

University of Southern Queensland
Faculty of Engineering and Surveying

Fractal Video Compression

A dissertation submitted by

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in fulfilment of the requirements of

Courses ENG4111 and ENG4112 Research Project

towards the degree of

Bachelor of Engineering (Computer Systems)

Submitted: October, 2005

Abstract

With the rapid increase in the use of computers and internet, the demand for higher transmission and better storage is increasing as well. One way to solve this problem is by using compression, in which a small amount of data can represent the much larger amount of original data.

This dissertation describes the different techniques for data (image-video) compression in general and, in particular, the new compression technique called Fractal Image Compression. Fractal image compression is based on self-similarity, where one part of an image is similar to the other part of the same image.

The most significant aspect of this project is the development of color images using fractal-based color image compression, since little work has been done previously in this area. The results obtained show that the fractal-based compression for the color images works as well as for the gray-scale images. Nevertheless, the encoding of the color images takes more time than the gray-scale images.

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Certification of Dissertation

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I further certify that the work is original and has not been previously submitted for assessment in any other course of institutions, except where specifically stated.

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0031137033

Signature

Date

Acknowledgments

This project would not have been possible without the assistance and support of many people. I would like to take this opportunity to thank my supervisor, Dr John Leis, my family and friends for their encouragement and guidance throughout the year.

Khalid Kamali

University of Southern Queensland

October 2005

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Chapter 1

Introduction

Day by day, the demands for higher and faster technologies are rapidly increasing for everyone. Although the technologies available now are considered more advanced than 30-40 years ago, people are still looking for improvements and enhancements. In the last twenty years, computers have been developed and their price has reached a level of acceptance where almost anyone can buy one. Nowadays, before purchasing computers, customers are concerned about two things: (1) the speed of the CPU; and (2) the storage and memory capacity.

Image and video compression helped to reduce the memory capacity and to have faster transmission rates. Many compression techniques have been introduced and developed, such as JPEG (joint photographic experts group) and GIF (graphics interchange format) for image compression, as well as MPEG (moving picture experts group) for video compression. Although some of these techniques showed significant and enhanced performance in decreasing the cost of the transmission and storing data, the search for alternatives continues. Fractal image or video compression is a new compression method which is based on self-similarity within the different portions of the image. It might be revolutionary in the world of data compression because of its high compression rate compared with other methods, however, it suffers from some problems such as the time taken for encoding. Until now, a lot of work has been done on fractal gray scale image compression, although unfortunately not too much effort has been devoted to fractal color image and video compression.

1.1 Project Objectives

The main objectives of this project are:

- 1- To investigate and have a general idea of several compression techniques.
- 2- To investigate and have a good understanding of fractal image compression.
- 3- To use MATLAB to implement fractal image compression for both gray-scale images and color images.
- 4- To discuss methods of improving the performance of fractal image compression.
- 5- To compare fractal image compression and some other compression techniques.
- 6- To investigate and use MATLAB to implement fractal video compression.

1.2 Image Quality

In this project PSNR (Peak Signal to Noise Ratio) is used to compute the quality of the reconstructed image compared with the original image by measuring the differences between the two images; the formula used to compute the PSNR is [8].

$$\text{PSNR} = 20 \log_{10} (b / rms) \quad (1.1)$$

where b is the highest pixel value (255) and rms is the root mean square differences between the two images. PSNR is measured in dB, the highest the PSNR, the better the quality of the reconstructed image. Nevertheless, this is not the only way used to measure the quality of images—there are other ways but PSNR is the most widely used.

One other technique is also used here in this project. However, this technique will not measure the quality of the image—in other words; it will not give us values. Rather, it shows the differences between the original image and the reconstructed image, and this is called the *error image*.

1.3 Test Images

Throughout this project, a set of images been used to test the quality and the performance of the fractal image compression and other types of compression. The gray-scale images are of size 256 x 256 and the only color image is of size 512 x 512. Figure 1.1 shows the full set of the test images.



Figure 1.1: Test Images
(Source: Kominck, J 2002)

1.4 Outline of the Dissertation

Chapter 2: Image and Video Compression. This chapter contains a brief description of types of compression and some compression techniques for both images and videos.

Chapter 3: Fractal Image Compression. This chapter gives an overview the history of fractal-based compression, together with some mathematical background and a brief outline of the concept of fractal-based compression.

Chapter 4: Performance of Fractal Image Compression. This chapter shows the performance of fractal image compression for gray-scale images in terms of quality and compression rates, together with a comparison between fractal-based compression and some other compression techniques.

Chapter 5: Fractal Color Image Compression. This chapter contains the methods used to encode a color image using a fractal-based compression, and looks at the performance of the decompression using different coordinate systems of the color image.

Chapter 6: Different Partitioning Methods. This chapter contains different partitioning approaches in fractal image compression, and details the advantages and disadvantages of using each approach.

Chapter 7: Faster Encoding. This chapter contains different procedures adopted to reduce encoding time in the case of fractal image compression.

Chapter 8: Fractal Video Compression. This chapter contains the two methods used to encode video sequences in fractal-based compression.

Chapter 2

Image and Video Compression

2.1 Image Compression Methods

Primarily, there are two types of image compression: one is called *lossless* and the other is *lossy*. Lossless method describes when the decompressed image is exactly as the original image and there is no loss of information. On the other hand, lossy has some information loss after decompressing the image, however, this loss of data is considered negligible (if the loss of data is acceptable), since the difference between the reconstructed image and the original image is not large and remains relatively unchanged.

This chapter looks at several compression techniques with a brief description of each one. Some of those compression techniques belong to lossless compression and others to lossy compression.

2.1.1 Huffman Coding

One of the oldest coding techniques is Huffman coding; it was first introduced by David Huffman in 1952.

Huffman coding follows lossless method of encoding data. The idea of Huffman coding is using short or a small number of bits to encode the data with the higher probabilities and using a large number of bits to encode data with the least probabilities.

For example, let's take the encoding of this page that you are reading, then first to be found is the probability for each character in this page; to make the example more simple, let's say this page only contains seven characters—each has a probability of what is shown in the table below:

Character	Probability
A	0.3
B	0.29
C	0.13
D	0.12
E	0.1
F	0.05
G	0.01

Table 2.1: The probability for each character.

Now, based on the probability of each character, a binary tree is formed, starting with two child nodes which represent the two characters with the lowest probability. Taking these two from the list we add the next lowest probability character to the tree, continuing to do so until we reach to the root of the tree. After that we label one side of the tree with 0s and the other with 1s (it does not matter which side is which).

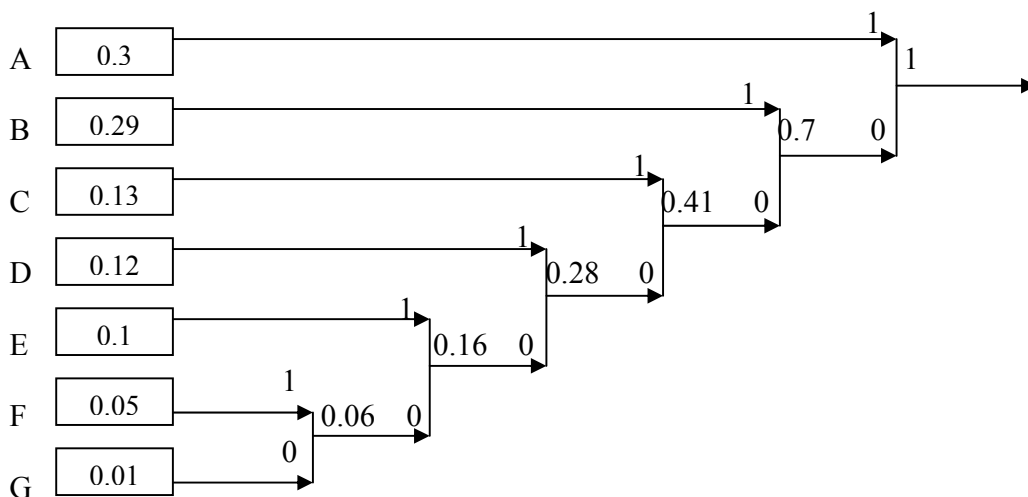


Figure 2.1: Huffman code construction.

2.1.2 Arithmetic Coding

Arithmetic coding is a lossless type of compression, and it is very similar to Huffman coding except for that it does not produce a single code for each symbol but, rather, it produces a specific codeword for the whole message. The codeword is in the form of a float point number between 0 and 1.

The idea in arithmetic coding is to generate a probability table in the same way as the Huffman coding, and then to assign each symbol with a range between 0 and 1. The symbols with higher probabilities will be assigned to wider ranges and the symbols with lower probabilities be assigned to smaller ranges [5, 6, 20, 22].

For illustration, we will encode the word 'TARA'. First, we form the following table showing the probability and range for each character:

Character	Probability	Range
A	0.5	0.0-0.5
R	0.25	0.5-0.75
T	0.25	0.75-1.0

Table 2.2: The probability and range for each character in the arithmetic coding.

Then we start coding using the following algorithm [6]:

LOW = 0.0

HIGH = 1.0

WHILE not end of input stream

Get next CHARACTER

RANGE = HIGH – LOW

*HIGH = LOW + RANGE * high range*

*Low = LOW + RANGE * low range*

END While

Output LOW

The following table (Table 2.3) shows how the range, high and low, changes as each character is processed:

T	Range=1	Low=0.75	High=1
A	Range=0.25	Low=0.75	High=0.875
R	Range=0.125	Low=0.8125	High=0.84375
A	Range=0.03125	Low=0.8125	High=0.828125

Table 2.3: The encoding steps in arithmetic coding.

Final output of the encoding will be equal to 0.815.

The decoding process is simply the inverse of the encoding.

2.1.3 Vector Quantization

Vector Quantization (VQ) follows the lossy type of data compression. In VQ the encoding process is quite complex, however, this complexity makes the decoding process very fast.

In the process of image compression using vector quantization, first the image is divided into smaller sections called *target vectors*. Then, each target vector will be encoded as a *code vector* which is introduced by matching the target vector with the codeword from the codebook (finding the minimum distortion). The codebook is formed from a collection of many representative images, therefore, it should contain all the possible representations of the target vectors. By transmitting only the index of each best matching codeword it is possible to obtain a good compression ratio because each section of the image will be only represented with an 8-bit codeword index. In the decoding process, the decoder should have the same codebook as the encoder, and then simply by using the saved codeword index and a look-up table the original image is reproduced [4, 8].

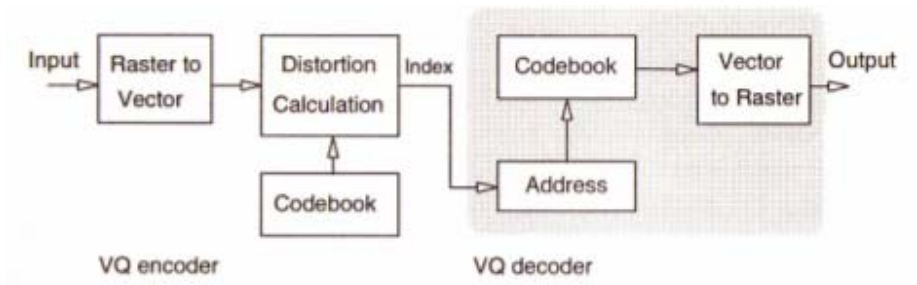


Figure 2.2: Block diagram of Vector Quantization image coder
(Source: Bhaskaran-et-al, V 1995)

2.1.4 DCT (Discrete Cosine Transform)

Most of the image and video compression standards such as JPEG and MPEG are based on the DCT coding. DCT compression is another type of lossy compression.

Discrete cosine transform is very much related to the well-known FFT (Fast Fourier Transform), in as much as they both change the data from its current domain to frequency domain, however, DCT only deals with real numbers—unlike FFT which suffers from complex multiplications.

In DCT coding system the blocks (usually 8 x 8 blocks are chosen) of image are to be transformed from spatial domain into a set of frequency domain representation. During the encoding and due to the quantization process, there will be some data loss which will not be recovered when the decoding process takes place (Inverse DCT). The quantization process will cause zeroing of many of the DCT coefficients, which means having fewer values representing the pixel. The nonzero values are then compressed using entropy coding, which a type of lossless compression [1, 4, 22].

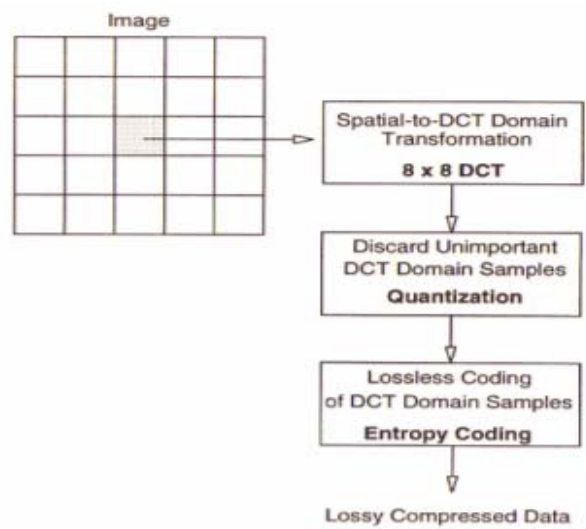


Figure 2.3: DCT-based coding system
(Source: Bhaskaran-et-al, 1995)

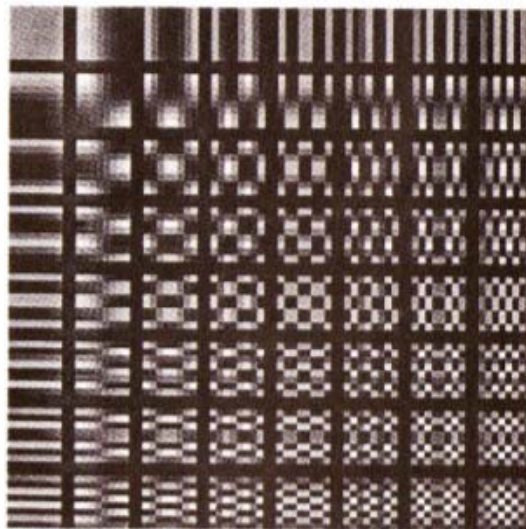


Figure 2.4: DCT basis
(Source: Barnley, MF 1993)

2.1.5 BTC (Block Truncated Coding)

Block truncation coding is another type of lossy image compression technique (mainly suited for gray scale images). Although this technique does not reach very high compression ratios, it has the advantage of been very simple to implement, and also

described as faster and more efficient than vector quantization. This is because in BTC the conversion of the blocks into simpler forms will preserve some statistics of the original blocks—unlike vector quantization which relies on the approximations of the blocks in the codebook.

