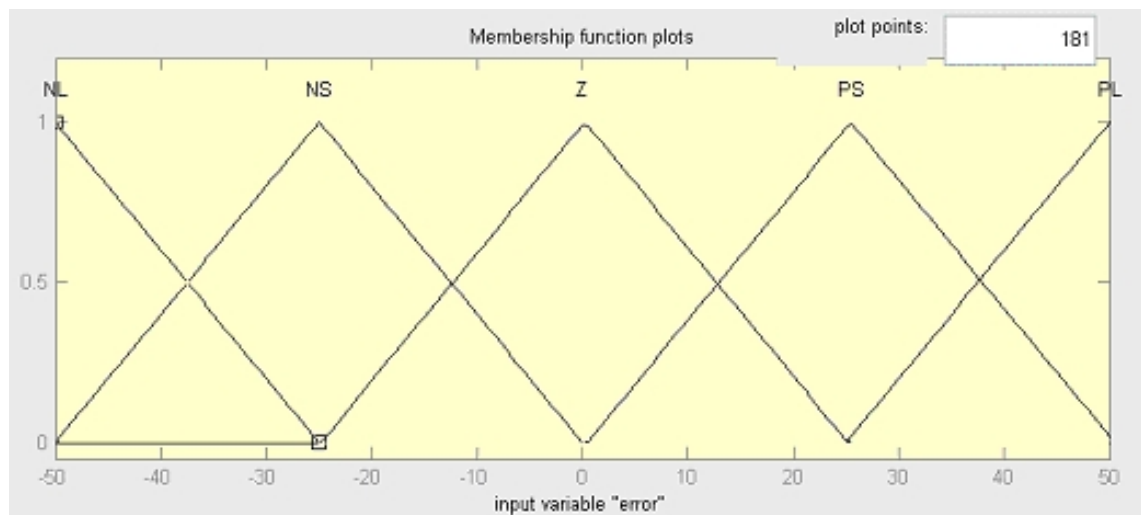


Figure 5.5: Membership function for the error input to the fuzzy controller

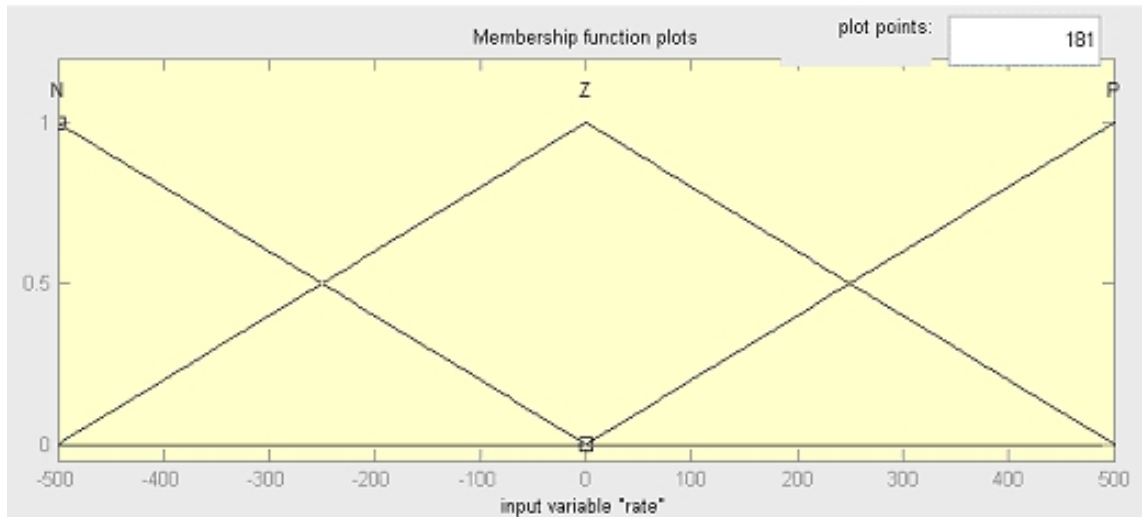


Figure 5.6: Membership function for the rate of change input to the fuzzy controller