In this method the image is divided into blocks of pixels each with the size of $n \times n$ (usually 4×4), then finding the mean or the average of the block to be used as a threshold. Pixels are then classified to high or low, depending on that threshold. For the reconstruction process of the block we will use two values, a and b —these two values will be close enough to the original values.

$$a = x_m - \sigma (q / (n - q))^{1/2} \quad (2.1)$$

$$b = x_m + \sigma ((n - q) / q)^{1/2} \quad (2.2)$$

where: x_m is the mean, σ is the variance, q is number of pixels greater than mean and n is the number of pixels in the block [5].

2.1.6 JPEG/JPG (Joint Photographic Experts Group)

Nowadays, JPEG/JPG is the most widely recognized compression; JPEG/JPG is a lossy type of compression developed by the *Joint Photographic Experts Group* and was first accepted as an international standard in 1992.

JPEG/JPG is based on DCT, and the encoding and the decoding of it consists of several steps that each contributes to compression. The first step of the encoding process is partitioning the image into blocks of 8×8 , then for each block the FDCT (*Forward Discrete Cosine Transform*) is computed. The 64 DCT coefficients are then scalar quantized using the quantization table, and those coefficients ordered in a zigzag format as shown in Figure 2.7. At the last stage of the JPEG/JPG encoding is the “*Entropy*

Encoder” which is used to send out the coefficients codes by using either Huffman Coding or Arithmetic Coding. The decoding, as illustrated in Figure 2.6, is simply the inverse process of the encoding [10, 15, 18, 22].

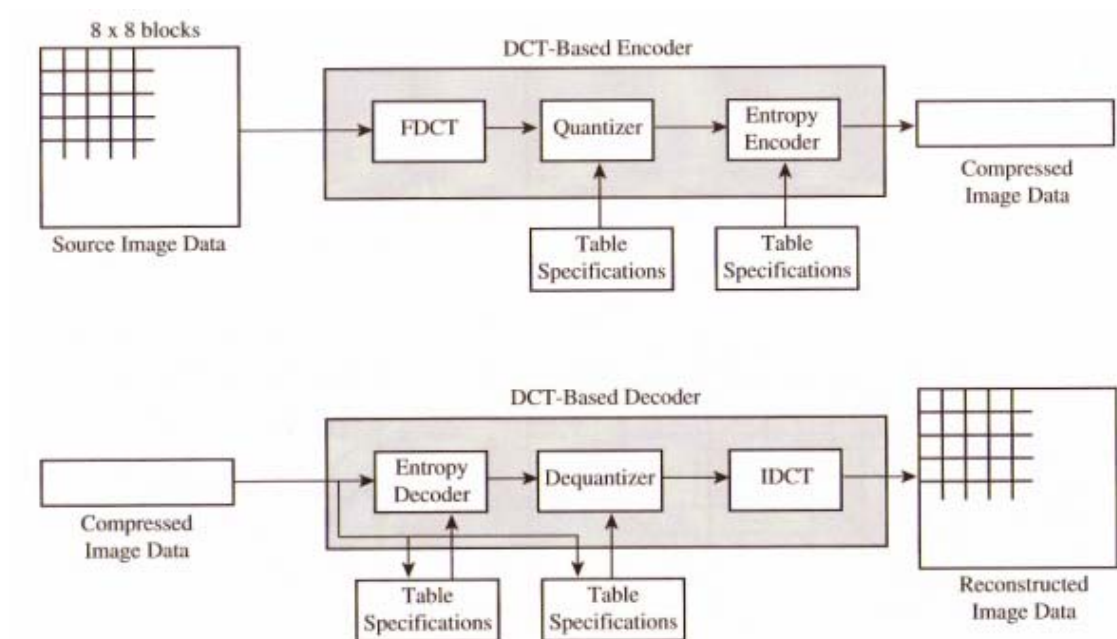


Figure 2.5: The basic JPEG/JPG Encoder and Decoder
(Source: Gibson-et-al, 1998)

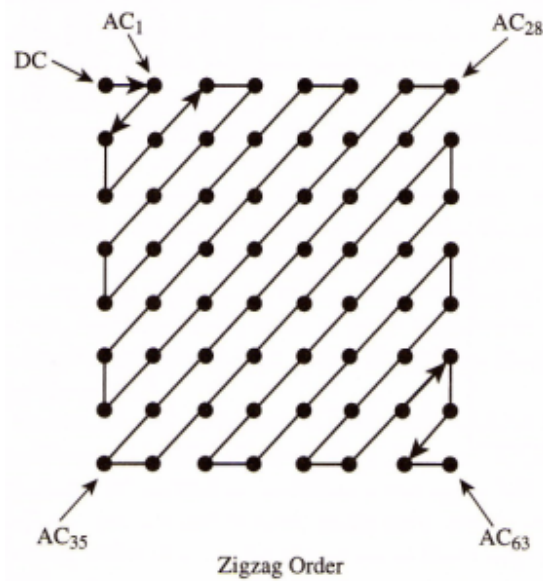


Figure 2.6: Zigzag Coefficient Ordering
(Source: Gibson-et-al, 1998)

2.2 Video Compression

Video is a number of images in sequence; therefore, the same way we compress images can be applied on the video by compressing each frame of the video separately (this is called *intra-frame* coding). Though this technique looks very simple, it is impractical because it requires a very large memory space to store data. The best way to achieve a better compression is in taking advantage of the similarities between the video frames (this advantage is also used in fractal video compression and discussed in chapter 8 of this dissertation). There are two main functions used in video coding [24]:

- 1- *Prediction*: create a prediction of the current frame based on one or more previously transmitted frames.
- 2- *Compensation*: subtract the prediction from the current frame to produce a ‘residual frame’.

2.2.1 Frame Differencing

The idea of frame differencing in video coding is to produce a residual frame by subtracting the previous frame from the current frame; the residual frame will then be a form of zero data, with light and dark areas (light indicates positive residual data and the dark indicates negative residual data). Nevertheless, there will be more portions in the residual frame with zero data than the light and dark; this is due to the similarity between the frames (most of the pixels in the previous frame will be equal to the pixels in the current frame). Therefore, since more of the residual frame is zero data, then the compression efficiency will be further improved if we compress the residual frame instead of the current frame. Frame differencing method in video coding faces a major problem which is best illustrated by the following example. In the encoding and the decoding process for the first frame, there will be no prediction, but the problem starts with the second frame when the encoder uses the first frame as the prediction and encodes the resulting residual frame. Here, we are dealing with a lossy type compression; the decoded first frame is not exactly the same as the input frame, which leads to a small error in the prediction of the second frame at the decoder. This error will increase as we continue with the following frames and the result will be low quality in the decoded video sequence.

There is one way in which this problem can be solved; the idea is that when we encode the first frame we decoded immediately and use the decoded frame to form the prediction for the second frame [24].

2.2.2 Motion-compensation Prediction

With frame differencing we can achieve a good compression, but only with frames of video which are very similar. When the differences between the previous frame and the current frame are not really similar, or there is a big change between the frames, then the

compression may not be significant. This is due to the movement in the video scene. So, in this type of frames we use another method of prediction called Motion-compensation Prediction; where the achievement of better prediction is by estimating the movement and compensating for it. Motion-compensation is similar to Frame differencing with two extra steps [4, 24]:

- 1- *Motion estimation*: comparing a region in the current frame with the neighboring regions of the previous decoded frame and finding the best match.
- 2- *Motion Compensation*: Subtracting the matching region from the current region.

The encoder will send the location of the best match to the decoder to perform the same motion compensation operation in the process of decoding the current frame.

The residual frame in motion-compensation contains less data compared with frame differencing (higher compression); however, motion-compensation is very computationally intensive.

2.3 Chapter Summary

Image and video compression is very necessary to reduce the capacity required for storing data and also to shorten the time required for sending data. Compression of image and video is divided into two categories—lossy and lossless. With lossless compression we cannot achieve high rates of compression but the decoded image or video will be exactly same as the input; on the other hand, with lossy type we obtain high compression rates but lose some data. However that loss of data is not significant since human visual systems cannot detect it.

The techniques used in image coding can be used for the video as well, but in video coding we take advantage of the similarity between the frames and, because of that, we obtain higher compression rates compared with image compression.

Chapter 3

Fractal Image Compression

3.1 History of Fractal Image Compression

The idea of fractals is not new—it goes back to the beginning of the last century. However, no one really understood or paid attention to the importance of fractals and its geometry until 1977 when Benoit Mandelbrot published his book *The Fractal Geometry of Nature*. Mandelbrot, in his book, tried to show the existence of fractals in nature—like clouds, mountains and trees. However, he did not think of fractals as a method for compression. Michael Barnsley was the first person to use the idea of fractals in image compression. It has been claimed that fractal coding may reach compression ratios up to 10000:1—which sounds like a very impressive rate of compression. However, it doesn't have a standard because it suffers from such problems as very expensive encoding time and, therefore, it is still under discussion and being studied.

3.2 How Does Fractal Image Compression Work?

To better understand how fractal image compression works, we take an example of a photocopier machine which takes an input image, reduces the image into half and then produces three copies of the original image at the output (see Figure 3.1 below):

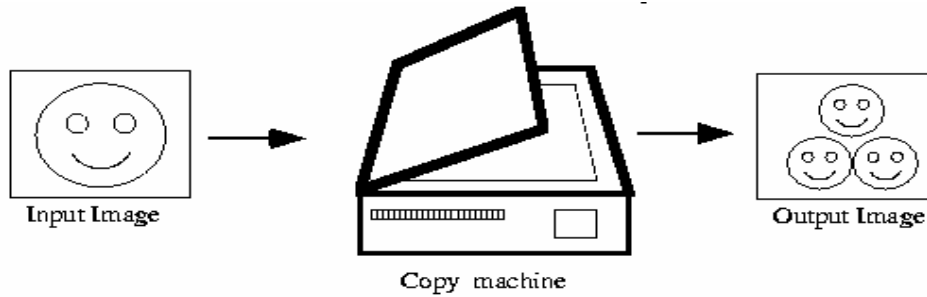


Figure 3.1: A copy machine.
(Source: Fisher, Y 1995)

Now if we feed back the output as an input and repeat the same process for two more times, we will end up with the shape showing in Figure 3.2 below:

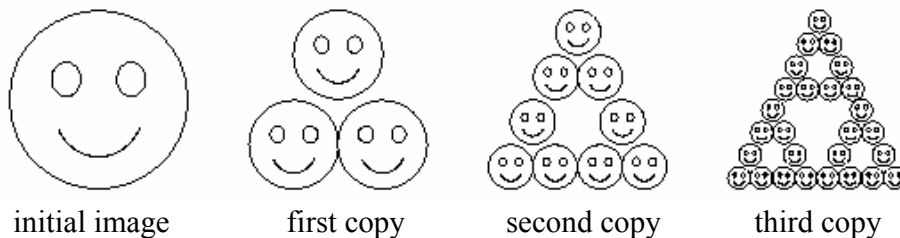


Figure 3.2: first three copies generated by the copy machine.
(Source: Fisher, Y 1995)

If we continue doing the same thing again and again for a number of times (n iterations), at some stage we will reach the following (see Figure 3.3 below):



Figure 3.3: The output of the copy machine after n iterations.
(Source: Fisher, Y 1995)

This final image is called the attractor for this copying machine. Now, notice that the last figure we ended up with is basically formed from the initial image (fractal), but this image does not affect the final shape of the final image (the attractor); in fact, the position

and the orientation of the copies will decide the final shape of the final image. In other words, the final image will be determined by the way the input image is transformed. This transformation of the input image must be *contractive* or, in other words, the transformation of two points in the input image must bring the points closer at the output [8].

3.3 The Contractive Mapping Fixed-Point Theorem

The contractive mapping fixed-point theorem state: A transformation $f : X \rightarrow X$ on a metric space (X, d) is called *contractive* if there is a constant $0 \leq s < 1$ such that [1, 2, 8, 16]:

$$d(f(x), f(y)) \leq s.d(x, y) \quad \forall \quad x, y \in X$$

(where $s = \text{contractivity factor}$)

And a point $a \in X$ is called a *fixed point* of the transformation f if $f(a) = a$.

In other words, the convergence to the fixed point or the attractor depends on the contractivity of the mappings. We say a map is contractive when it brings points closer to each other. For example, let's consider the map $f(x) = x/2$ —this map is contractive because if we take any initial value to be x , then by computing $f(x), f(f(x)), f^3(x), \dots$, the convergence will be toward the fixed point which is 0.

3.4 Fractal in Image Compression

The idea of fractal image compression is based on the self-similarity sets of the image (mainly works for natural images), where some parts of the image called range blocks is

similar to larger parts of the same image called domain blocks. Fractal image compression is of the lossy type, since the decompressed image is not the same as the original image.