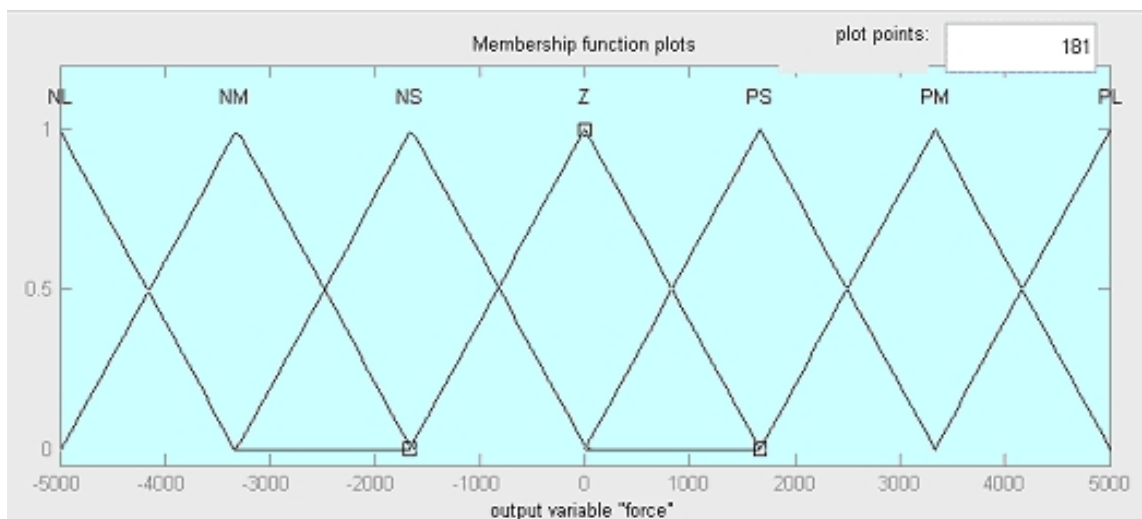


Figure 5.7: Membership function for the output of the fuzzy controller

The next phase of the design process is to enter the rules into the fuzzy controller rules editor and simulate the system. The rule set is displayed in Table 5.2.

Table 5.2: Linguistic rules of the fuzzy controller for the accumulator

Item	Rule
1.	If (error is NL) and (rate is N) then (force is NL)
2.	If (error is NL) and (rate is Z) then (force is NL)
3.	If (error is NL) and (rate is P) then (force is NL)
4.	If (error is NS) and (rate is N) then (force is NM)
5.	If (error is NS) and (rate is Z) then (force is NM)
6.	If (error is NS) and (rate is P) then (force is NS)
7.	If (error is Z) and (rate is N) then (force is NS)
8.	If (error is Z) and (rate is Z) then (force is Z)
9.	If (error is Z) and (rate is P) then (force is PS)
10.	If (error is PS) and (rate is N) then (force is PS)
11.	If (error is PS) and (rate is Z) then (force is PM)
12.	If (error is PS) and (rate is P) then (force is PM)
13.	If (error is PL) and (rate is N) then (force is PL)
14.	If (error is PL) and (rate is Z) then (force is PL)
15.	If (error is PL) and (rate is P) then (force is PL)

The system response of the fuzzy control system and the accumulator model to a step input is displayed in Figure 5.8.