3.5 Fractal Compression and the Self-similarity

The self-similarity in images is unlike the self-similarity in fractals—in fractals self-similarity is almost perfect or exactly the same. However, in real images, such as the images of people, trees, clouds and nature, there are different types of self-similarity, where one part of the image is similar to the other but not the entire image. Even the self-similarity may not be identical. Figure 3.4 below shows the self-similarity of two parts of the image with different scales; the small squares represent the ranges and the large squares represent the domains:

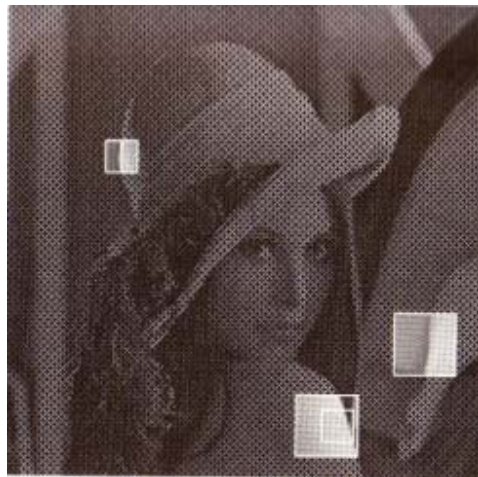


Figure 3.4: Lena image showing ranges and domains
(Source: Fisher, Y 1995)

3.6 Affine Transformation

The general form of the Affine Transformation in \mathbf{R}^2 is [1, 9, 33]:

$$w(x) = w \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \quad (3.1)$$

The matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is equal to $\begin{pmatrix} r_1 \cos \theta_1 & -r_2 \sin \theta_2 \\ r_1 \sin \theta_1 & r_2 \cos \theta_2 \end{pmatrix}$

So, equation (1) can be written as:

$$w(x) = w \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} r_1 \cos \theta_1 & -r_2 \sin \theta_2 \\ r_1 \sin \theta_1 & r_2 \cos \theta_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \quad (3.2)$$

Affine transformations are able to skew, stretch, scale, rotate and translate an input image. There are thousands or millions of transformations, therefore, to make the compression process easier (reducing the domain pool, decreasing the searching time); Jacquin (Arnaud Jacquin, one of Barnsley's students) suggested having the range and domain blocks to be always square and the domain size to be twice the size of the range. Also, he suggested having only eight transformations for the domain blocks:

1. Rotation by 0.
2. Rotation by 90.
3. Rotation by 180.
4. Rotation by 270.
5. Flip about horizontal median.
6. Flip about vertical median.
7. Flip about forward diagonal.

8. Flip about reverse diagonal.

In dealing with gray scale images, intensity of pixels should be treated as a third spatial dimension, thus the affine transformation for the gray scale images will become:

$$w(x) = w \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & s \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} e \\ f \\ o \end{pmatrix} \quad (3.3)$$

where s represents the contrast (luminance) and o represents the offset to the pixel contrast which is the brightness of the transformation.

3.7 Iterated Function System

Iterated Function System (IFS) was first introduced by John Hutchinson in 1981 and developed by Michael Barnsley in 1988. IFS play the main rule in fractal image compression. From the word 'iteration' we know that it is something repeating. IFS takes the output of the first iteration as the input to the second iteration and continues doing that until it reaches the last iteration. Fractal image compression is based on Iterated Function System; the idea of IFS is to have a finite set of contraction mappings in which they are written as affine transformations. By applying IFS to a seed image (which is the same size of the original image) and provided the mapping is contractive (reducing the distance and bringing the points together), ultimately the final result is the *attractor* image or the *fixed point*. Iterated function system is not really practical in terms of it being hard to find the IFS for the entire image. PIFS (Partitioned Iterated Function Systems) was thus the breakthrough in fractal image compression. In 1988, Arnaud Jacquin developed the theory of Partitioned Iterated Function Systems (PIFS). The theory states that instead of finding the IFS of the entire image, it is possible to partition the

image into non-overlapping blocks of ranges and then apply IFS on each block of ranges [22].

3.8 Fractal Image Compression algorithm

3.8.1 Encoding

The encoding process of fractal image compression is not complicated, but very time consuming. In this project, the MATLAB codes written for this project (Appendix B) deal with both gray-scaled and color images, and the algorithm used is the classical way where the range and domain are square (later in the dissertation we will look at different types of partitioning). We can sum the encoding process in the following steps:

- 1- Read an image (the images used in this project are of the square size).
- 2- Partition the image into non-overlapped blocks of ranges of square sizes covering the whole image and non-overlapped blocks of domains (and we can take overlapped blocks), twice the size of range block.
Note: Using non-overlapped domain blocks is much better since we will have a larger domain pool to be compared with each of the range blocks for best matching. However, it will take a very long time compared with non-overlapping blocks of domain. Later in the dissertation it will be demonstrated that even with using non-overlapped domain blocks the resulting decoded images are acceptable in terms of quality.
- 3- Rescale the domain blocks (to the same size of range blocks) and find the eight possible transformations for each block.
- 4- Compare each range block with whole domain blocks to find best match.
- 5- Save the following coefficients:
 - The location of the domain.
 - Best transformation.
 - Contrast (scaling factor).

- Offset.
- 6- Continue doing the same for the rest of the range blocks until the last one is reached.

3.8.2 Decoding

The decoding in fractal compression is much faster compared with the encoding; here the time depends on the number of iterations, however, we will see that only a few iterations (usually after 4-8 iterations) are required to reach the fixed point or the attractor. In this project, we will see that only after 4 iterations we are getting to the fixed point, as illustrated in Figure 3.5 and Figure 3.6.

The following are the steps used in the decompressing process of fractal-based compression:

- 1- Load the saved coefficients.
- 2- Create memory buffers for the range and domain screens.
- 3- Apply the Affine coefficients on the domain screen.
- 4- Copy the content of the domain screen to the range screen.
- 5- Take the output of first iteration (range screen) to be the input of the next iteration.
- 6- Repeat doing the same until reaching the desired attractor.



Original Lena Image



Lena after 1 iteration



Lena after 2 iterations



Lena after 4 iterations



Lena after 8 iterations

Figure 3.5: Showing the original Lena image and the Decoded Images after 1, 2, 4 and 8 iterations.

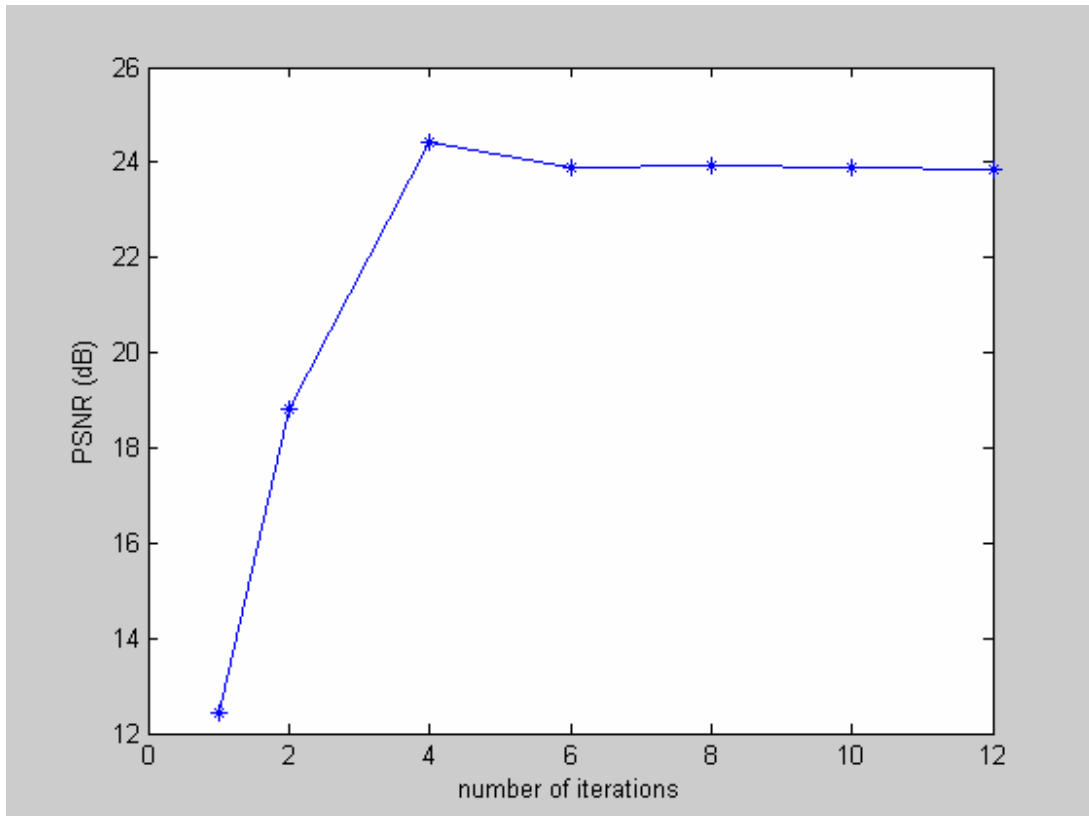


Figure 3.6: The Convergence towards the fixed point for Lena image.

3.9 Chapter Summary

Fractal image compression is mainly based on the self-similarity within the image; the image is divided into blocks of ranges and domains. The main task in fractal image compression is to find for each range block a corresponding domain block. The search for the best match between the range and domain blocks makes the encoding computationally very expensive, however, the following chapters explore some methods which have been adopted to reduce the searching time.

The decoding in fractal image compression is very fast compared with encoding and it depends on the number of iterations. The convergence of an image to its fixed-point usually takes no more than 4-8 iterations.

Chapter 4

Performance of Fractal Image Compression

This chapter focuses on the performance of fractal image compression in terms of compression rate, image quality and comparison with two other compression techniques; BTC (Block Truncated Coding) and JPEG/JPG (Joint Photographic Experts Group). The codes used for fractal image compression were implemented using MATLAB (Appendix B). BTC (Source: Lucey, 1998) and JPEG/JPG (Source: Skiljan, 2005) compression were implemented using MATLAB and *irfanview* respectively.

4.1 Compression Rate

In fractal image compression the achievement of different levels of compression ratios are dependent on the size of the range blocks. Compression occurs because each block of range is represented by only five values. These values are [33]:

- e & f : Represent the location of the corresponding domain block and each takes 8 bits.
- M : Is the transformation of the domain block and takes 3 bits
- s : Is the contrast and takes 5 bits.
- o : is the offset and takes 6 bits.

In this project, three different sizes of the range blocks are taken as the following; 4 x 4, 8 x 8 and 16 x 16. The compression ratio is then calculated as follows:

In the case of 4 x 4 range blocks, the compression rate is:

$$8 + 8 + 3 + 5 + 6 = 30 \text{ bits}/(4 \times 4) \text{ pixels (16 bytes)}$$
$$= 1.875/\text{pixel. } (\approx 4.25:1)$$

In the case 8 x 8 range blocks, the compression rate is:

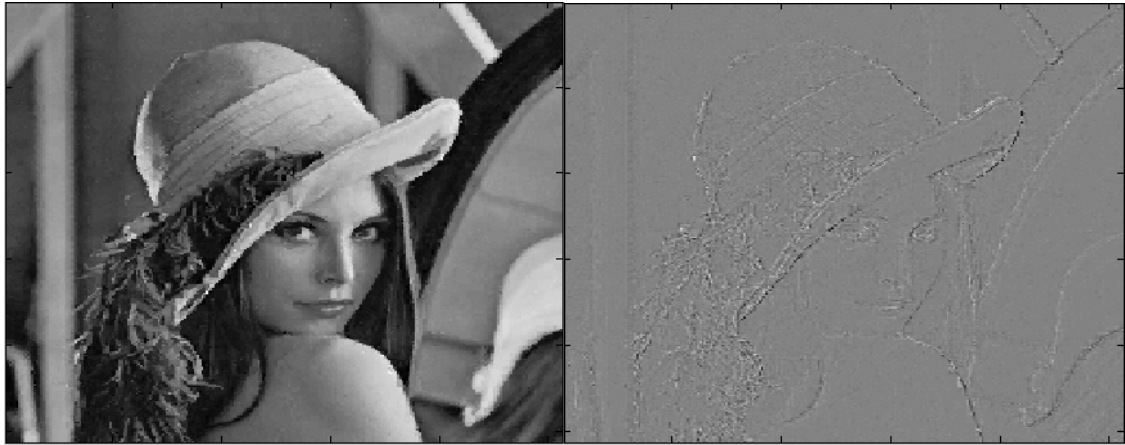
$$8 + 8 + 3 + 5 + 6 = 30 \text{ bits}/(8 \times 8) \text{ pixels (64 bytes)}$$
$$= 0.46875/\text{pixel. } (\approx 17.00:1)$$

And in the case of 16 x 16 range blocks, the compression rate is:

$$8 + 8 + 3 + 5 + 6 = 30 \text{ bits}/(16 \times 16) \text{ pixels (256 bytes)}$$
$$= 0.117/\text{pixel. } (\approx 68.00:1)$$

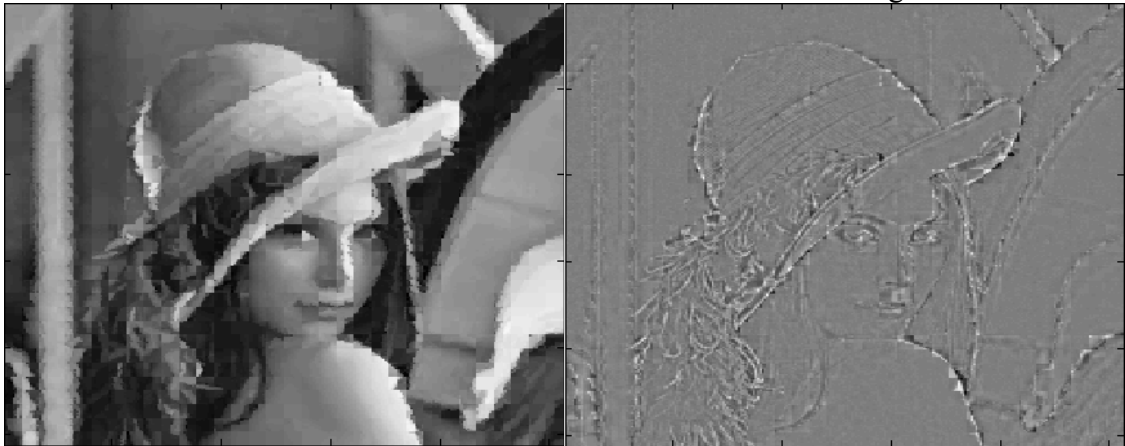
It is important to mention that experiments have shown that it is not necessary to have the same value of s (contrast) used in the encoding to be used in the decoding process; instead we can use a set of values for s between 0 and 1 in the encoding process and use only a single value or the same set for the decoding process, therefore, it is not necessary to save the s values, which means obtaining a higher compression rate [7].

Unfortunately, the relationship between the quality and the compression rate is inversely proportional. In other words, as the compression rate increases, the quality of the image decreases. This is illustrated in Figure 4.1 which shows how the Lena image results in poorer quality as the range size gets bigger (higher compression rates).



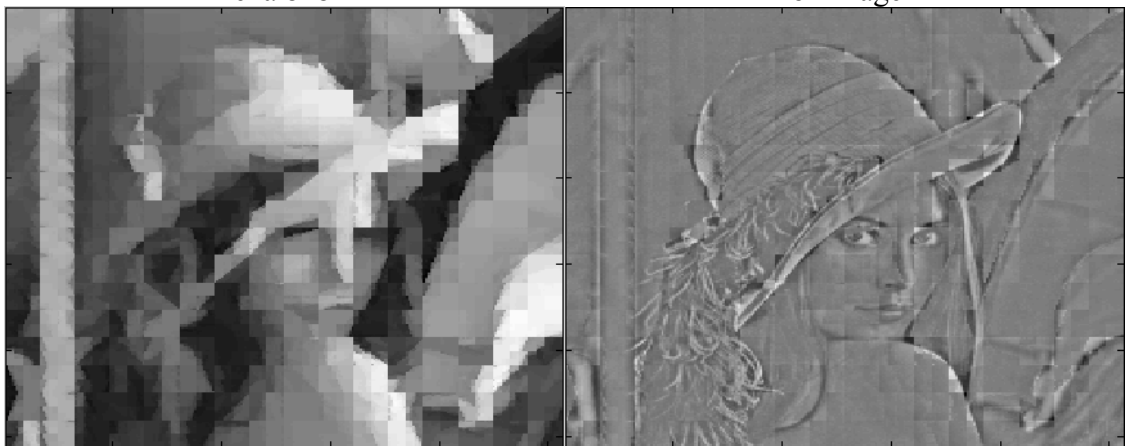
Lena 4x4

Error Image



Lena 8x8

Error Image



Lena 16x16

Error Image

Figure 4.1: Decoded Lena image with error image using 4x4, 8x8 and 16x16 range blocks.

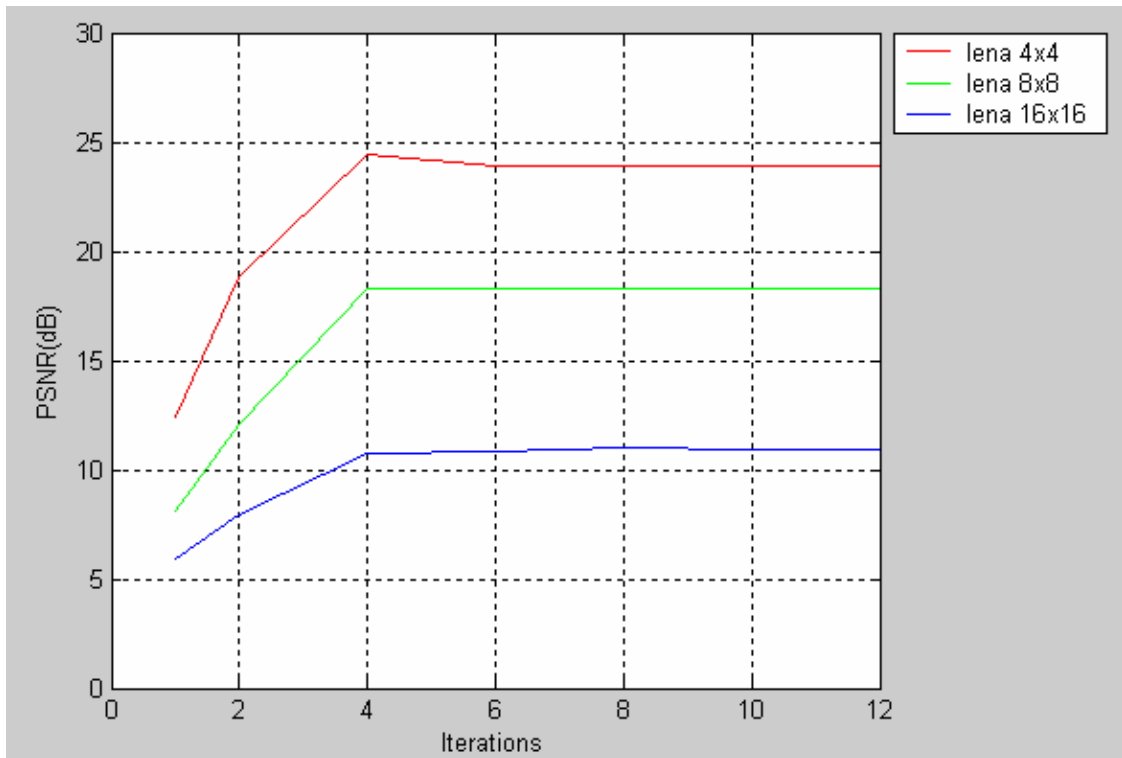


Figure 4.2: The convergence of Lena image using 4x4, 8x8 and 16x16 range blocks.

4.2 Image Quality

The quality in fractal image compression depends on many factors. One factor—mentioned in the previous section—is the size of the range block, but the main factor is the type of the image. Using the test images for this project it can be seen that the qualities of the output images are not all the same and in some cases the quality is very poor—for example decoded text image, which substantiates the findings of chapter 2. Fractal-based compression mainly works well with natural images, where there is more possibility of self-similarity within the image. In Figure 4.3 it can be seen how much the PSNR value of decoded text image is low compared with other images, and Figure 4.4 illustrates that decompressed image of text is very low in quality (decompressed text is hard to read) even with a small size of range blocks (4 x 4).

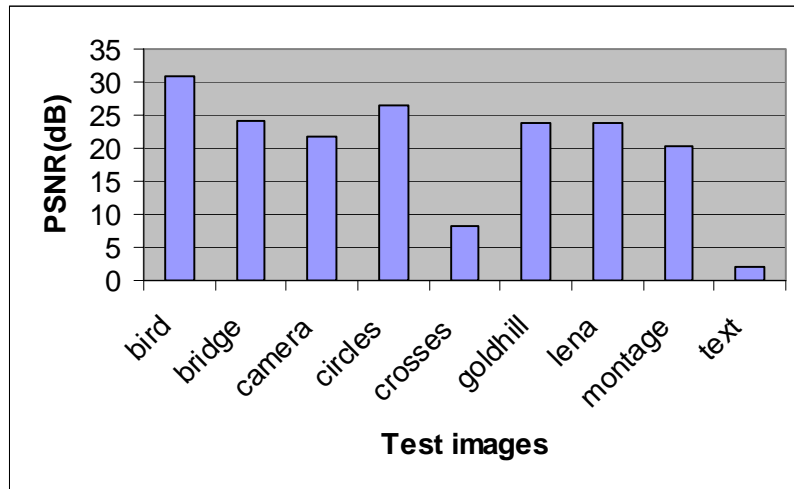


Figure 4.3: Quality of the decoded test images after 12 iterations

Another fact worth reiterating is that the image will not reach the fixed point (the final output image) until after some number of iterations. In this project, the results have shown that at least four iterations are required in order to gain a good output image (reaching the fixed-point). On the other hand, in the case of the text image it does not matter how many iterations are applied, the output image will always be low in quality and unreadable.

Fractal-based compression based on the self-similarity, then the size of the domain pool, also plays a significant role in the quality of the output image. With bigger domain pool and more possible transformations of the domain blocks, each range block will have more chance of finding the best/right corresponding domain block (best match). Consequently, the more the domain block is similar to the range block, the more the quality of the output image increases.

```

MT-LEVEL
{spinet}/home/u/rjkroeger/vfs
{spinet}/home/u/rjkroeger/vfs
Makefile compress.c fi
Makefile~ display.c fra
{spinet}/home/u/rjkroeger/vfs
{spinet}/home/u/rjkroeger/vfs
Makefile compress.c fi
Makefile~ display.c fra
{spinet}/home/u/rjkroeger/vfs
Makefile display.c imi
Makefile~ fileio.c ima
compress.c fractal.h imy
{spinet}/home/u/rjkroeger/vfs
{spinet}/home/u/rjkroeger/vfs
{spinet}/home/u/rjkroeger/vfs

```

Original text image

```

MT-LEVEL
{spinet}/home/u/rjkroeger/vfs
{spinet}/home/u/rjkroeger/vfs
Makefile compress.c fi
Makefile~ display.c fra
{spinet}/home/u/rjkroeger/vfs
{spinet}/home/u/rjkroeger/vfs
Makefile compress.c fi
Makefile~ display.c fra
{spinet}/home/u/rjkroeger/vfs
Makefile display.c imi
Makefile~ fileio.c ima
compress.c fractal.h imy
{spinet}/home/u/rjkroeger/vfs
{spinet}/home/u/rjkroeger/vfs
{spinet}/home/u/rjkroeger/vfs

```

Decompressed text image using fractal 4x4

Figure 4.4: Original and Decompressed text image

One other important feature regarding the quality of fractal image compression is *Resolution Independency*. Usually when zoom occurs at any point in a digitalized image, after some stages the image becomes blocky. Nevertheless, in fractal image compression

the decoded image will not be blocky and it will contain details at every scale (see Figure 4.5). The decoded process creates an artificial detail which was not presented in the original image. This is due to the iterations in the decoding process; after each iteration the details on the decoded image becomes smoother [5, 8, 16, 22].

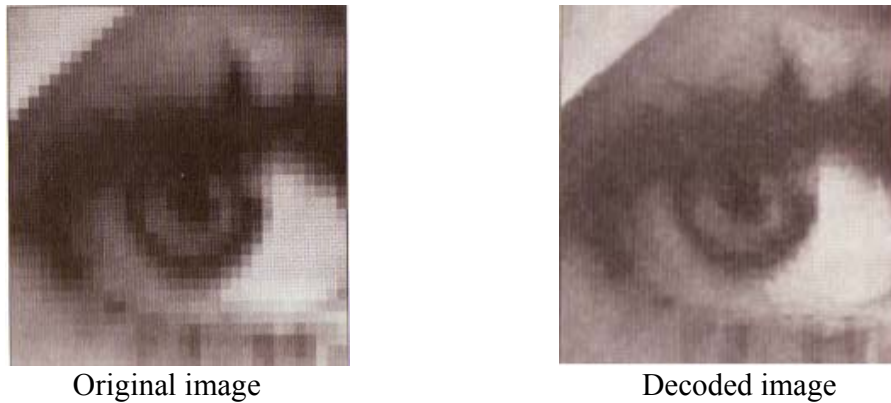


Figure 4.5: Lena original and decoded zoomed in.
(Source: Lu, N 1997)

4.3 Fractal Vs JPEG/JPG and BTC

In this section there will be a comparison between the fractal-based compression and two other lossy type of compressions: JPEG/JPG (Joint Photographic Experts Group) and BTC (Block Truncated Coding). However, since JPEG/JPG is widely used and one of the most famous around the world, it will be discussed further in this section and contrasted in terms of quality and compression ratios.

In Figure 4.6, using an approximated compression ratio for the three techniques, it is obvious that the performance of JPEG/JPG is the best when compared with the other two compressions, except for the text image. Though the PSNR value is not as low as in fractal compression, it is the only case where it gives lower PSNR value than the BTC coding. The BTC coding results (quality) are almost the same for the whole test images and they are very close to the results of fractal image compression, except for some images (crosses and text).

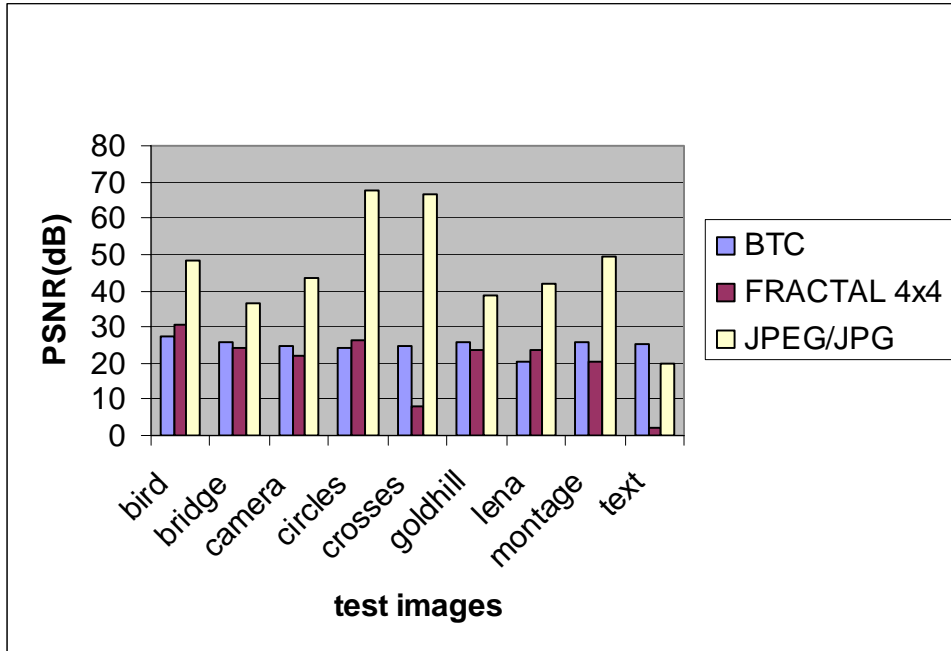


Figure 4.6: PSNR values for the decoded text images using the three techniques with an approximated compression rates for the three techniques.