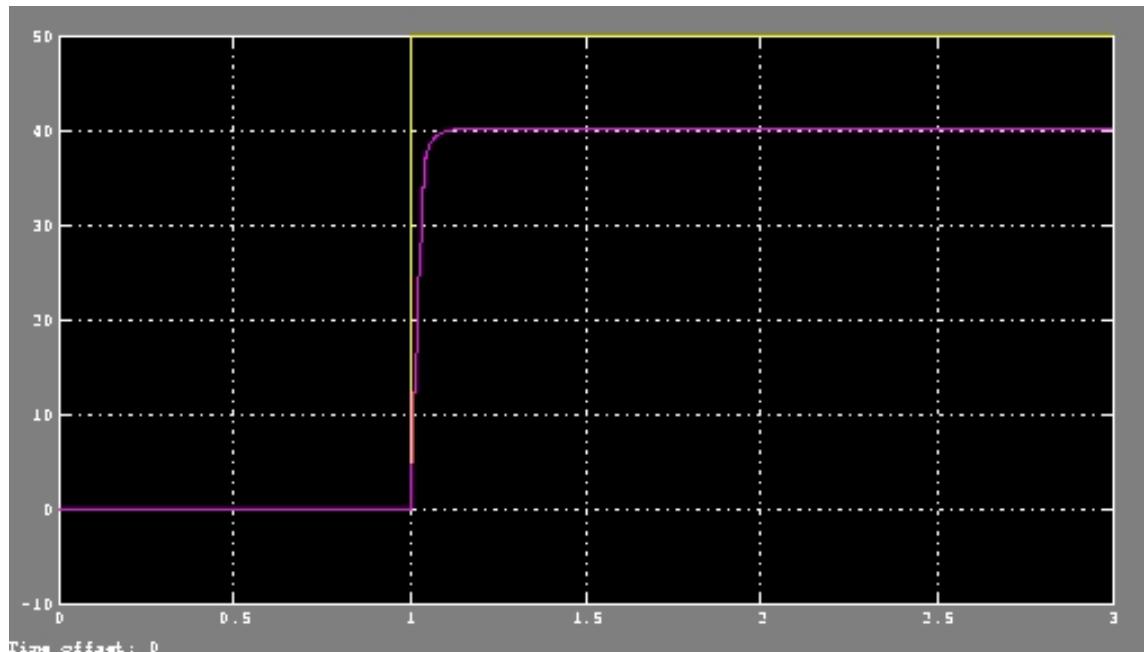


Figure 5.8: Unit step response of the fuzzy control system

5.4 Tuning

Tuning of the fuzzy controller was difficult in the fact that established procedures do not exist. The performance results suffered as a result of this lack of clarification about how to counteract deficiencies identified within the system. The final outcome was not able to satisfy the required performance specifications. The system has a 20 percent steady state error when subjected to a step input as displayed in Figure 5.8.

5.5 Summary

In summary, this chapter detailed the design of a fuzzy controller for the application of controlling an accumulator in a continuous processing line. The basis of the fuzzy controller is a combination of natural language rules to perform the required control tasks. The process of constructing the fuzzy controller is relatively simple but unfortunately the design and testing process is complicated. The design process is complicated by the fact that procedures are not as advanced as in classical control and are less intuitive. Essentially, the consequences of altering values in the rules table is uncertain until after tests are completed and the affects of the change assessed. This tends to extend the duration of the design process when compared to other techniques that offer greater intuition into the consequence of parameter adjustment within the controller.

Chapter 6

Simulation Results and Analysis

6.1 Introduction

Design performance analysis for the PID controller was detailed in chapter 4 and chapter 5 for the fuzzy controller. This section discusses the performance results during a coil changeover cycle when the accumulator carriage is moving. A coil changeover forces the accumulator carriage to move in an upward and downward motion. Whilst the carriage is in motion, the control system must operate to maintain the desired tension within the accumulator. This operation challenges the control system as the movement introduces mechanical disturbances into the control loop. The control system must maintain stability and react to any errors between the desired setpoint and the actual feedback.

6.2 Methodology

The methodology for this performance analysis is to simulate a coil changeover by setting the exit section speed within the simulation model to zero. This forces the

accumulator carriage into motion to maintain the desired tension due to the mismatch between the speeds of the material on either side of the accumulator.

6.3 Performance Results

Figure 6.1 displays the performance of the PID controller during motion of the accumulator carriage.