The BTC code used in this project uses 4 x 4 blocks only, with a compression rate of 2 bits per pixel. On the other hand, the JPEG/JPG code used here can achieve different levels of compression ratios. Therefore, this advantage is taken to compare between fractal compression and JPEG/JPG on different levels of compression ratios. The compression ratios for fractal compression are based on the size of the range and calculated as in section 4.1.

The main common characteristic between the JPEG/JPG and fractal-based compression that the performance of the both techniques depends on is the type of the image,. Fractal image compression works very well with natural images, whilst JPEG/JPG compression works best with geometric images such as crosses and circles.

In figure 4.7 and 4.8 below it is shown how the JPEG/JPG compression works better than fractal-based compression for different levels of compression ratio. In general, this is the main advantage of JPEG/JPG compression, where it works very well—even with high compression ratios and it starts losing quality with very high compression ratio.

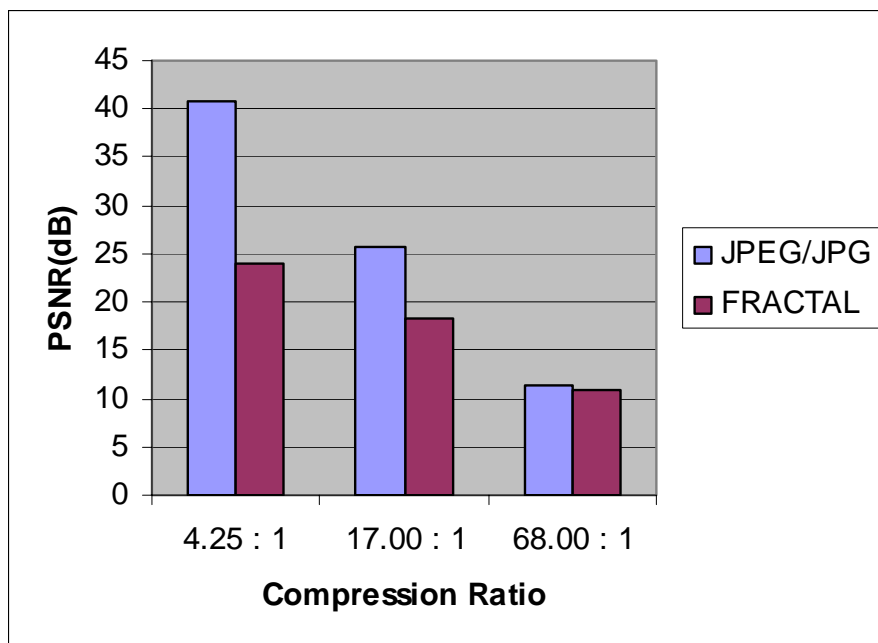


Figure 4.7: PSNR values for decompressed Lena image using different levels of compression ratio for both fractal-based compression and JPEG/JPG.



Figure 4.8: Lena image decompressed using fractal-based compression and JPEG/JPG with different compression ratios.

4.4 Chapter Summary

The advantage of using fractal image compression is that for each range block we have to save only five coefficients, which will give the ability of obtaining a very high compression ratio. The compression rate depends on the size of the range blocks—bigger range blocks leads to higher compression ratio. However, the cost will be degradation in the decoded image.

Fractal image compression does not work well for all types of images—it mainly works well with natural types of images such as trees, mountains and humans. On the other hand, the decompressed images of the fractal compression for unnatural images are very poor in quality.

Chapter 5

Fractal Color Image Compression

Little work has been done of fractal color image compression compared with the work done on fractal gray-scaled image compression. This chapter looks at how the color images are formed and how to use fractal-based compression for color images. Additionally, there will be a demonstration and comparison between the uses of different coordinate systems in the color images.

All the MATLAB codes used for this chapter are detailed in Appendix B, and only one color image is used for illustration and comparison (Lena color image, with size of 512 x 512, range size of 8 x 8).

5.2 Red, Green and Blue

The human visual system is sensitive to three primary colors: red, green and blue. Therefore, these three colors are used to represent colors in digitalized images or videos. Usually 8 bits are reserved for each color, so in theory the number of colors possible equals 2^{24} . In general, many systems use a palette-based system to indirectly produce the colors, using a look up table as depicted in Figure 5.1:

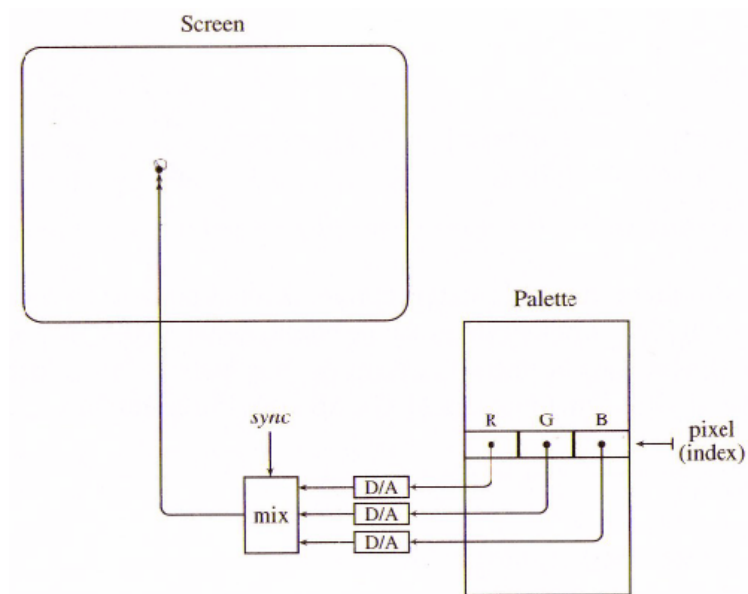


Figure 5.1: color palette as a lookup table to produce a pixel
(Source: Leis, J 2002)

In a compressing process of a color image the main idea is to divide the image into its three different layers or components (red, green and blue). It is then possible to compress each of these layers separately, in other words, handle each of the layers as an independent image [1, 8, 15, 16], therefore, the data needed to be stored and the time of encoding will be three times what it takes for the gray-scaled image.

The figure in the next page shows the image of Lena separated into red, green and blue components:



Red component



Green Component



Blue component

Figure 5.2: The Red, Green and Blue components of Lena color image.

In this project, the method outlined above is used to encode a color image using a fractal-based compression. The code (Appendix B) used takes the color image and then separates it into three different components and then encodes each of them in the same manner used for the gray-scale images. In the decoding process each of the components will be decoded separately and at the final stage all will be added together. The results of using RGB components are illustrated in Figure 5.3 and Figure 5.6.

5.3 YIQ and YUV

RGB coordinate system is not the only system used; there are two other coordinate systems which are universally used, namely, YUV and YIQ, where Y is the *luminance* or *brightness*, I-U is the *hue* and Q-V is the *saturation* (the combination of the hue and saturation is called *chrominance*).

These two systems are related to the R,G and B by a linear transformation as shown below [1, 8, 16]:

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.528 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (5.1)$$

$$\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.289 & 0.436 \\ 0.615 & -0.515 & -0.100 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (5.2)$$

The most important reason for using YIQ and YUV is that with these two it is possible to significantly reach much higher compression ratios compared with using RGB, and with only a small or negligible amount of degradation in the output image. The main reason is that the human visual system is more sensitive to the luminance than the chrominance, therefore, the I-U and Q-V can be decimated to one-half or one-quarter of their original size.

From the following figures it is obvious that compressing color image using YIQ and YUV components is not much different than using the RGB components—almost in the three cases the convergence is to the same fixed-point (see Figure 5.6); the output of the three methods is very similar.

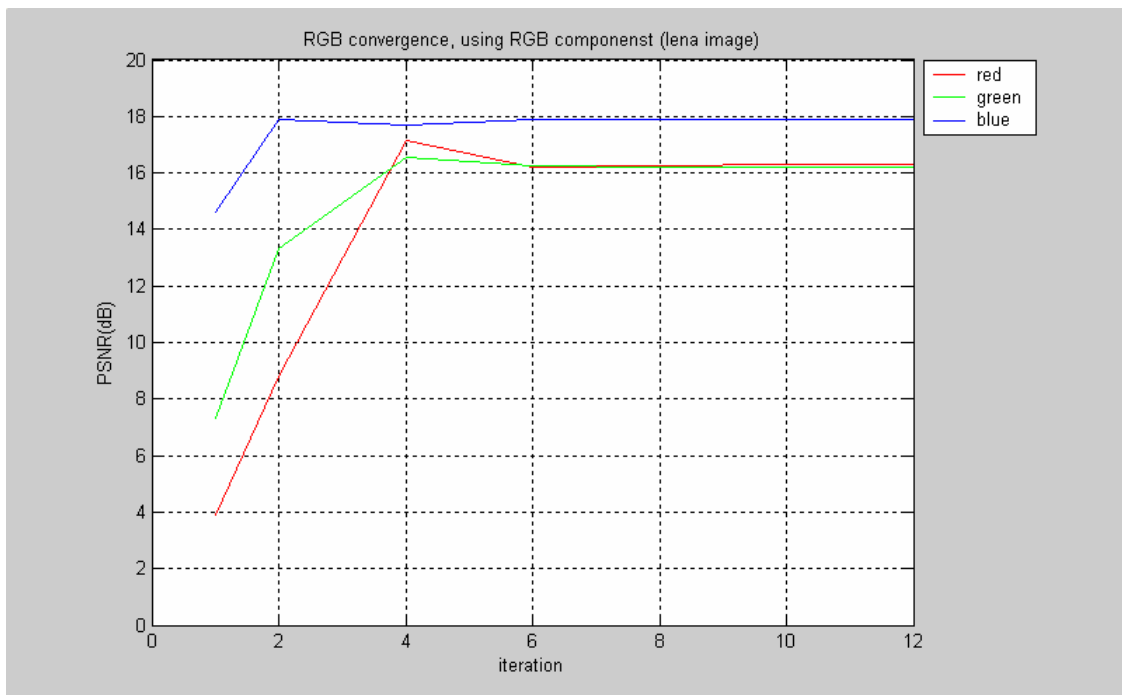


Figure 5.3: The convergence of the Red, Green and Blue images using the RGB components.

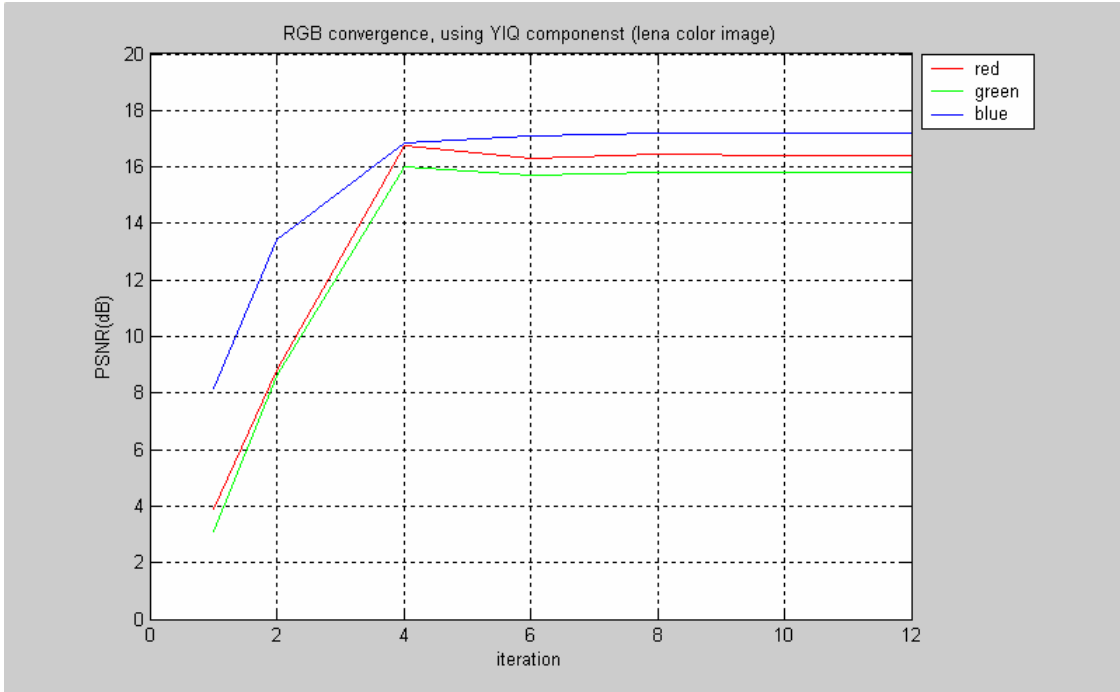


Figure 5.4: The convergence of the Red, Green and Blue images using the YIQ components.