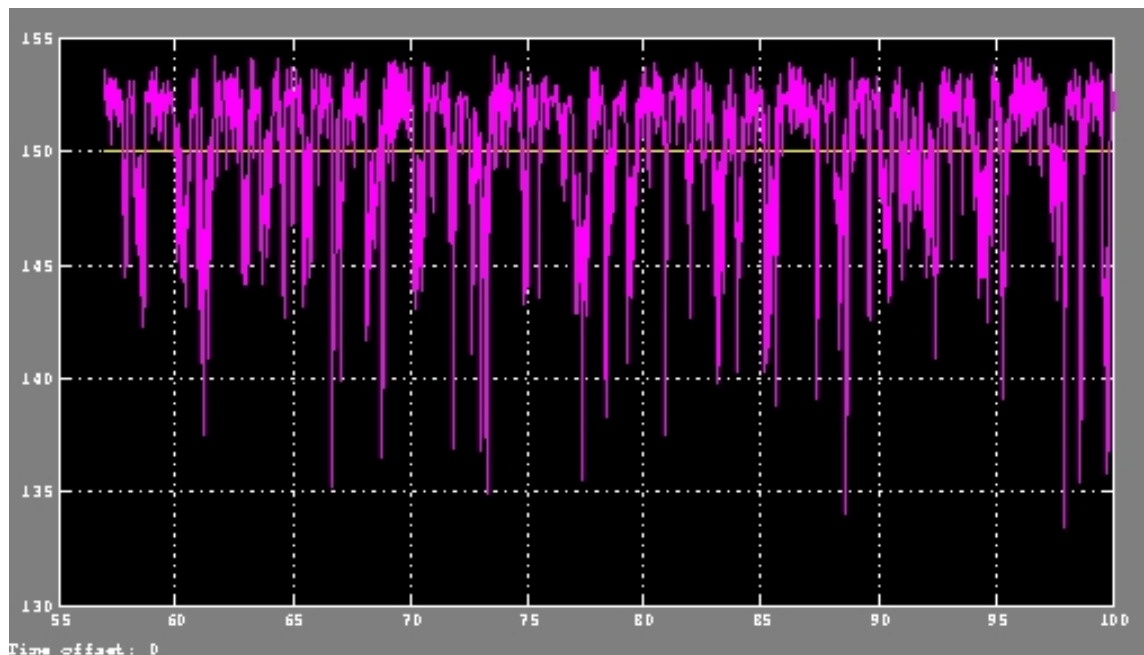


Figure 6.1: Response of the PID controller during accumulator carriage motion

From Figure 6.1, the PID control system shows that it is capable of controlling tension whilst the accumulator carriage is in motion. It is evident from the step response plot that there is a maximum 10 percent deviation from the desired set point. It is possible that the performance of the system could be improved with

tighter control performance specifications. Although, in reality a 10 percent deviation may be acceptable but the real impact of this would require assessment on an operational continuous processing line.

Figure 6.2 displays the performance of the fuzzy controller during motion of the accumulator carriage.

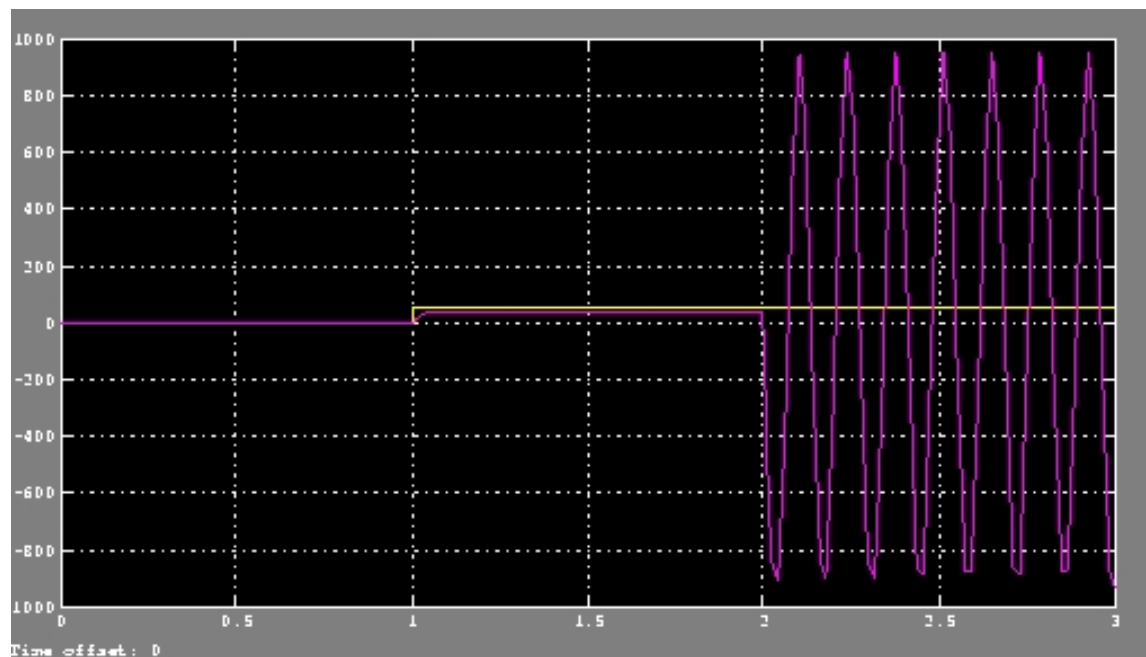


Figure 6.2: Response of the fuzzy controller during accumulator carriage motion

From Figure 6.2 it can be seen that once the accumulator carriage commences movement the control system becomes unstable. The control system becomes unstable as the system characteristics change into pure proportional control. This is the contributing factor to the wild oscillations displayed in the response plot. In reality, this condition is totally unacceptable and could not be used to control tension within the accumulator.

6.4 Chapter Summary

Initial performance specifications indicated that both the PID controller and fuzzy controller may be suitable for an accumulator control system. The simulation results obtained in this chapter do not concur and highlight a deficiency in the fuzzy controller with the current parameters. The lack of intuitive tuning and design procedures hinder further improvements to the system.

Chapter 7

Conclusion

7.1 Dissertation Summary

This dissertation provided a comprehensive analysis of controlling an accumulator in a continuous processing line (CPL). This research has been motivated by the need to maintain desired tension control of the material within an accumulator. During filling and emptying cycles, the accumulator has been identified as a possible source of tension disturbance that can propagate through a CPL. Therefore, the control of the accumulator has been singled out for attention. Several objectives were undertaken for this project and are discussed in the following.

Current control methods applied in industry. The most widely implemented controller in industry is the proportional + integral + derivative (PID) controller. Classical design techniques and well established tuning procedures contribute to the preferred application of this controller. In addition, the controller exhibits robust performance over a wide range of operating conditions.

Model the dynamic system of the accumulator. The model of the accumulator system was developed using MATLAB[®] Simulink[®]. The dynamic system equations for the model were taken from research conducted in this same area. Concerns are

raised that the model does not accurately display the natural characteristics of an accumulator in a CPL. This conclusion is drawn from the non existence of a natural damping mechanism within the system and the subsequent oscillatory nature of the accumulator. For this reason, further investigation to increase the exactness of the simulation model would benefit future research in this area.

Simulation of different controllers. The main emphasis of this dissertation involved the application to a real industrial process. Control system performance has been demonstrated through simulation with a model of the accumulator. The controllers selected for simulation are the industry standard PID plus the fuzzy controller as an alternative. The fuzzy controller was selected for analysis due to the existence of considerable research into the subject and it is suited for operational type applications similar to a continuous processing line.

Critically evaluate the controllers. Performance specifications were stated in chapter 4 for both design and evaluation purposes. Pole placement and the root locus, are classical design techniques that were employed for successful design of the PID controller to meet the required performance specifications. These techniques are well entrenched in control engineering circles and literature such as texts and research papers. In addition, the design process is relatively intuitive when applying the visual design tool of the root locus. The PID controller also performed adequately during a changeover cycle where tension control can suffer from external influences when the accumulator carriage is moving. On the other hand, techniques for designing fuzzy controllers do not have the same level of rigour. Also apparent, is the fact that the design techniques are not as visually accommodating or intuitive as that of the PID controller. These limitations tend to complicate the design process. In addition, the duration of the design and test process is extended due to the number of iterations required to arrive at a solution. Another contributing factor affecting the duration is the lack of established tuning rules. The fuzzy controller as determined during the process of this dissertation was found to exhibit an inferior performance to the PID controller. Therefore, the PID controller is better suited than the fuzzy controller in the application of controlling an accumulator in a CPL.

7.2 Further Work and Recommendations

Previous sections eluded to the fact that the dynamic system model should be investigated for accuracy. Specific areas to target could be components of the accumulator that should exhibit natural damping characteristics. For instance, the weight of the strip and the drive mechanism. The PID controller's capability of adequately controlling the accumulator has not been disproved in the simulation study. However, there is still merit in examining the remaining alternative controllers for superior performance. Deficiencies of the fuzzy controller have been highlighted in the analysis. These deficiencies are mainly attributed to the lack of established design and tuning procedures. This is an area that could benefit from further development.

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Appendix A

Project Specification

University of Southern Queensland
FACULTY OF ENGINEERING AND SURVEYING

ENG 4111/4112 Research Project

PROJECT SPECIFICATION

FOR: Michael Crawford-Ross

TOPIC: MODELLING AND CONTROL ANALYSIS OF AN ACCUMULATOR
IN A CONTINUOUS PROCESSING LINE


SUPERVISOR: Dr. Paul Wen

ENROLMENT: ENG4111 – S1, 2008:
ENG4112 – S2, 2008

PROJECT AIM: To investigate methods of controlling an accumulator in a continuous
processing line, model the dynamic system and simulate the response with
different control parameters.