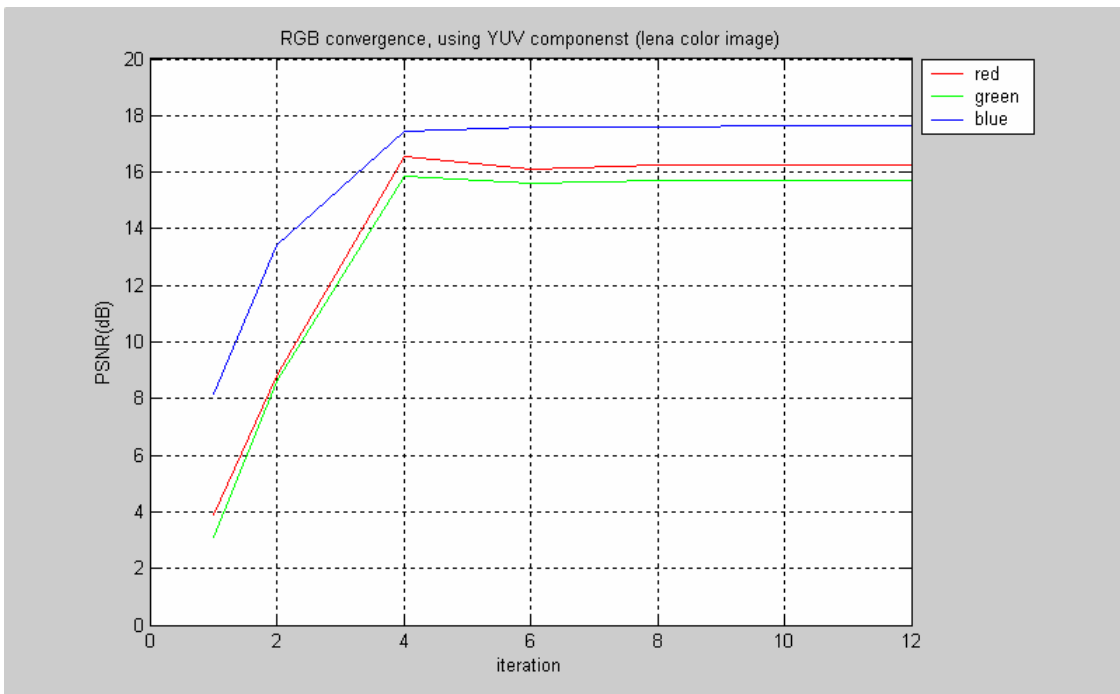


Figure 5.5: The convergence of the Red, Green and Blue images using the YUV components.



Figure 5.6: (a) Original Lena color image. (b) Decompressed Lena color image using the RGB components. (c) Decompressed Lena color image using the YIQ components. (d) Decompressed Lena color image using the YUV components.

5.4 Chapter Summary

In the fractal-based compression, the technique used to encode a color image is the same as for the gray-scale image, except that color images are made from three layers instead of one and each of these layers is to be compressed separately. RGB, YIQ and YUV coordinate systems are used in the color images. With YIQ and YUV it is possible to obtain higher compression ratios with a slight degradation in the output image compared with RGB. YIQ and YUV are not the only alternatives available; there are other coordinate systems like *simplified color coordinate system* LMN and *equalized color coordinate system* EMN in which both perform as well as the YIQ and YUV [16].

Chapter 6

Different Partitioning Methods

Image partitioning is one of the important issues in fractal image compression. The naive or classical way of partitioning an image was having the range blocks in a fix square size and the domain blocks being twice the size of the range block. However, other partitioning methods have been used in which have a lesser number of blocks causing a shorter encoding time, or to have more flexible partitioning leading to higher compression rates. In this chapter, three different partitioning methods are presented:

6.2 Quadtree Partitioning

One of the most common used partitioning methods is the quadtree partition, in this method the image is represented by number of layers in a tree structure as it is shown in the figure below:

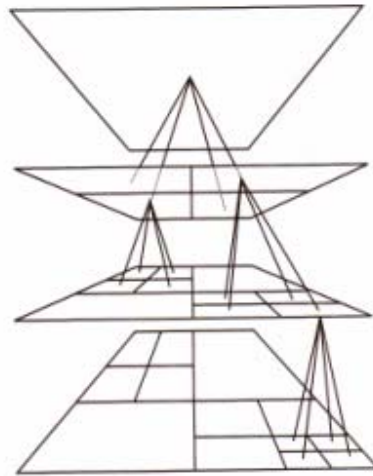


Figure 6.1: Quadtree Partition
(Source: Fisher, Y 1995)

The original image (the root image) is first broken into four quadrants, then for each range block it is compared with the domain block (transformed domain). If the distance or the *rms* value between the range and the domain is below preselected threshold, then no further partition is required. If this is not the case, then each range block will be subdivided into four quadrants, and the process repeats until it reaches the preselected maximum depth of the quadtree [8, 10, 16].

6.3 Horizontal-Vertical Partitioning

HV-partitioning is similar to quadtree partitioning; however, it splits the image in rectangles instead of squares. Figure 6.2 illustrates the H-V partitioning.

The original image is to be considered as a rectangular image, and then the cutting of the image will be either horizontally or vertically, depending on the splitting measure. At each stage, there will be a check through all rectangles to decide which are to remain the same and which need to be cut into two smaller rectangles. This continues in this way until there is no more cutting possible. The advantage of HV-partitioning over quadtree partitioning is its flexibility because the position of each block is changeable [8, 16].

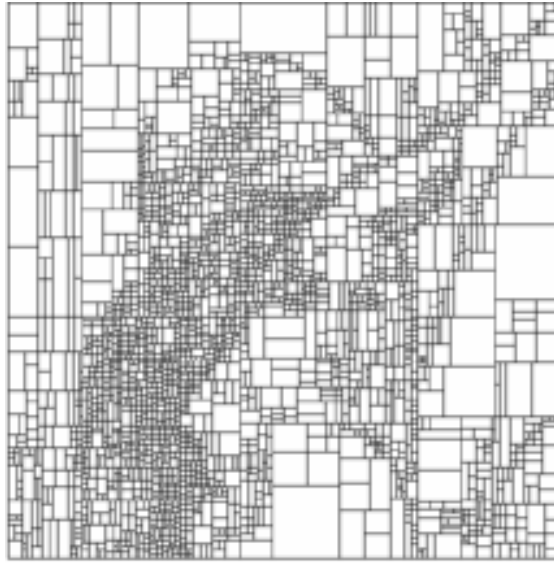


Figure 6.2: Partitioning Lena image using H-V partition.
(Source: Fisher, Y 1995)

6.4 Triangular Partitioning

The triangular partitioning is also similar to the quadtree partitioning in which each block splits into smaller pieces whenever it is necessary. From the name, it is recognised that the partition of the image is of the triangular shape. Starting with the original image to be divided into two triangles, each can then be subdivided into four triangles; this process will continue until no more partitioning is possible.

Due to the triangular shape of the partitioning, it gives higher and better flexibility of the blocks compared with the H-V partitioning and quadtree partitioning [8, 16].

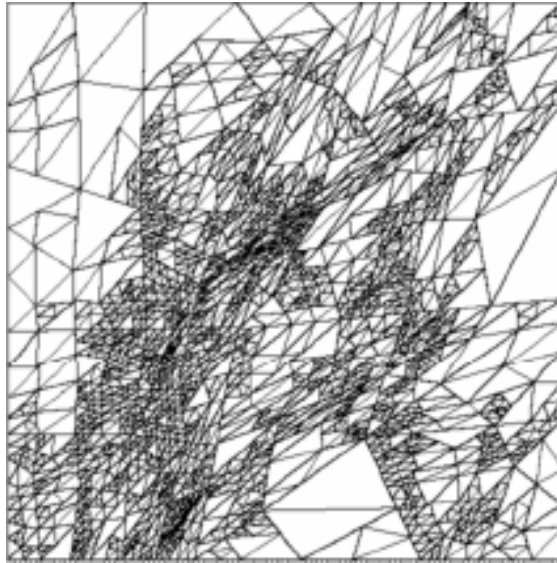


Figure 6.3: Partitioning Lena image using Triangular partition.
(Source: Fisher, Y 1995)

6.5 Chapter Summary

The traditional way of image partitioning was square blocks of range and domain blocks. Other partitioning methods have since been adopted to reduce the number of blocks in order to improve the encoding time, like the Quadtree partitioning, and to have more flexible partitioning to gain higher compression rates, like the H-V and Triangular Partitioning.

There are other types of partitioning such as Fuzzy Hexagonal Partitioning, Mixed Square Partitioning, Hexagonal Partitioning, Polygonal Partitioning and Split and Merge partitioning [8, 16, 25, 26].

Chapter 7

Faster Encoding

Fractal image compression has one main disadvantage: it is computationally expensive. The time taken for each range to be compared with the domains in the domain pool is very lengthy. Consider an image of the size 256 x 256. If the range blocks is of the size 4 x 4 then the number of range blocks will be $(256/4)^2 = 4096$ blocks. Domain blocks are overlapped, so, the number of domain blocks will be $(256 - 2 \times 4 + 1)^2 = 62001$ blocks. If there are only eight possible transformations for each domain block then the total number will be 496,008 blocks, so, in total $496,008 \times 4096 = 2,031,648,768$ comparisons. Therefore, most of the work on fractal coding involved investigating methods in order to reduce the encoding time [11, 26]:

- 1- One method used to reduce the encoding time is the *classification scheme*, where the range blocks and the domain blocks are grouped in classes in which the ranges and domain of the same characteristics will be in the same class. Consequently, during the encoding, comparison process takes place with the range and the domain of the same class—not the whole domain blocks. Therefore, encoding time will be reduced.
- 2- One of the latest methods to improve the encoding time of fractal image compression is by discarding all the domain blocks with high entropy. It has been noted that not all domain blocks are used in fractal encoding; most of the domain blocks used are located at the regions with high degree of structure, for instance,

along the edges and high contrast regions of the image. As a result, having less domain blocks to be compared with the range blocks will improve encoding time. Figure 7.1 shows the domain blocks of the size 8×8 used in the encoding process for the Lena image:



Figure 7.1: Lena image with domain blocks used in black.
(Source: Hassaballah-et-al, 2005)

- 3- Another method of reducing the encoding time is comparing the range block with neighboring domain blocks (domain that overlap the range blocks). It has been observed that the best suitable blocks of domain to be compared with the range blocks are the block which overlap with the range block. This will reduce the search for the corresponding domain block for the range block from the entire image to only some parts close to the range block.

Those methods mentioned above are not the only methods adopted to reduce the encoding time in the fractal-based compression. There are other experiments that showed significant results using wavelet-based block classification, DCT domain block matching and Genetic Algorithm. For more information, read [19, 31, 34].

Chapter 8

Fractal Video Compression

For fractal video compression, there are two extensions of still image compression. They are frame-based compression and cube-based compression.

In frame-based compression, video clips and motion pictures are naturally divided into segments according to scene changes [30]. Each segment, beginning with an initial frame, is called an intra-coded frame, or I-frame. Each frame then can be coded mainly using the motion codes by referencing its preceding frame called a P-frame, as a predicted frame from its predecessor. The I-frames and the P-frames are also called coarse frames and the frames that are added between any two of the I-frames and P-frames are called bidirectional frames or B-frames (see Figure 8.1). Each B-frame is coded using the prediction from both coarse frames immediately before and after it [3, 16].

In a 2-dimensinal fractal video compression system, the I-frames are compressed using image compression technique. The I-frames are coded by 2-to-1 local and global self-referencing fractal codes. While decoding such an I-frame, a hidden frame has been created in each iteration. For example, if an I-frame F is created by 10 iterations from some initial image F_0 with self reference fractal codes, 10 consecutive P-frames will be set that have the same set of codes and are identical to the I-frame fractal codes, but will be referenced to the preceding frame instead of itself. Then, starting from the same initial image F_0 , by the end, the tenth P-frame is clearly the same I-frame obtained in the first

procedure. As a result, a fractal represented I-frame can be replaced by a sequence of P-frames if a time delay is allowed [16].

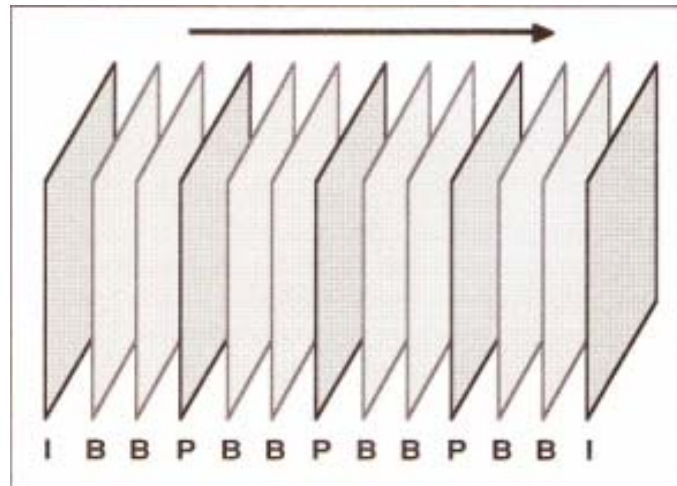


Figure 8.1: Video clip frames.
(Source: Lu, N 1997)

In a cube based compression, image sequences are portioned into groups of frames, and every group of frames is portioned into non-overlapped cubes of ranges and domains (see Figure 8.2). The compression codes are computed and stored for every cube.

Every group of frames is called GOF. Each GOF can be compressed and decompressed separately as an entity. Assuming temporal axis along the sequence, every GOF can be considered as a large cuboid. In fractal compression, each GOF is portioned into non-overlap small cuboids. Each cuboid is called as a range cuboid and denoted as R. The sizes of edges of R may be different; especially the edge in the temporal direction may vary from the horizontal direction and the vertical direction. In order to obtain the approximate transformation of R, another overlap partition is necessary whose small parts are called domain cuboids. The horizontal and the vertical edges of the domain cuboids are twice as large as the range cuboids respectively. But the temporal edge of the domain cuboids is the same as the one of the range cuboids. [30]

The cuboid algorithm of fractal video compression is given below [30]:

Partition the motion image sequence to a series of GOF.

1. For each GOF the following steps have been done:
 - i. Partition the GOF into range cuboids and domain cuboids. The horizontal and the vertical edges of the domain cuboids are twice as large as the ones of the range cuboids respectively. The temporal edge of the domain cuboids is the same size as the one of the range cuboids.
 - ii. For each range cuboid R the following steps have been done:
 - a. All domain cuboids are shrunk to codebook cuboids that are the same sizes as R in three directions.
 - b. For each codebook cuboid D , compute the scale factor and the offset factor α, β of D and the rms error between R and $\alpha D + \beta I$.
 - c. Choose the optimal approximation $R \approx \alpha D + \beta I$ that have the minimal rms error of all codebook cuboids.
 - d. Store α, β and the location of D of the optimal approximation as the compression codes of R .

The frame-based compression can obtain high compression ratio, but the compression of the current frame is related to the previous decompressed image, so there is a delay between frames when decompressed and error may spread between frames. The cube-based method can obtain the decompressed images with high qualities. Some studies use adaptive partitioning to improve the compression ratio. However, if transmission error is considered, the adaptive partition is not suitable because the partition information may be lost during transmission [30].

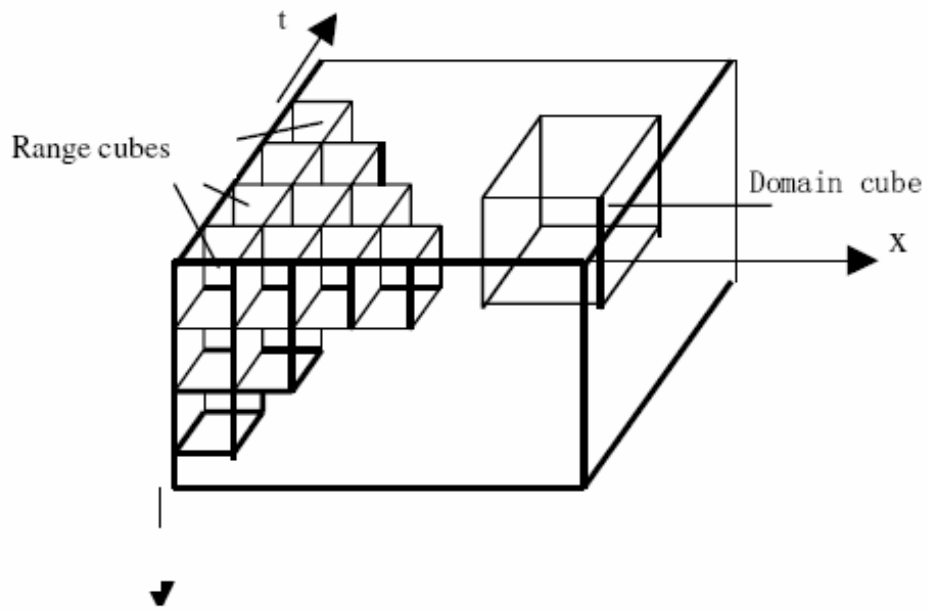


Figure 8.2: Range cubes and domain cube in a GOF
(Source: Wang, M 2004)

Chapter 9

Conclusions and Further Work

9.1 Achievement of Objectives

In this project, the topics of image and video compression using different techniques were investigated, in particular, the new compression method called fractal image-video compression. Through this investigation the following was found:

- 1- All compression techniques belong to two types of compression, one called lossless compression and the other called lossy compression. With lossy compression, there is a loss in some data, but high compression rate is achieved; with lossless compression, no data is lost but it is hard to achieve a high compression rate.
- 2- Fractal image compression is a lossy type of compression based on self-similarity within the image and mainly works well with natural type of images. It has the ability to achieve high compression rates, however, with very high compression rates it starts losing quality.
- 3- Though fractal compression works very well with gray-scale and color images, it suffers from one main weakness. Fractal compression is computationally very expensive. In this project, and using a personal computer (Pentium 4, CPU 2.40 GHz, RAM 480 MB), the fractal encoding time for a gray-scale image was around one hour and for the color image was around three and half hours.

However, as the speed of the CPU is rapidly increasing every year, maybe after 10 or 20 years the encoding time will not be such an issue.

- 4- Compared with fractal image compression, very little work has been done on fractal video compression. Primarily, there are two methods in fractal video compression: one is frame-based compression and the other is cube-based compression.
- 5- Many methods have been adopted to improve the performance of fractal image compression, such as using different partitioning and reducing the domain pool, however, more investigation and study is required for further improvement.

9.2 Further Work

Unfortunately, less investigation has been carried out on fractal video compression when compared with what has been done on still images, especially gray-scale images. This, combined with time constraints, made it unfeasible to investigate this area more comprehensively for this project, neither was it possible to use MATLAB to implement fractal video compression based on the algorithm represented in Chapter 8.

Future work should concentrate on fractal video compression, trying to implement fractal-based video compression and comparing the results with other video compression methods such as MPEG. On the other hand, the improvement of encoding time remains a significant challenge.

References

- [1] Barnsley, MF & Hurd, LP 1993, *Fractal image compression*, AK Peters, USA.
- [2] Barnsley, MF 1993, *Fractals everywhere*, 2nd edn, Academic Press, USA.
- [3] Barthel, KU & Voye, T, 'Three-dimensional fractal video coding', viewed 5th June 2005, <<http://portal.acm.org/citation.cfm?id=839284.841398>>.
- [4] Bhaskaran, V & Konstantinides, K 1995, *Image and video compression standards-algorithm and architecture*, Kluwer Academic Publisher, USA.
- [5] Clarke, RJ 1995, *Digital compression of still images and video*, Academic Press Limited, London.
- [6] Crane, R 1997, *A simplified approach to image processing*, Prentice Hall, New Jersey.
- [7] Finlay, M & Blanton, KA 1993, *Real-world fractals*, M&T Books, New York.
- [8] Fisher, Y (ed) 1995, *Fractal image compression-theory and application*, Springer-Verlag, New York.
- [9] Gharavi-Alkhansari, M & Huang, TS 1996, 'Fractal-based image and video coding', in L Torres & M Kunt(eds), *Video coding-the second generation approach*, Kluwer Academic, Boston.
- [10] Gibson, JD, Berger, T, Lookabaugh, T, Lindbergh, D & Baker, RL 1998, *Digital compression for multimedia-principles & standards*, Morgan Kaufmann, San Francisco.

- [11] Hassaballah, M, Makky, MM & Mahdy, YB 2005, 'A fast fractal image compression method based entropy', *Electronic Letters on Computer Vision and Image Analysis*, vol.5, no. 1, pp. 30-40.
- [12] Kominek, J 2001, *Waterloo fractal compression project*, < <http://links.uwaterloo.ca>>
- [13] Lalitha, EM & Satish, L 1998, 'Fractal Image Compression for Classification of PD Sources', *IEEE Transactions on Dielectrics and Electrical Insulation*, vol.5, no.4, August, viewed 20 July 2005, <<http://eprints.iisc.ernet.in/archive/00001459/01/image.pdf>>.
- [14] Leis, J 2002, *Digital signal processing-a matlab-based tutorial approach*, Research Studies Press Ltd, England.
- [15] Li, ZN & Drew, MS 2004, *Fundamentals of multimedia*, Prentice Hall, Upper Saddle River, NJ.
- [16] Lu, N 1997, *Fractal imaging*, Academic Press, USA.
- [17] Lucey, S 1998, *IEEE transaction on communication*, vol. com-27, no. 9, September 1979.
- [18] Miano, J 1999, *Compressed image file formats*, ACM Press, New York.
- [19] Mitra, SK, Murthy, CA & Kundu, MK 1998, 'Technique for fractal image compression using genetic algorithm', *IEEE Transaction on Image Processing*, vol. 7, no. 4, viewed 12 march 2005, <http://www.isical.ac.in/~malay/Papers/IP_4_1998.pdf>.
- [20] Moffat, A & Turpin A 2002, *Compression and coding algorithm*, Kluwer Academic Publishers, Boston.

[21] Morris, B 2004, *Fractal image compression for video*, Bachelor of electrical & electronic, University of Southern Queensland, Queensland.

[22] Nelson, M & Gailly JL 1996, *The data compression book*, 2nd edn, M&T Books, USA.

[23] Puram, VG 1999, *Image coding based of the fractal theory contractive image transformation*, Project, University of Kentucky, Kentucky, viewed 7 March 2005, <<http://voip.netlab.uky.edu/~venu/cs635/project.doc>>.

[24] Richardson, IEG 2002, *Video codec design-developing image and video compression systems*, John Willey & Sons , USA.

[25] Ruhl, M, Hartenstein, H & Saupe, D, 1997, ‘Adaptive partitioning for fractal image compression’, *IEEE International Conference on Image Processing*, Santa Barbara, October.

[26] Saupe, D & Ruhl, M 1996, *Evolutionary fractal image compression*, Proceeding of IEEE International Conference on Image Processing, Lausanne, pp. 129-132.

[27] Saupe, D, 1996, ‘Lean domain pools for fractal image compression’, *SPIE Electronic Imaging’96*, San Jose, January.

[28] Skiljan, I 2005, *Irfanview*, <<http://www.irfanview.com>>.

[29] Soberano, LA 2000, *The mathematical foundation of image compression*, University of North Carolina, North Carolina, viewed 9 March 2005, <http://people.uncw.edu/hermanr/signals/ImgComp_Soberano.PDF>.

[30] Wang, M, 2004, ‘Cubid method of fractal video compression’, *International Conference on Information Technology for Application*, China.

[31] Wohlberg, B & Jager, G, 1995, 'Fast image domain fractal compression by DCT domain block matching', *Electronic Letter*, vol. 31, pp. 869-870, May.

[32] Wong, B 1993, *Fractal image compression*, Bachelor of engineering degree in electrical engineering-computer systems , The University of Queensland, Queensland.

[33] Yang, GZ, *Image compression*, Department of Computing-Imperial College, viewed 15 March 2005, <<http://www.doc.ic.ac.uk/~twl1/multimedia/mm-notes-3.pdf>>.

[34] Zhang, Y & Po, LM 1996, 'Speeding up fractal image encoding by wavelet-based block classification', *Electronics Letters*, November, vol.32, no.23, pp.2140-2140, viewed 10 April 2005, <http://www.ee.cityu.edu.hk/~lmpo/publications/1996_EL_Fractal.pdf>.

Appendix A

Project Specification

University of Southern Queensland
Faculty of Engineering and Surveying

ENG 4111/4112 Research Project
PROJECT SPECIFICATION

FOR : Khalid Kamali

TOPIC: Fractal Video Compression

SUPERVISOR: Dr. John Leis

ENROLMENT: ENG 4111 – S1, D, 2005
ENG 4112 – S2, D, 2005

PROJECT AIM: The aim of this project is to investigate the variety of image and video compression methods, especially fractal compression for both images and video.

PROGRAMME: Issue A, 4th April 2005

1. Conduct a literature review of different image compression techniques.
2. Investigate the compression rates and computational efficiency of different compression techniques.
3. Research fractal image compression, and compare fractal image compression with other compression techniques in terms of compression rate, image quality, compression/decompression time, and memory requirements.
4. Investigate the use of fractal compression for color images.
5. Use MATLAB to implement several techniques for still image compression, and in particular fractal compression.
6. Investigate the possible applications of fractal compression to video sequences. Implement the proposed methods in MATLAB.

And as time permits....

7. Investigate methods of reducing the computational complexity of the domain/range search required in fractal compression.

AGREED: _____ (student) _____ (supervisor)

DATE: ___/___/___

Appendix B

MATLAB Codec for Fractal Gray-Scale & Color Image Compression

B.1 main_enc.M

```
function main_enc
% Syntax:    <main_enc>
% Description: It asks the user for the image name and the size of the
%              range. It does all necessary checking. Then it encodes
%              the image using Fractal Image Compression.
% Inputs:    Nill
% Outputs:   The decompressed file
% Functions Used: fractal_enc, rgb2yuv, rgb2yiq
% Written by: Khalid Kamali
% Date:      9/7/2005
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
clc;

% Taking the file name
file_name=input('Enter the file name:','s');

% Reading the image
readimage=imread(file_name);
[l w h]=size(readimage);

% Checking whether the image is square image or not.
while (l ~= w)
    disp('Image must be square (256x256 or 512x512 or 1024x1024 etc)');
    choice=input('Do you want to continue ( y for yes or n for no):','s');
    if choice == 'y'
        file_name=input('Enter the file name:','s');
        readimage=imread(file_name);
        [l w h]=size(readimage);
```

```

else
    disp('Program terminated....');
    return;
end
end

% Setting range sizes to 4, 8 or 16
ranges=[4 8 16];

% Getting the range size from the user.
choice = menu('Choose range size','4','8','16');
rangesize=ranges(choice);

% Compression of the gray image
if (h==1)
    disp('This is a grayscale image')
    choice=0;
    gray=readimage;
    coeff=fractal_enc(rangesize,gray);

% Compression of the color image
elseif(h==3)
    disp('This is a color image')
    red=double(readimage(:, :, 1));
    green=double(readimage(:, :, 2));
    blue=double(readimage(:, :, 3));

% Asking the user for the compression type of the color image
choice = menu('Choose the type of the components','Red Green Blue','YUV','YIQ');

% Compressing the color image using red, green and blue components

```

```

if choice==1
    coeff1=fractal_enc(rangesize,red);
    coeff2=fractal_enc(rangesize,green);
    coeff3=fractal_enc(rangesize,blue);

    % Compressing the color image using Y,U and V components
elseif choice==2
    [ Y, U, V ]= rgb2yuv( red,green,blue); % Converting R,G,B into Y,U,V
    coeff1=fractal_enc(rangesize,Y);
    coeff2=fractal_enc(rangesize,U);
    coeff3=fractal_enc(rangesize,V);

    % Compressing the color image using Y,I and Q components
else
    [ Y, I, Q ]= rgb2yiq( red,green,blue); % Converting R,G,B into Y,I,Q
    coeff1=fractal_enc(rangesize,Y);
    coeff2=fractal_enc(rangesize,I);
    coeff3=fractal_enc(rangesize,Q);
end
else
    disp('!!! Not a known image !!!')
end

% Clearing all unnecessary variables
clear readimage w h ranges gray red green blue Y U V I Q fractal_enc;

% Saving image name, image size, range size, choice and the coefficient matrix
uisave;

%%%%%%%%%%%%%% End of file %%%%%%%%%%%%%%%