PROGRAMME: Issue A, 20th February 2008

1. Research the current control methodologies used in industry for accumulators in continuous processing lines.
2. Model the dynamic system of an accumulator using Matlab Simulink.
3. Simulate the operation of an accumulator with different controllers and analyse the system response.
4. Critically evaluate the controllers with respect to their performance for controlling the system.
5. Investigate the possibility of using different methodologies for controlling the accumulator.
6. Submit an academic dissertation on the research.

AGREED  (student) _____ (supervisor)

Date: 20/2/2008 Date: / /2008

Examiner/Co-examiner: _____

Appendix B

Some Supporting Information

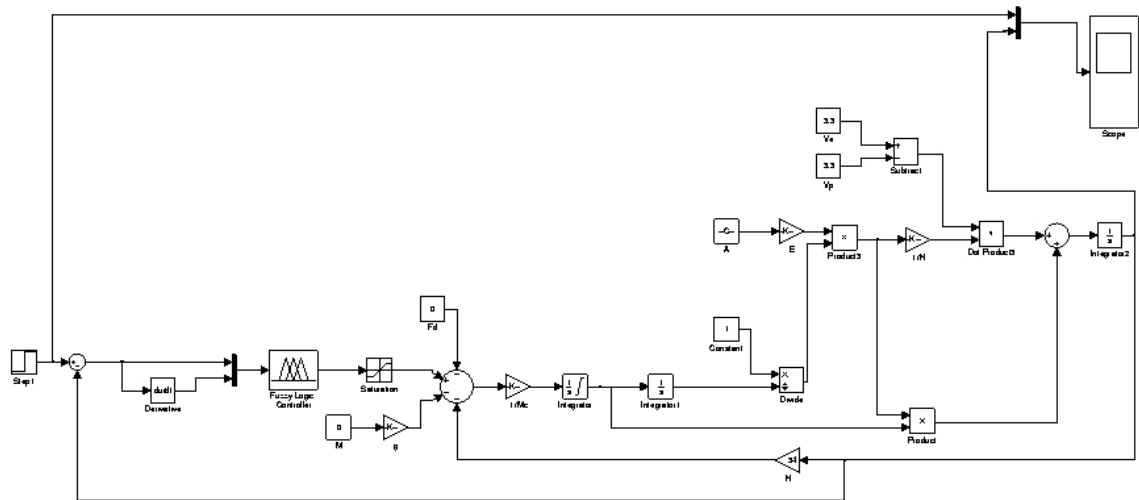


Figure B.2: Simulink model of the fuzzy controller and the dynamic system for the accumulator

B.3 Root Locus Plot Software and Output

```
% M-file to calculate and plot the root locus to assist with control system design
%
% Author: Michael Crawford-Ross
% Date: 1st September 2008
%
%
function design
num = [1089];
den = [1 0 1089];
[a,b] = size(num);
[c,d] = size(den);
if ((a > 1) | (c > 1))
    error('Numerator and denominator must be a single matrix')
end
sys = tf(num,den);
rlocus(sys)
```

```
zero = input('enter compensating zero for transient response: ')
contr = [1/zero 1];
numc = num*contr;
sys1 = tf(numc,den);
rlocus(sys1)
input('Press enter when ready to progress: ');
sys_cl = feedback(sys1,1)
zero2 = input('enter compensating zero for steady state error if required: ');
numc = [1/zero2 1];
denc = [1 0];
contr = tf(numc,denc);
sys2 = sys1*contr
rlocus(sys2)
input('Press enter when ready to progress: ');
sys_cl = feedback(sys2,1)
rlocus(sys_cl)
input('Press enter when ready to progress: ');
```

```
>> design
```

```
enter compensating zero for transient response: 11.5
```

```
zero =
```

```
11.5000
```

```
Press enter when ready to progress:
```

```
Transfer function:
```

```
94.7 s + 1089
```

```
-----
```

```
s^2 + 94.7 s + 2178
```

enter compensating zero for steady state error if required: 11

Transfer function:

$$8.609 s^2 + 193.7 s + 1089$$

$$s^3 + 1089 s$$

Press enter when ready to progress:

Transfer function:

$$8.609 s^2 + 193.7 s + 1089$$

$$s^3 + 8.609 s^2 + 1283 s + 1089$$

Press enter when ready to progress:

>>