```

B.2 main_dec.M

```
function main_dec
% Syntax:    <main_dec>
% Description: It asks the user for the number of iteration and then
%             it decodes any grayscale or color image that is
%             encoded using Fractal Image Compression.
% Inputs:    Nill
% Outputs:   Display the decompressed image
% Functions Used: fractal_dec, yuv2rgb, yiq2rgb
% Written by: Khalid Kamali
% Date:      9/7/2005
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
clc;

% Loading the decompressed file
uiload

% Reading the original image
[readimage, i_map]=imread(file_name);

% Getting the number of iteration
iter_mat=[1,2,4,6,8,10,12];
index=menu('Choose the number of iteration','1','2','4','6','8','10','12');;
num_iter=iter_mat(index);

% Calculating the domain size
domainsize=rangesize*2;

% Decompressing the grayscale image
```

```

if choice==0
    rangeimage=fractal_dec(num_iter,l,rangesize,coeff);

    % Decompressing the color image using red, green and blue components
elseif choice==1
    red=fractal_dec(num_iter,l,rangesize,coeff1);
    green=fractal_dec(num_iter,l,rangesize,coeff2);
    blue=fractal_dec(num_iter,l,rangesize,coeff3);
    rangeimage(:,1)=red;
    rangeimage(:,2)=green;
    rangeimage(:,3)=blue;

    % Decompressing the color image using Y, U and V components
elseif choice==2
    Y=fractal_dec(num_iter,l,rangesize,coeff1);
    U=fractal_dec(num_iter,l,rangesize,coeff2);
    V=fractal_dec(num_iter,l,rangesize,coeff3);

    % Converting Y,U,V into R,G,B
    [ red, green, blue ]= yuv2rgb( Y, U, V );
    rangeimage(:,1)=red;
    rangeimage(:,2)=green;
    rangeimage(:,3)=blue;

    % Decompressing the color image using Y, I and Q components
else
    Y=fractal_dec(num_iter,l,rangesize,coeff1);
    I=fractal_dec(num_iter,l,rangesize,coeff2);
    Q=fractal_dec(num_iter,l,rangesize,coeff3);

    % Converting Y,I,Q into R,G,B

```

```

[ red, green, blue ]= yiq2rgb( Y, I, Q );
rangeimage(:,:,1)=red;
rangeimage(:,:,2)=green;
rangeimage(:,:,3)=blue;
end
if choice==0
    error_image=double(readimage)-double(rangeimage);
else
    error_image(:,:,1)=double(readimage(:,:,1))-double(rangeimage(:,:,1));
    error_image(:,:,2)=double(readimage(:,:,2))-double(rangeimage(:,:,2));
    error_image(:,:,3)=double(readimage(:,:,3))-double(rangeimage(:,:,3));
end
subplot(1,3,1); imagesc(uint8(readimage)); colormap(i_map);
title('The original image');
subplot(1,3,2); imagesc(uint8(rangeimage));
title('The Decompressed image');
subplot(1,3,3); imagesc(uint8(error_image));
title('The Error image');
set(gcf,'Position',[200,200,600,300]);
choice=0;
while(choice~=5)
    choice = menu('Choose image','Show Original image','Show Compressed image','Show
Error image','Show all','Exit');hold on;
    switch choice
    case 1
        clf;
        imagesc(uint8(readimage));
        title('The original image');
        set(gcf,'Position',[200,150,400,400]);
    case 2
        clf;

```

```

    imagesc(uint8(rangeimage));
    title('The Decompressed image');
    set(gcf,'Position',[200,150,400,400]);
case 3
    clf;
    imagesc(uint8(error_image));
    title('The Error image');
    set(gcf,'Position',[200,150,400,400]);
case 4
    clf
    subplot(1,3,1); imagesc(uint8(readimage));
    title('The original image');
    subplot(1,3,2); imagesc(uint8(rangeimage));
    title('The Decompressed image');
    subplot(1,3,3); imagesc(uint8(error_image));
    title('The Error image');
    set(gcf,'Position',[200,200,600,300]);
case 5
    close all
    break;
end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% End of file %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

B.3 fractal_enc.M

```
function [coeff] =frac_enc(r,plate)
% Syntax:    <[rangeimage]=fractal_dec(num_iter,l,r,c)>
% Description: It encodes any grayscale or color image that is
%              encoded using Fractal Image Compression.
% Inputs:    r - The range size
%            plate - Image matrix
% Outputs:   coeff - The coefficient matrix of the compressed
%            Fractal-based image.
% Written by: Khalid Kamali
% Date:      23/4/2005
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[l w h]=size(plate);
plate=double(plate);
rangesize=r;

% Calculating the domain size
domainsize=rangesize*2;

i1=1:rangesize:l;
i2=rangesize:rangesize:l;
j1=1:rangesize:l;
j2=rangesize:rangesize:l;

a1=1:domainsize:l;
a2=domainsize:domainsize:l;
b1=1:domainsize:l;
b2=domainsize:domainsize:l;
coeff=[];
```

```

for i=1:l/rangesize
  for j=1:l/rangesize
    range=plate(i1(i):i2(i),j1(j):j2(j));

    % o is the offset, the average of each range
    o=round(mean(mean(range)));
    range=range - o;

    % Initialize minerror1
    minerror1=100;

    % Partitioning the domain blocks(non-overlapping).
    for a=1:l/domainsize
      for b=1:l/domainsize
        D_unscaled=plate(a1(a):a2(a),b1(b):b2(b));

        % Now scaling the domain
        D_scale=D_unscaled(1:2:domainsize,1:2:domainsize);

        % Average of each domain block
        ave=mean(mean(D_scale));
        Domain=D_scale - ave;

        % Scaled domain with a 0 deg rotation
        DT_MAT(1:rangesize,1:rangesize,1)=Domain;
        % Scaled domain with a 90 deg rotaion
        DT_MAT(1:rangesize,1:rangesize,2)=rot90(Domain);
        % Scaled domain with a 180 deg rotaion
        DT_MAT(1:rangesize,1:rangesize,3)=rot90(rot90(Domain));
        % Scaled domain with a 270 deg rotaion
        DT_MAT(1:rangesize,1:rangesize,4)=rot90(rot90(rot90(Domain)));

```

```

% Scaled domain with a Horizontal flip
DT_MAT(1:rangesize,1:rangesize,5)=flipud(Domain);
% Scaled domain with a Vertical flip
DT_MAT(1:rangesize,1:rangesize,6)=fliplr(Domain);
% Scaled domain with a flip about forward diagonal
DT_MAT(1:rangesize,1:rangesize,7)=transpose(Domain);
% Scaled domain with a flip about reverse diagonal
DT_MAT(1:rangesize,1:rangesize,8)=rot90(rot90(transpose(Domain)));

% Now comparing each range with domain blocks, with each
% possible transformation of the domain
for Loop=1:8
    Domain=DT_MAT(1:rangesize,1:rangesize,Loop);

    % Calculating the contrast
    ss = sum(sum(range.*Domain))/sum(sum(Domain.^2));

    % Checking if contrast value is greater or equal to zero
    % and less than one.
    if ss >= 0 && ss < 1
        minerror=mae(ss*Domain - range);
        if minerror < minerror1
            minerror1 = minerror;
            e=a1(a);
            f=b1(b);
            M=Loop;
            s=round(ss*10); % Save s as integer
        end
    end
end
end
end
end

```

```
end
coeff= [coeff; e f M o s]; % Saving the coefficients
end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% End of file %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

B.4 fractal_dec.M

```
function [rangeimage]=fractal_dec(num_iter,l,r,c)
% Syntax:      <[rangeimage]=fractal_dec(num_iter,l,r,c)>
% Description: It decodes any grayscale or color image that is
%              encoded using Fractal Image Compression.
% Inputs:      num_iter - The number of iteration.
%              l - The size of the image
%              r - The range size
%              c - The coefficient matrix of the compressed
%              Fractal-based image.
% Outputs:     rangeimage - The decompressed matrix after applying
%              IFS.
% Written by:  Khalid Kamali
% Date:        23/4/2005
%%%%%%%%%%%%%%

rangesize=r;
coeff=c;
domainsize=rangesize*2; % Calculating the domain size

i1=1:rangesize:l;
i2=rangesize:rangesize:l;
j1=1:rangesize:l;
j2=rangesize:rangesize:l;

a1=1:domainsize:l;
a2=domainsize:domainsize:l;
b1=1:domainsize:l;
b2=domainsize:domainsize:l;
```

```

% Creating memory buffers for the domain and range screens
domainimage=zeros(1);
rangeimage=zeros(1);

% Implementing IFS on the compressed image
for iteration=1:num_iter
    for i=1:l/rangesize
        for j=1:l/rangesize

            % Reading data
            row=(i-1)*(l/rangesize)+j;

            % Location of domain
            e=coeff(row,1);
            % Location of domain
            f=coeff(row,2);
            % Affine transformation
            M=coeff(row,3);
            % Offset
            o=coeff(row,4);
            % Scaling factor(contrast)
            s=coeff(row,5);

            % Rescaling the domain blocks
            Domain=domainimage(e:e+domainsize-1,f:f+domainsize-1);
            Domain=Domain-mean(mean(Domain));

            % Transforming the domain
            switch M
                case 1

```

```

        % If there is 0 deg rotation
        Domain=Domain;
    case 2
        % If there is 90 deg rotaion
        Domain=rot90(Domain);
    case 3
        % If there is 180 deg rotaion
        Domain=rot90(rot90(Domain));
    case 4
        % If there is 270 deg rotaion
        Domain=rot90(rot90(rot90(Domain)));
    case 5
        % If there is a Horizontal flip
        Domain=flipud(Domain);
    case 6
        % If there is a Vertical flip
        Domain=fliplr(Domain);
    case 7
        % If There is a flip about forward diagonal
        Domain=transpose(Domain);
    case 8
        % If there is a flip about reverse diagonal
        Domain=rot90(rot90(transpose(Domain)));
end

% Rescale domain, convert each domainsize x domainsize to
% rangesize x rangesize
D_scale=Domain(1:2:domainsize,1:2:domainsize);
rangeimage(i1(i):i2(i),j1(j):j2(j))=((s/10) * D_scale + o);
end
end

```

```
domainimage=rangeimage;  
end  
%%%%%%%%%%%%%% End of file %%%%%%%%%%%%%%%
```

B.5 rgb2yuv.M

```
function [ Y, U, V ]= rgb2yuv( R,G,B)
% Syntax:      <[ Y, U, V ]= rgb2yuv( R,G,B)>
% Description: It converts red, green and blue components of the color image
%              into Y,U and V components.
% Inputs:     R - Red components of the color image.
%              G - Green components of the color image.
%              B - Blue components of the color image.
% Outputs:    Y - Luminance components of the color image.
%              U - Hue components of the color image.
%              V - Saturation components of the color image.
% Written by: Khalid Kamali
% Date:       11/8/2005
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[m ,n]=size(R);
Y=zeros(m,n);
U=zeros(m,n);
V=zeros(m,n);
C=[ 0.299 0.587 0.114 ; -0.147 -0.289 0.436 ; 0.615 -0.515 -0.100];
for i=1:m
    for j=1:n
        a=C*[R(i,j) ; G(i,j); B(i,j)];
        Y(i,j)=a(1,1);
        U(i,j)=a(2,1);
        V(i,j)=a(3,1);
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% End of file %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

B.6 yuv2rgb.M

```
function [ R, G, B ]= yuv2rgb( Y, U, V )
% Syntax:      <[ R, G, B ]= yuv2rgb( Y, U, V )>
% Description: It converts Y,U and V components of the color image
%              into red, green and blue components.
% Inputs:     Y - Luminance components of the color image.
%              U - Hue components of the color image.
%              V - Saturation components of the color image.
% Outputs:    R - Red components of the color image.
%              G - Green components of the color image.
%              B - Blue components of the color image.
% Written by: Khalid Kamali
% Date:       11/8/2005
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[m ,n]=size(Y);
R=zeros(m,n);
G=zeros(m,n);
B=zeros(m,n);
C=[ 0.299 0.587 0.114 ; -0.147 -0.289 0.436 ; 0.615 -0.515 -0.1 ];
for i=1:m
    for j=1:n
        a=inv(C)*[Y(i,j) ; U(i,j); V(i,j)];
        R(i,j)=a(1,1);
        G(i,j)=a(2,1);
        B(i,j)=a(3,1);
    end
end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% End of file %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```


B.7 rgb2yiq.M

```
function [ Y, I, Q ]= rgb2yiq( R,G,B)
% Syntax:      <[ Y, I, Q ]= rgb2yiq( R,G,B)>
% Description: It converts red, green and blue components of the color image
%              into Y,I and Q components.
% Inputs:      R - Red components of the color image.
%              G - Green components of the color image.
%              B - Blue components of the color image.
% Outputs:     Y - Luminance components of the color image.
%              I - Hue components of the color image.
%              Q - Saturation components of the color image.
% Written by:  Khalid Kamali
% Date:        11/8/2005
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[m ,n]=size(R);
Y=zeros(m,n);
I=zeros(m,n);
Q=zeros(m,n);
C=[ 0.299 0.587 0.114 ; 0.596 -0.275 -0.321 ; 0.212 -0.528 0.311];
for i=1:m
    for j=1:n
        a=C*[R(i,j) ; G(i,j); B(i,j)];
        Y(i,j)=a(1,1);
        I(i,j)=a(2,1);
        Q(i,j)=a(3,1);
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
End of file %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

B.8 yiq2rgb.M

```
function [ R, G, B ]= yiq2rgb( Y, I, Q )
% Syntax:      <[ R, G, B ]= yiq2rgb( Y, I, Q )>
% Description: It converts Y,I and Q components of the color image
%              into red, green and blue components.
% Inputs:     Y - Luminance components of the color image.
%              I - Hue components of the color image.
%              Q - Saturation components of the color image.
% Outputs:    R - Red components of the color image.
%              G - Green components of the color image.
%              B - Blue components of the color image.
% Written by: Khalid Kamali
% Date:       11/8/2005
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[m ,n]=size(Y);
R=zeros(m,n);
G=zeros(m,n);
B=zeros(m,n);
C=[ 0.299 0.587 0.114 ; 0.596 -0.275 -0.321 ; 0.212 -0.528 0.311 ];
for i=1:m
    for j=1:n
        a=inv(C)*[Y(i,j) ; I(i,j); Q(i,j)];
        R(i,j)=a(1,1);
        G(i,j)=a(2,1);
        B(i,j)=a(3,1);
    end
end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% End of file %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```